Scan Conversion

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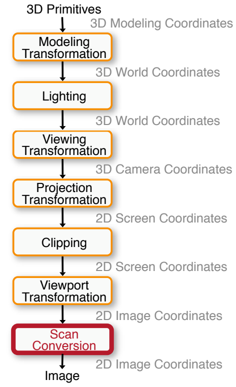
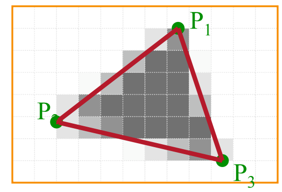
COS 426, Spring 2008

3D Rendering Pipeline (for direct illumination)





Scan Conversion & Shading

Overview

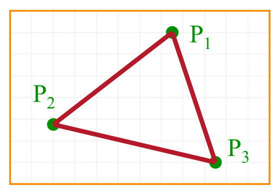
- Scan conversion
  - Figure out which pixels to fill
- Shading
  - Determine a color for every filled pixel
- Texture mapping
  - Describe shading variation within polygon interiors
- Visible surface determination
  - Figure out which surface is front-most at every pixel

Scan Conversion

- Render an image of a geometric primitive by setting pixel colors

```
void SetPixel(int x, int y, Color rgba)
```

- Example: Filling the inside of a triangle



Scan Conversion

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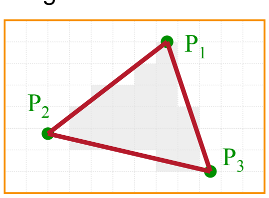
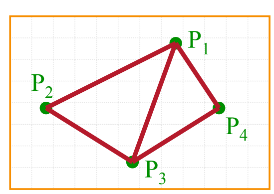
- Example: Filling the inside of a triangle



Triangle Scan Conversion

- Properties of a good algorithm
  - Symmetric
  - Straight edges
  - Antialiased edges
  - No cracks between adjacent primitives
  - MUST BE FAST!



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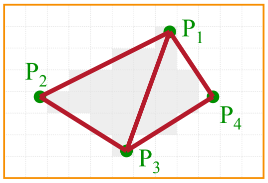
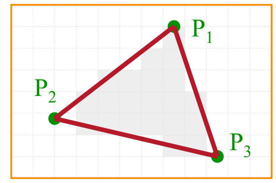


Simple Algorithm

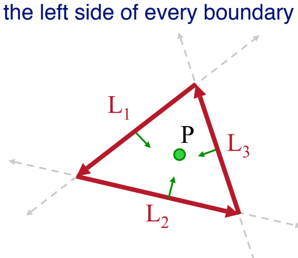
- Color all pixels inside triangle

```
void ScanTriangle(Triangle T, Color rgba){
   for each pixel P at (x,y){
      if (Inside(T, P))
         SetPixel(x, y, rgba);
   }
}
```



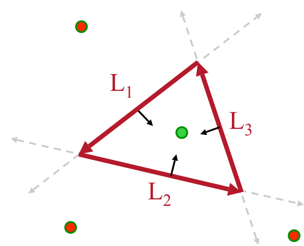
Inside Triangle Test

- A point is inside a triangle if it is in the positive halfspace of all three boundary lines
  - Triangle vertices are ordered counter-clockwise
  - Point must be on the left side of every boundary line



Inside Triangle Test

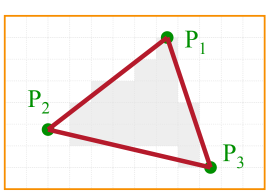
```
Boolean Inside(Triangle T, Point P)
{
   for each boundary line L of T {
      Scalar d = L.a*P.x + L.b*P.y + L.c;
      if (d < 0.0) return FALSE;
   }
   return TRUE;
}
```



Simple Algorithm

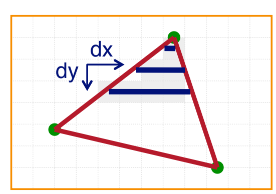
- What is bad about this algorithm?

```
void ScanTriangle(Triangle T, Color rgba){
   for each pixel P at (x,y){
      if (Inside(T, P))
         SetPixel(x, y, rgba);
   }
}
```



Triangle Sweep-Line Algorithm

- Take advantage of spatial coherence
  - Compute which pixels are inside using horizontal spans
  - Process horizontal spans in scan-line order
- Take advantage of edge linearity
  - Use edge slopes to update coordinates incrementally

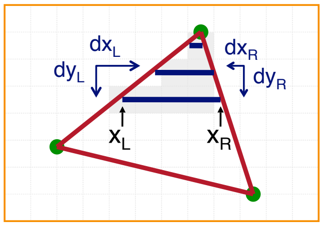


Triangle Sweep-Line Algorithm

```
void ScanTriangle(Triangle T, Color rgba){
  for each edge pair {
     initialize xL, xR;
     compute dxL/dyL and dxR/dyR;
     for each scanline at y
        for (int x = xL; x <= xR; x++)
           SetPixel(x, y, rgba);
        xL += dxL/dyL;
        xR += dxR/dyR;
  }
}
```

