Latent Semantic Indexing

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Introduction

- Vector model => use of theory of linear algebra
- Look at matrix formulation:

$$\begin{pmatrix} w_{11} & \dots & w_{t1} \\ & & \\ w_{1N} & \dots & w_{tN} \end{pmatrix} \bullet \begin{pmatrix} w_{1q} \\ \\ w_{tq} \end{pmatrix} = \begin{pmatrix} s_{1q} \\ \\ s_{Nq} \end{pmatrix}$$

document vector   query vector   scores

$$s_{xq} = \Sigma^{t}_{i=1}(w_{ix} * w_{iq})$$

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Goals

- # terms t large - large dimension
  $\Rightarrow$reduce dimension
- find some semantic relationship
  - correlate terms to find structure
    - synonomy
    - polysomy

  "people choose same main terms <20% time"

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Summary: Singular Value Decomposition (SVD)
Review set-up

M - number of terms N - number of documents

C the $M \times N$ (term×doc.) matrix of weights *(*our old w_{ij}) ≥ 0

- of rank r ($r \leq \min(M,N)$)
- documents are *columns* of C
- compare query to all documents: C^T**q**
  [N×M][M×1]

CC^T and C^TC:

- symmetric,
- share the same eigenvalues $\lambda_1, \lambda_2, \ldots$
  - $\lambda_1, \lambda_2, \ldots$ are indexed in decreasing order
- $CC^T(i,j)$ measures strength co-occurrence terms i and j
- $C^TC(i,j)$ measures similarity documents i and j

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Summary: Singular Value Decomposition (SVD) **Theorem**

matrix C has a *singular value decomposition*

$$C = U\Sigma V^T$$

Where:

U M×M matrix
  with columns = orthogonal eigenvectors of CC^T

V N×N matrix
  with columns = orthogonal eigenvectors of C^TC

Σ M×N diagonal matrix:
  $\Sigma(i,i) = \sqrt{\lambda_i}$ for $1 \le i \le r$
  $\Sigma(i,j) = 0$ otherwise

$\sqrt{\lambda_i}$ *called singular values*

$$\begin{array}{ccccccc}\sqrt{\lambda_1} & & & & & & \\ & \sqrt{\lambda_2} & & & & & \\ & & \cdots & & & & \\ & & & \sqrt{\lambda_r} & & & \\ & & & & 0 & & \\ & & & & & \cdots & \\ & & & & & & 0\end{array}$$

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Summary: **Reduced Rank Approximation of C**

- Reduce rank of Σ from r to **k**
  keep only k largest singular values

  Σ_K is M×N diagonal matrix: $\Sigma(i,i) = \sqrt{\lambda_i}$ for $1 \le i \le$ **k**
  $\Sigma(i,j) = 0$ otherwise

$$\begin{array}{cccccccc}\sqrt{\lambda_1} & & & & & & & \\ & \cdots & & & & & & \\ & & \sqrt{\lambda_k} & & & & & \\ & & & \sqrt{\lambda_{k+1}} & & & & \\ & & & & \cdots & & & \\ & & & & & \sqrt{\lambda_r} & & \\ & & & & & & 0 & \\ & & & & & & & \cdots \\ & & & & & & & 0\end{array} \rightarrow \begin{array}{cccccccc}\sqrt{\lambda_1} & & & & & & & \\ & \cdots & & & & & & \\ & & \sqrt{\lambda_k} & & & & & \\ & & & \mathbf{0} & & & & \\ & & & & \cdots & & & \\ & & & & & 0 & & \\ & & & & & & 0 & \\ & & & & & & & \cdots \\ & & & & & & & 0\end{array}$$

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Summary:
Reduced Rank Approximation of C

- Approximation:

$$C_k = U\Sigma_k V^T$$

- Theorem:

  C_k is the best rank-k approximation to C under the least square fit (Frobenius) norm

$$= \sqrt{\Sigma^M_{i=1} \Sigma^N_{j=1} (C(i,j) - C_k(i,j))^2}$$

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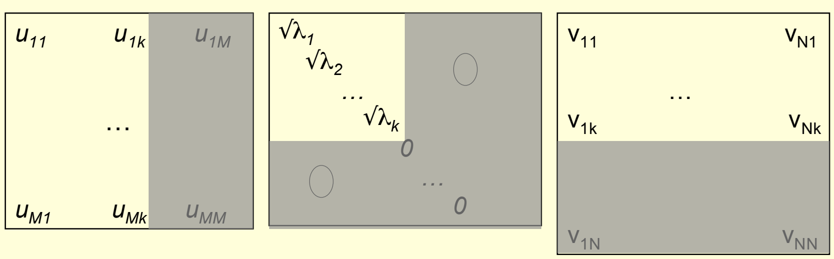
Query Approximation
using reduce dimension matrices



