Mesh Representations & Subdivision Surfaces

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COS 426, Spring 2008

Selected material courtesy of Scott Schaefer

3D Object Representations

- Raw data
  - Voxels
  - Point cloud
  - Range image
  - Polygons
- Solids
  - Octree
  - BSP tree
  - CSG
  - Sweep
- Surfaces
  - Mesh
  - Subdivision
  - Parametric
  - Implicit
- High-level structures
  - Scene graph
  - Application specific

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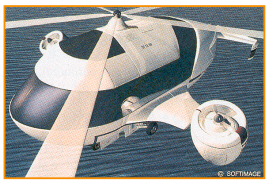
Surfaces

- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



H&B Figure 10.46

Surfaces

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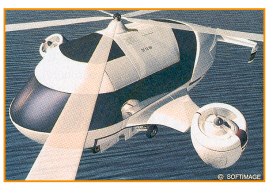

H&B Figure 10.46

Subdivision

- How do you make a smooth curve?

- More on smooth curves on Thursday…

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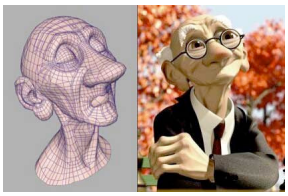
Subdivision Surfaces

- Used in movie and game industries
- Supported by most 3D modeling software

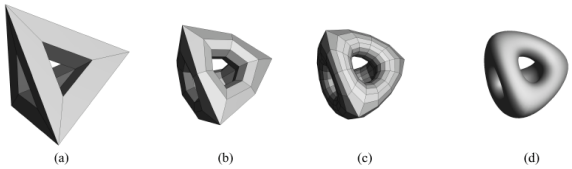
Toy Story © Disney / Pixar



Geri's Game © Pixar Animation Studios

Subdivision Surfaces

- Set of rules, recursively applied to a *control mesh*
- Smooth surface defined as limit of this process
- Works on arbitrary surface topology



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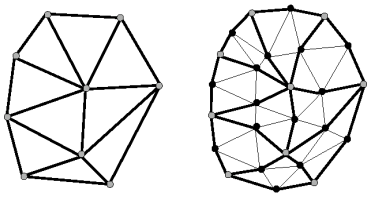
Subdivision Rules

- Repeated application of
  - Mesh refinement
  - Weighted averaging of vertex positions
- Special treatment of surface boundaries



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Subdivision Surfaces – Examples

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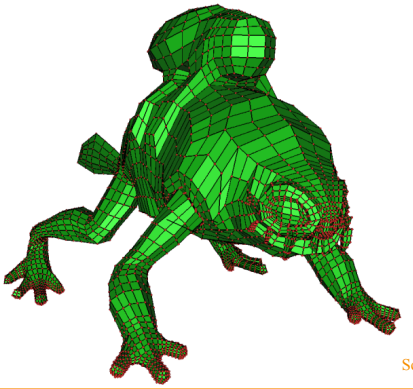
Subdivision Surfaces – Examples



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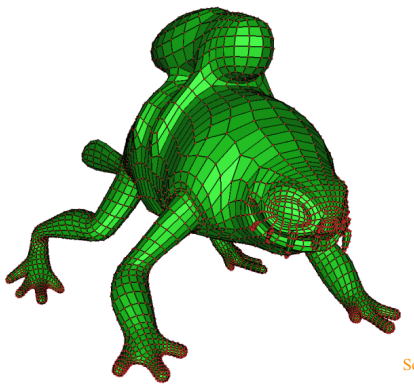
Subdivision Surfaces – Examples



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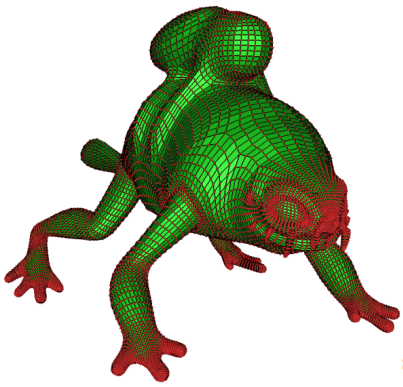

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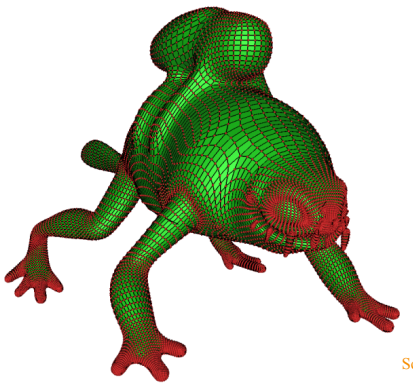

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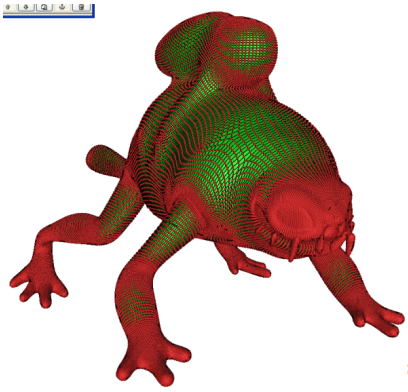