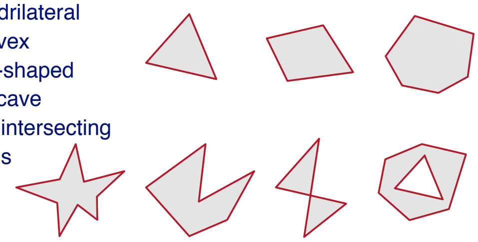
Historical note:

Bresenham's Algorithm integer-only version of line calculation (good for hardware)



Polygon Scan Conversion

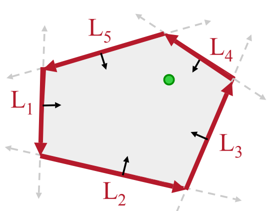
- Fill pixels inside a polygon
  - Triangle
  - Quadrilateral
  - Convex
  - Star-shaped
  - Concave
  - Self-intersecting
  - Holes



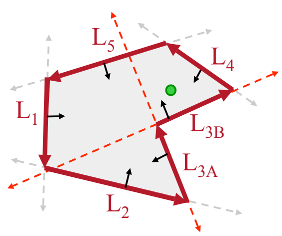
What problems do we encounter with arbitrary polygons?

Polygon Scan Conversion

- Need better test for points inside polygon
  - Triangle method works only for convex polygons



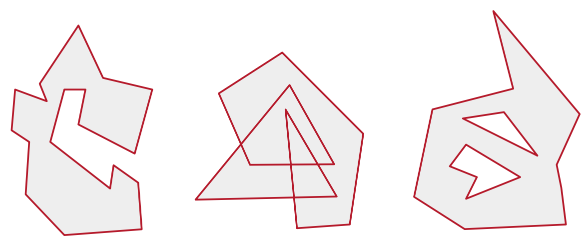
Convex Polygon



Concave Polygon

Inside Polygon Rule

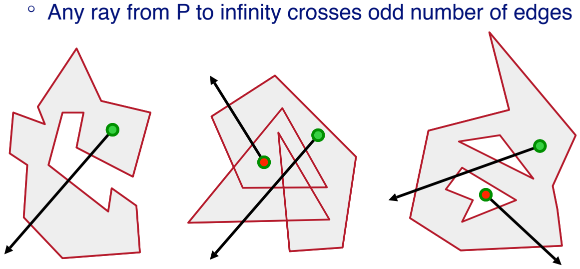
- What is a good rule for which pixels are inside?



Concave   Self-Intersecting   With Holes

Inside Polygon Rule

- Odd-parity rule
  - Any ray from P to infinity crosses odd number of edges

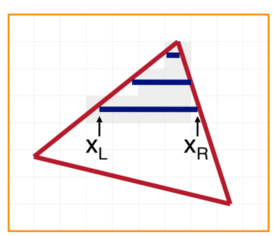


Concave   Self-Intersecting   With Holes

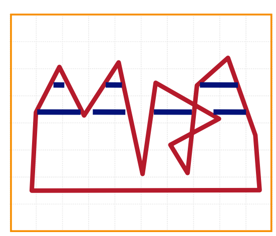
  - What if you hit a vertex?

Polygon Sweep-Line Algorithm

- Incremental algorithm to find spans, and determine insideness with odd parity rule
  - Takes advantage of scanline coherence



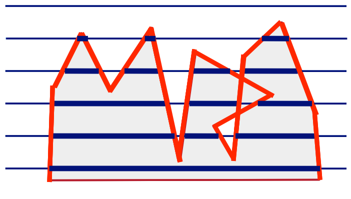
Triangle



Polygon

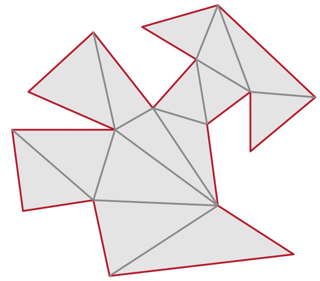
Polygon Sweep-Line Algorithm

```
void ScanPolygon(Triangle T, Color rgba){
  sort edges by maxy
  make empty "active edge list"
  for each scanline (top-to-bottom) {
     insert/remove edges from "active edge list"
     update x coordinate of every active edge
     sort active edges by x coordinate
     for each pair of active edges (left-to-right)
        SetPixels(xi, xi+1, y, rgba);
     }
}
```