C_k =	U'_k	Σ'_k	V'^T_k
M×N	M×k	k×k	k×N

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Using the Approximation

- View V'^T_k as a representation of documents in a k-dimensional space
  - a "concept space"?
- Transform query vector **q** into that space:

$$C_k^T C_k = (U'_k \Sigma'_k V'^T_k)^T (U'_k \Sigma'_k V'^T_k) = (V'_k \Sigma'^T_k U'^T_k)(U'_k \Sigma'_k V'^T_k)$$

$$= V'_k (\Sigma'_k)^2 (V'_k)^T \qquad \text{compares documents}$$

$$\Rightarrow \quad C_k^T \boldsymbol{q} = V'_k (\Sigma'_k)^2 \boldsymbol{q}_k \qquad \text{compares doc. to query}$$

$$\Rightarrow \quad \boldsymbol{q}_k = (\Sigma'^{-1}_k)^2 V'^T_k C_k^T \boldsymbol{q} = (\Sigma'^{-1}_k)^2 V'^T_k V'_k \Sigma'^T_k U'^T_k \mathbf{q}$$

$$= (\Sigma'_k)^{-1} (U'_k)^T \boldsymbol{q}$$

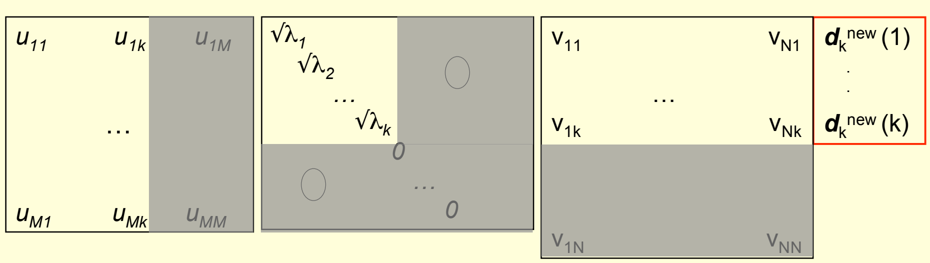
recalling $(V'^T_k)(V'_k) = (U'^T_k)(U'_k) = I$

Adding a new document

add new document $\boldsymbol{d}^{new}$ to C_k => add column $\boldsymbol{d}_k^{new}$ to V'^T_k

Transform $\mathbf{d}^{new}$ into the k-dimensional space version $\boldsymbol{d}_k^{new}$

$$V'^T_k = (\Sigma'_k)^{-1} (U'_k)^T C_k \qquad \Rightarrow \qquad (\Sigma'_k)^{-1} (U'_k)^T \boldsymbol{d}^{new} = \boldsymbol{d}_k^{new}$$



$C_k =$ M×(N+1) $\quad U'_k$ M×k $\quad \Sigma'_k$ k×k $\quad V'^T_k$ k×(N+1)

Original LSI paper:

Deerwester, Dumais, et. al.
Indexing by Latent Semantic Analysis
Journal of the Society for Information Science, 41(6), 1990, 391-407.

Example from that paper follows

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Deerwester, Dumais et. al. Table:

Terms	**Documents**								
	c1	c2	c3	c4	c5	m1	m2	m3	m4
	—	—	—	—	—	—	—	—	—
human	1	0	0	1	0	0	0	0	0
interface	1	0	1	0	0	0	0	0	0
computer	1	1	0	0	0	0	0	0	0
user	0	1	1	0	1	0	0	0	0
system	0	1	1	2	0	0	0	0	0
response	0	1	0	0	1	0	0	0	0
time	0	1	0	0	1	0	0	0	0
EPS	0	0	1	1	0	0	0	0	0
survey	0	1	0	0	0	0	0	0	1
trees	0	0	0	0	0	1	1	1	0
graph	0	0	0	0	0	0	1	1	1
minors	0	0	0	0	0	0	0	1	1

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Deerwester, Dumais et. al. example, cont.:

Matrix V^T for k=2

0.20	0.61	0.46	0.54	0.28	0.00	0.02	0.02	0.08
-0.06	0.17	-0.13	-0.23	0.11	0.19	0.44	0.62	0.53

Deerwester, Dumais, et al Figure 1