COS 226	Algorithms and Data Structures	Fall 2008

Midterm Solutions

1. **8 sorting algorithms.**

   0 6 5 2 4 9 3 8 7 1

2. **Sorting equal keys.**

Insertion	A_0 A_1 A_2 A_3 A_4 A_5 A_6
Selection	A_0 A_1 A_2 A_3 A_4 A_5 A_6
Shellsort	A_0 A_1 A_2 A_3 A_4 A_5 A_6
Mergesort	A_0 A_1 A_2 A_3 A_4 A_5 A_6
Quicksort	A_4 A_3 A_5 A_6 A_0 A_2 A_1
Heapsort	A_1 A_2 A_3 A_4 A_5 A_6 A_0

3. **Analysis of algorithms.**

   (a) I and II only

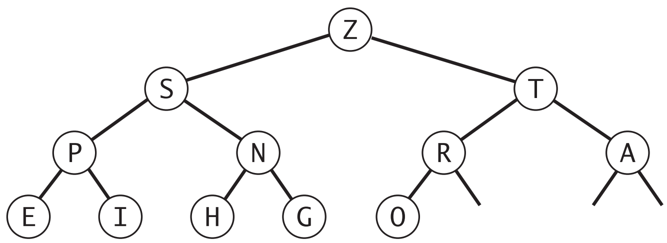
   Big-Oh notation and tilde notation both suppress lower order terms.

   (b) I only

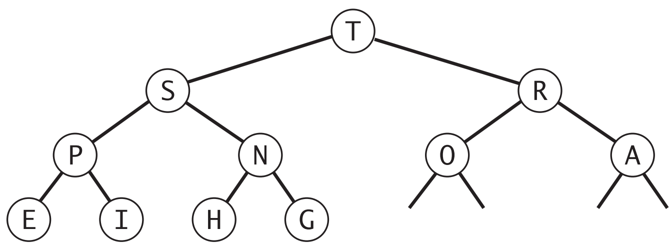
   Amortized analysis provides a worst-case guarantee on any sequence of operations starting from an empty data structure.

4. **Binary heaps.**

(a)



(b)

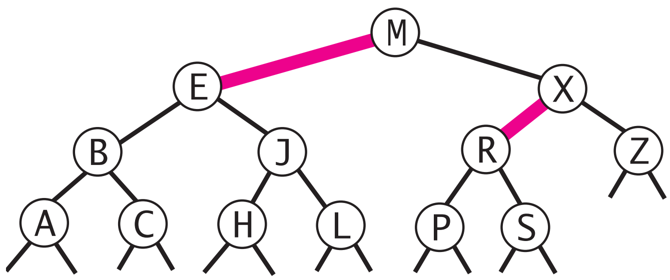


(c) True.

5. **Ordered-array implementation of a set.**

`add(key)`	*add the key to the set*	N
`contains(key)`	*is the key in the set?*	$\log N$
`ceiling(key)`	*smallest key in set* $\geq$ *given key*	$\log N$
`rank(key)`	*number of keys in set* $<$ *given key*	$\log N$
`select(i)`	*ith largest key in the set*	1
`min()`	*minimum key in the set*	1
`delete(key)`	*delete the given key from the set*	N
`iterator()`	*iterate over all* N *keys in the set in order*	N

6. **Red-black trees.**



7. **Line intersection.**

   (a) There are two cases:

   - If the two lines have the same slope ($a_0 = a_1$), then return no intersection.
   - Otherwise, the point (x, y) of intersection is given by:

$$x = -\frac{b_1 - b_0}{a_1 - a_0}, \quad y = a_0 x + b_0$$

   (b) To determine whether the ith line is involved in an intersection with 3 (or more) lines:

   - Create a symbol table with key = point, value = list (say, a queue) of lines.
   - For each line $j \neq i$ in order:
     - Compute the intersection point p between line i and line j.
     - If they don't intersect, continue.
     - If the key p is not already in the symbol table, add an entry to the symbol table with key = p and value = empty list.
     - Add line j to the end of the list associated with p.
   - For each key in the symbol table, if it's list contains 2 (or more) lines, they correspond to 3 (or more) lines intersecting at a single point (line i, plus the lines in the list).

   Implement the symbol table using a separate-chaining (or linear-probing) hash table so that each insert/search takes $O(1)$ time. Thus, the overall subroutine takes $O(N)$ time.

   To determine whether any 3 (or more lines) intersect at a point, run the previous subroutine N times, once for each line i. The total running time is $O(N^2)$.

   (c) Only print out a set of lines in the last step of the subroutine if the index of the first line in the list is greater than i. This guarantees we only find a set of lines once, when using the line with the smallest index as the base line.