Hardware Scan Conversion

- Convert everything into triangles
  - Scan convert the triangles



Overview

- Scan conversion
  - Figure out which pixels to fill
- Shading
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- Texture mapping
  - Describe shading variation within polygon interiors
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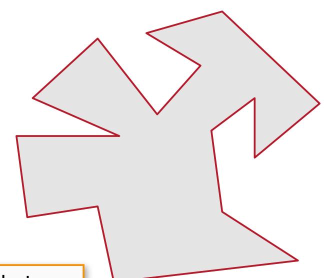
  - Figure out which surface is front-most at every pixel

Shading

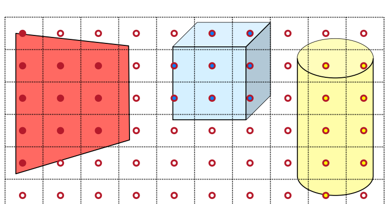
- How do we choose a color for each filled pixel?



Emphasis on methods that can be implemented in hardware

Ray Casting

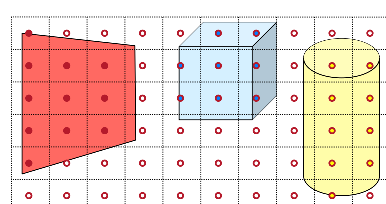
- Simplest shading approach is to perform independent lighting calculation for every pixel



$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

Polygon Shading

- Can take advantage of spatial coherence
  - Illumination calculations for pixels covered by same primitive are related to each other



$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

Polygon Shading Algorithms

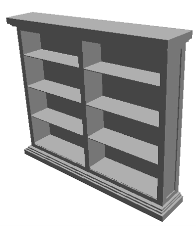
- Flat Shading
- Gouraud Shading
- Phong Shading

Polygon Shading Algorithms

- **Flat Shading**
- Gouraud Shading
- Phong Shading

Flat Shading

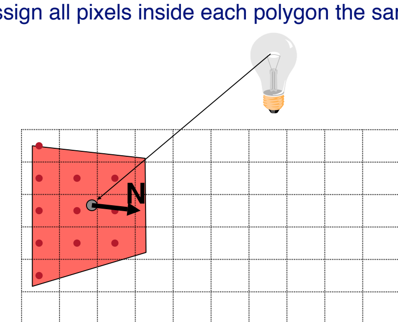
- What if a faceted object is illuminated only by directional light sources and is either diffuse or viewed from infinitely far away



$$I = I_E + K_A I_{AL} + \sum_i (K_D(N \bullet L_i)I_i + K_S(V \bullet R_i)^n I_i)$$

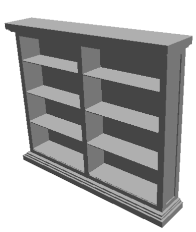
Flat Shading

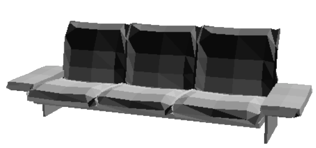
- One illumination calculation per polygon
  - Assign all pixels inside each polygon the same color



Flat Shading

- Objects look like they are composed of polygons
  - OK for polyhedral objects
  - Not so good for smooth surfaces



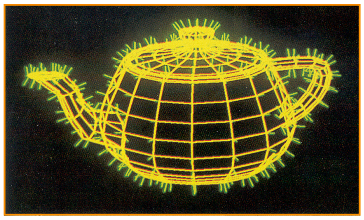


Polygon Shading Algorithms

- Flat Shading
- **Gouraud Shading**
- Phong Shading

Gouraud Shading

- What if smooth surface is represented by polygonal mesh with a normal at each vertex?

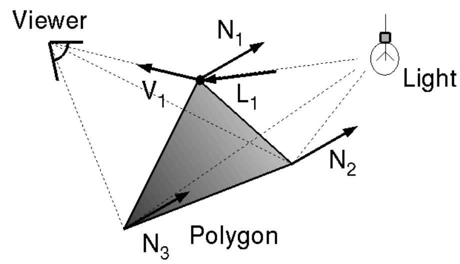


Watt Plate 7

$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

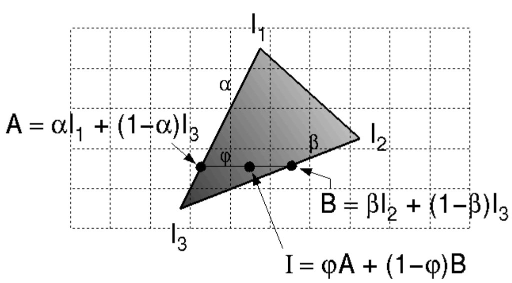
Gouraud Shading

- Method 1: One lighting calculation per vertex
  - Assign pixels inside polygon by interpolating colors computed at vertices



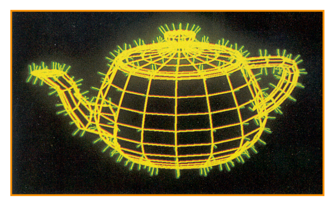
Gouraud Shading

- Bilinearly interpolate colors at vertices down and across scan lines



Gouraud Shading

- Smooth shading over adjacent polygons
  - Curved surfaces
  - Illumination highlights
  - Soft shadows

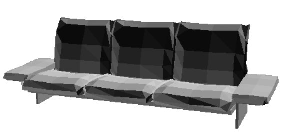


Mesh with shared normals at vertices

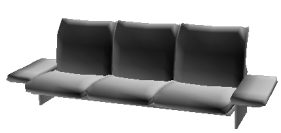
Watt Plate 7

Gouraud Shading

- Produces smoothly shaded polygonal mesh
  - Piecewise linear approximation
  - Need fine mesh to capture subtle lighting effects



Flat Shading



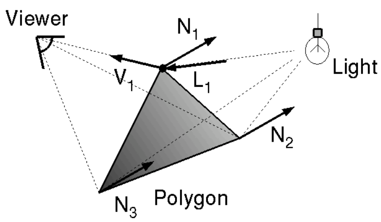
Gouraud Shading

Polygon Shading Algorithms

- Flat Shading
- Gouraud Shading
- **Phong Shading**

Phong Shading

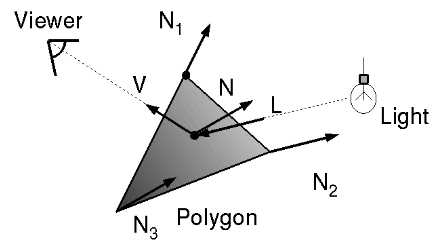
- What if polygonal mesh is too coarse to capture illumination effects in polygon interiors?



$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

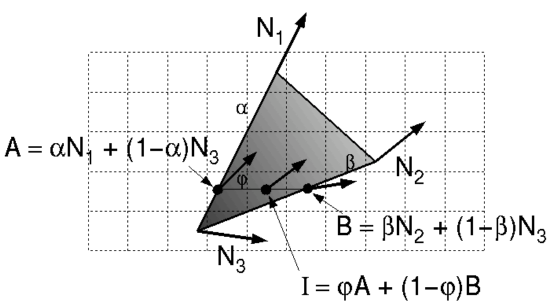
Phong Shading

- One lighting calculation per pixel
  - Approximate surface normals for points inside polygons by bilinear interpolation of normals from vertices

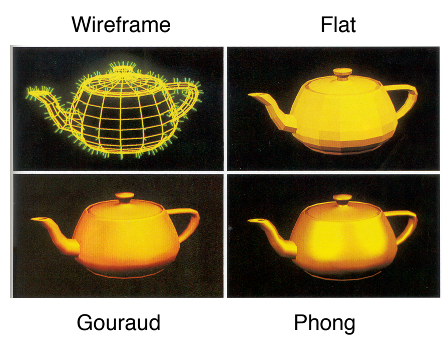


Phong Shading

- Bilinearly interpolate surface normals at vertices down and across scan lines



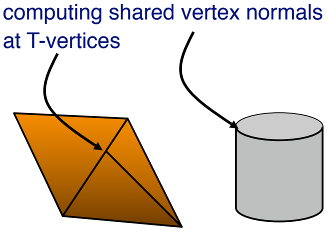
Polygon Shading Algorithms



Watt Plate 7

Shading Issues

- Problems with interpolated shading:
  - Polygonal silhouettes
  - Perspective distortion
  - Orientation dependence (due to bilinear interpolation)
  - Problems computing shared vertex normals
  - Problems at T-vertices



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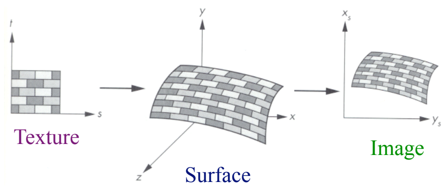
Textures

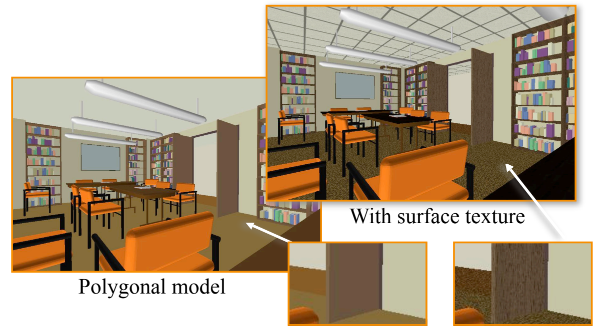
- Describe color variation in interior of 3D polygon
  - When scan converting a polygon, vary pixel colors according to values fetched from a texture



Angel Figure 9.3

Surface Textures

- Add visual detail to surfaces of 3D objects

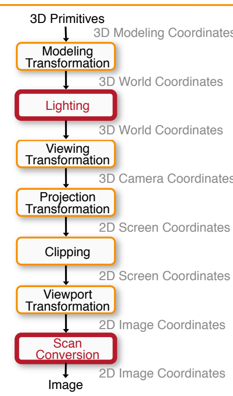


Surface Textures

- Add visual detail to surfaces of 3D objects

[Daren Horley]

3D Rendering Pipeline (for direct illumination)



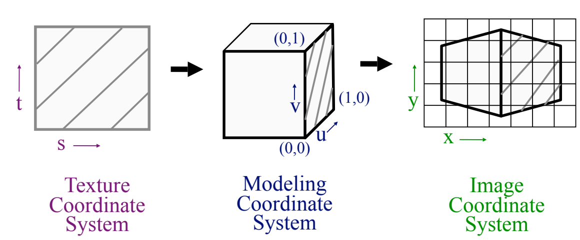
Texture mapping

Texture Mapping Overview

- Texture mapping methods
  - Mapping
  - Filtering
  - Parameterization
- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering
  - Non-photorealistic rendering

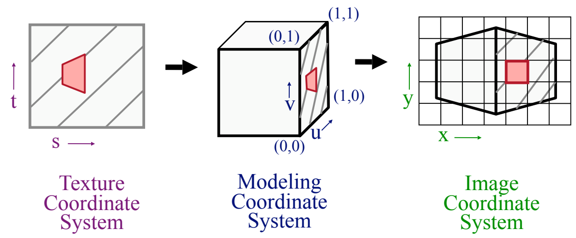
Texture Mapping

- Steps:
  - Define texture
  - Specify mapping from texture to surface
  - Lookup texture values during scan conversion



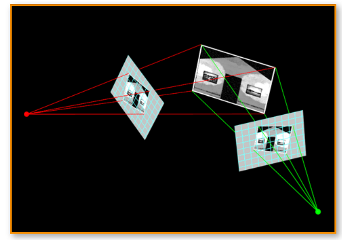
Texture Mapping

- When scan convert, map from …
  - image coordinate system (x,y) to
  - modeling coordinate system (u,v) to
  - texture image (t,s)



Texture Mapping

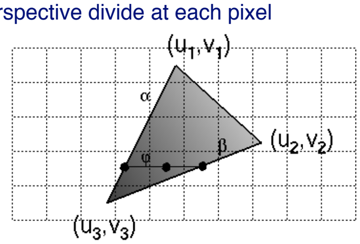
- Texture mapping is a 2D projective transformation
  - texture coordinate system: (t,s) to
  - image coordinate system (x,y)



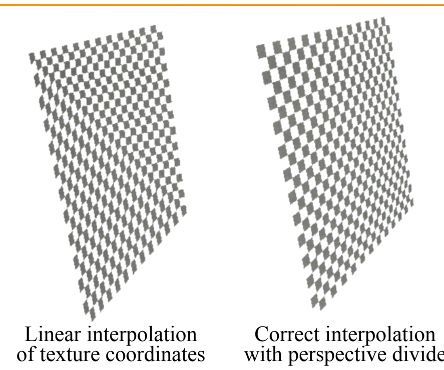
[Allison Klein]

Texture Mapping

- Scan conversion
  - Interpolate texture coordinates down/across scan lines
  - Distortion due to bilinear interpolation approximation
    - » Cut polygons into smaller ones, or
    - » Perspective divide at each pixel



Texture Mapping



Hill Figure 8.42

Texture Mapping Overview

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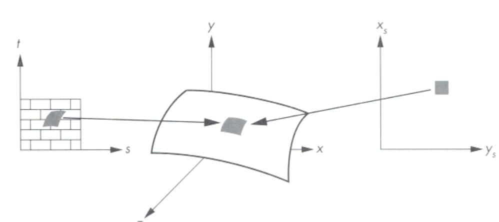
Texture Filtering

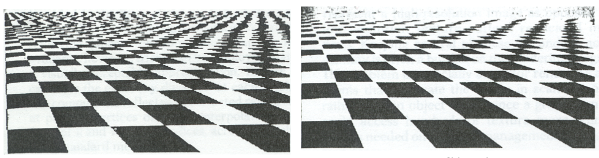
- Must sample texture to determine color at each pixel in image



Angel Figure 9.4

Texture Filtering

- Aliasing is a problem

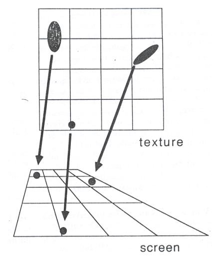


Point sampling　　Area filtering

Angel Figure 9.5

Texture Filtering

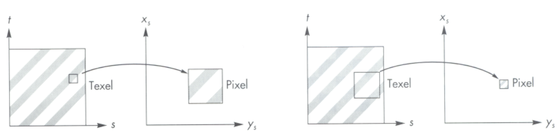
- Ideally, use elliptically shaped convolution filters



In practice, use rectangles

Texture Filtering

- Size of filter depends on projective warp
  - Can prefiltering images
    - » Mip maps
    - » Summed area tables

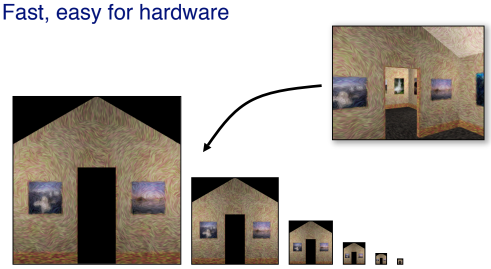


Magnification　　Minification

Angel Figure 9.14

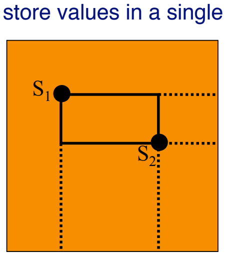
Mip Maps

- Keep textures prefiltered at multiple resolutions
  - For each pixel, linearly interpolate between two closest levels (e.g., trilinear filtering)
  - Fast, easy for hardware



Summed-area tables

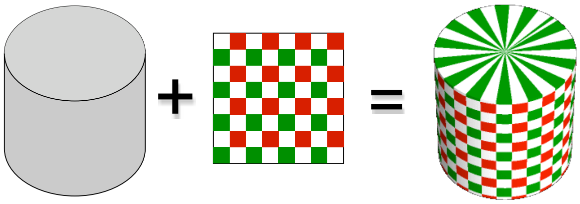
- At each texel keep sum of all values down & right
  - To compute sum of all values within a rectangle, simply subtract two entries
  - Better ability to capture very oblique projections
  - But, cannot store values in a single byte



Texture Mapping Overview

- Texture mapping methods
  - Mapping
  - Filtering
  - Parameterization
- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering
  - Non-photorealistic rendering

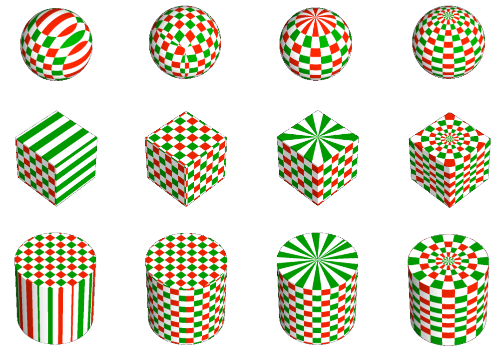
Parameterization



geometry image texture map

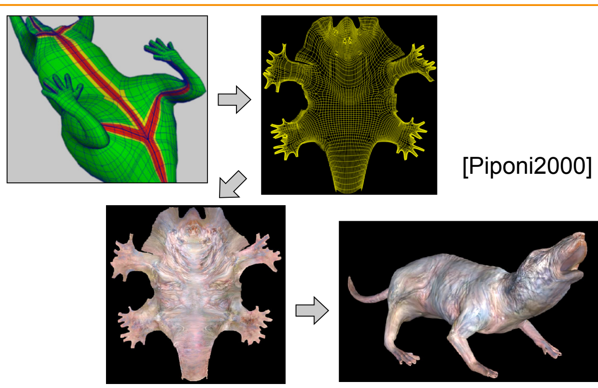
- Q: How do we decide *where* on the geometry each color from the image should go?

Option: Varieties of projections



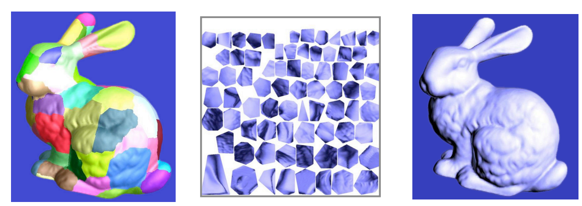
[Paul Bourke]

Option: unfold the surface



[Piponi2000]

Option: make an atlas



charts atlas surface

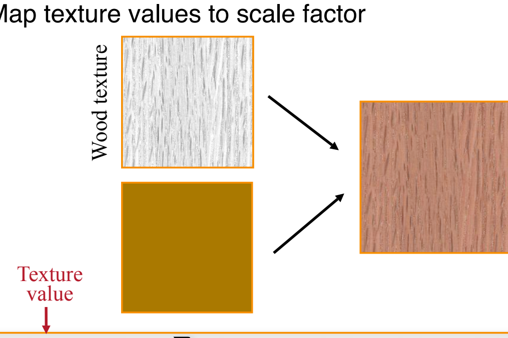
[Sander2001]

Overview

- Texture mapping methods
  - Mapping
  - Filtering
  - Parameterization
- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering

Modulation textures

Map texture values to scale factor



$$\boxed{I = T(s,t)(I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_T I_T + K_S I_S)}$$

Illumination Mapping

Map texture values to surface material parameter

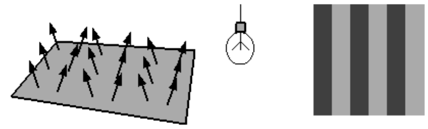
- K_A
- K_D
- K_S
- K_T
- n

$K_T = T(s,t)$

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_T I_T + K_S I_S$$

Bump Mapping

Texture values perturb surface normals

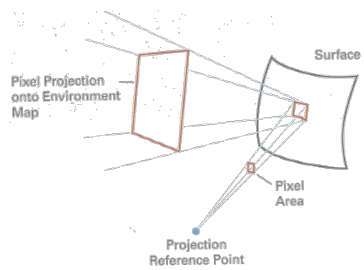


Bump Mapping

H&B Figure 14.100

Environment Mapping

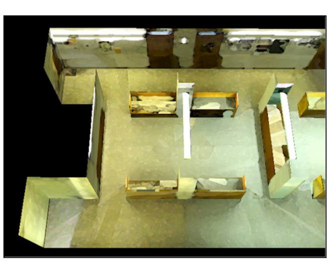
Texture values are reflected off surface patch



H&B Figure 14.93

Image-Based Rendering

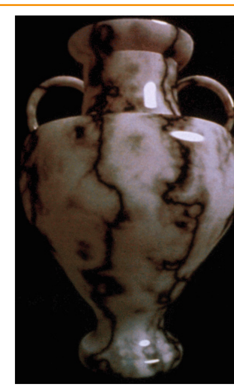
Map photographic textures to provide details for coarsely detailed polygonal model



Solid textures

Texture values indexed by 3D location (x,y,z)

- Expensive storage, or
- Compute on the fly, e.g. Perlin noise →



Texture Mapping Summary

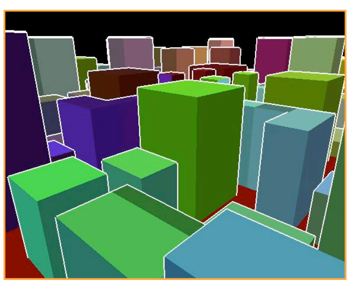
- Texture mapping methods
  - Parameterization
  - Mapping
  - Filtering
- Texture mapping applications
  - Modulation textures
  - Illumination mapping
  - Bump mapping
  - Environment mapping
  - Image-based rendering
  - Volume textures

Overview

- Scan conversion
  - Figure out which pixels to fill
- Shading
  - Determine a color for each filled pixel
- Texture mapping
  - Describe shading variation within polygon interiors
- Visible surface determination
  - Figure out which surface is front-most at every pixel

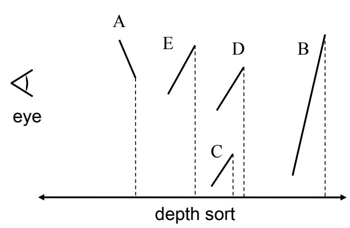
Visible Surface Determination

- Make sure only front-most contributes to color at every pixel

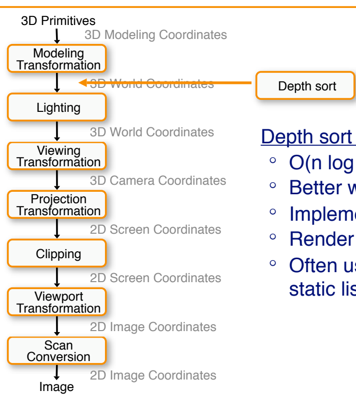


Depth sort

- "Painter's algorithm"
  - Sort surfaces in order of decreasing maximum depth
  - Scan convert surfaces in back-to-front order, overwriting pixels



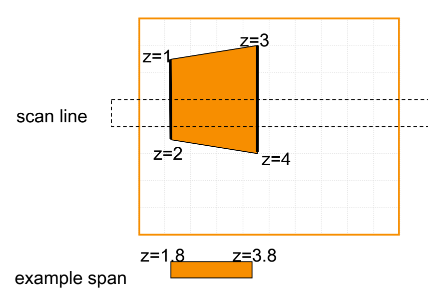
3D Rendering Pipeline



Depth sort comments
- O(n log n)
- Better with frame coherence?
- Implemented in software
- Render every polygon
- Often use BSP-tree or static list ordering

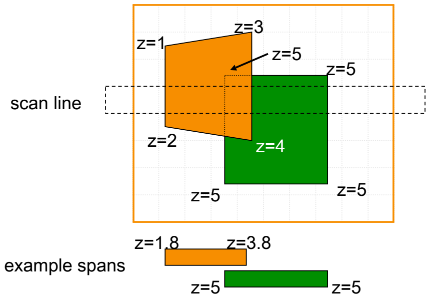
Scan-Line Algorithm

- For each scan line, construct and sort spans

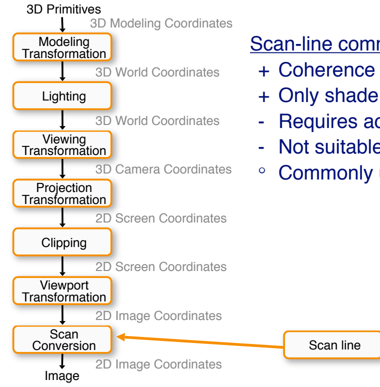


Scan-Line Algorithm

- For each scan line, construct and sort spans
  - Sort by depths within each scan line



Scan-Line Algorithm

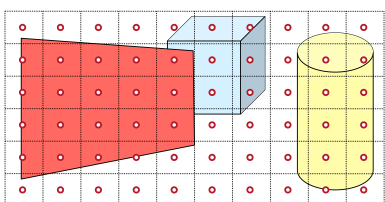


Scan-line comments

\+ Coherence among along scan lines
\+ Only shade each pixel once
\- Requires access to all polygons
\- Not suitable for hardware pipeline
∘ Commonly used in software

Z-Buffer

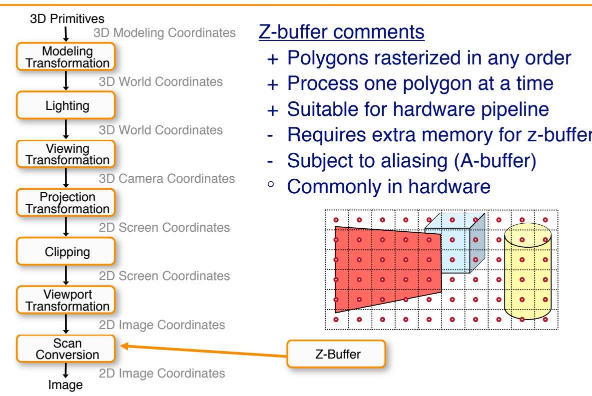
- Color & depth of closest object for every pixel
  - Update only pixels whose depth is closer than in buffer
  - Depths are interpolated from vertices, just like colors



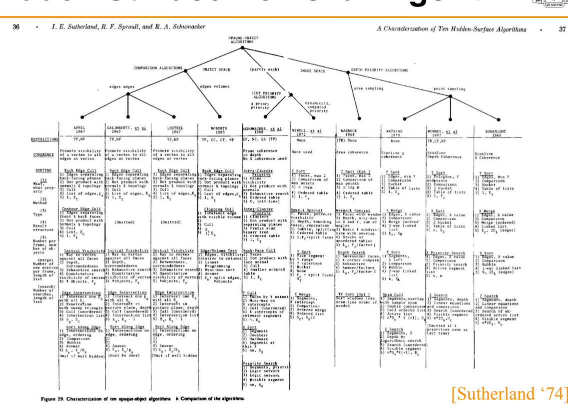
Z-Buffer

Z-buffer comments

\+ Polygons rasterized in any order
\+ Process one polygon at a time
\+ Suitable for hardware pipeline
\- Requires extra memory for z-buffer
\- Subject to aliasing (A-buffer)
∘ Commonly in hardware



Hidden Surface Removal Algorithms



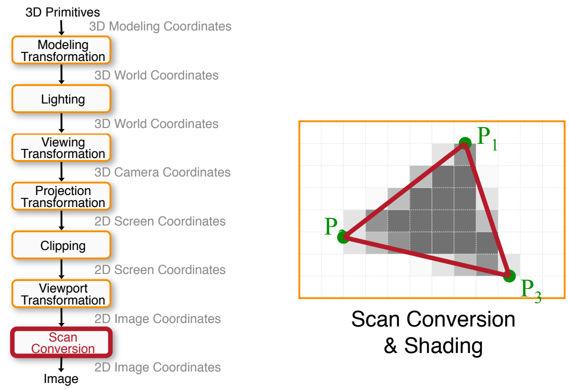
[Sutherland '74]

Summary

- Scan conversion
  - Sweep-line algorithm
- Shading algorithms
  - Flat, Gouraud
- Texture mapping
  - Mipmaps
- Visibility determination
  - Z-buffer

This is all in hardware

Summary



Scan Conversion
& Shading

PS: Programmable GPUs