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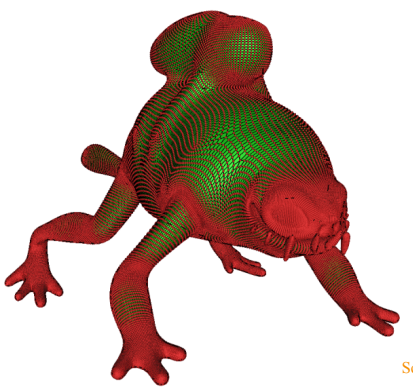

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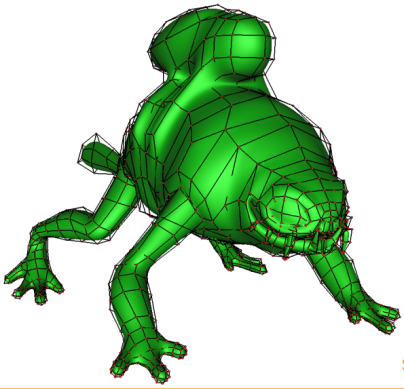

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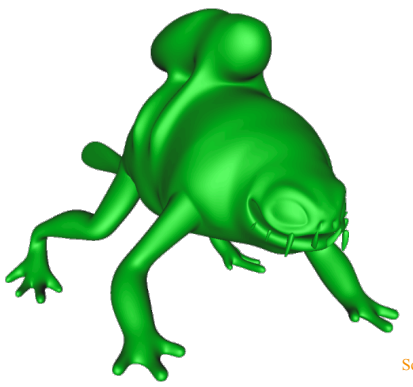

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Subdivision Surfaces – Examples

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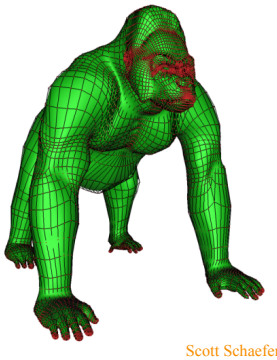
Subdivision Surfaces – Examples



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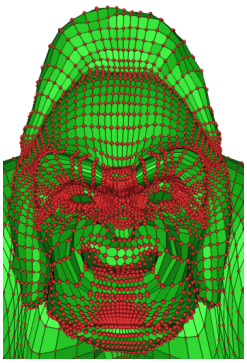
Subdivision Surfaces – Examples



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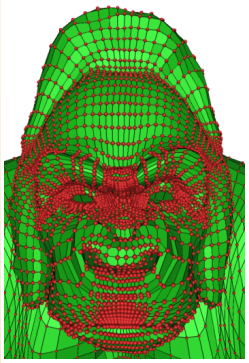
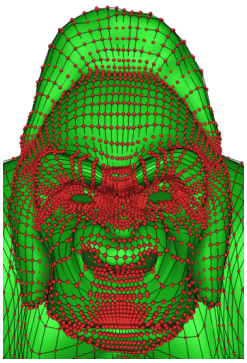
Subdivision Surfaces – Examples





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Subdivision Surfaces – Examples

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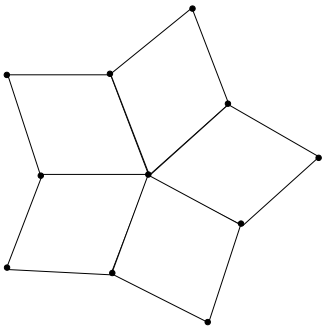
Key Questions

- Which subdivision rules?
  - Refinement
  - Weighted averaging
  - Boundaries
- Design goal:
  - Aim for smoothness (C^0, C^1, C^2, …?)
- Implementation:
  - Aim for efficiency for implementing subdivision rules

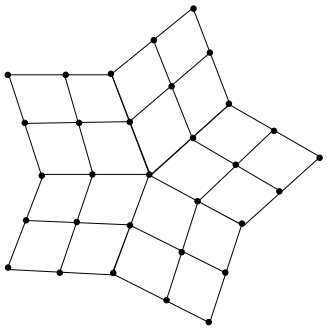
Repeated Averaging

- Very simple subdivision scheme
- Operates on quad-meshes
  - Input mesh consists of four-sided polygons (*quads*)
  - Any number of quads may touch each vertex
- Subdivision rule:
  - Linear subdivision
  - Simple averaging

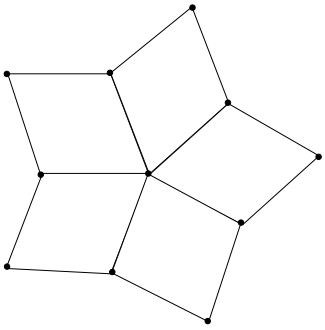
Linear Subdivision



Linear Subdivision



Averaging



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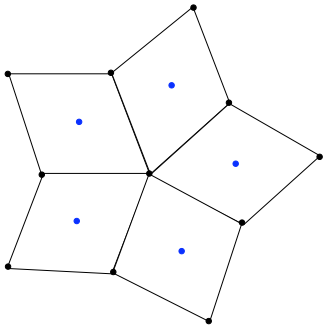

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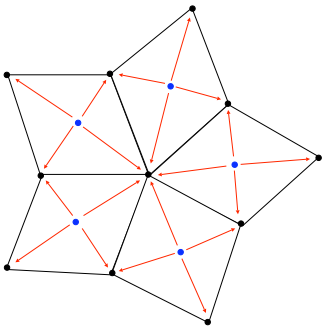
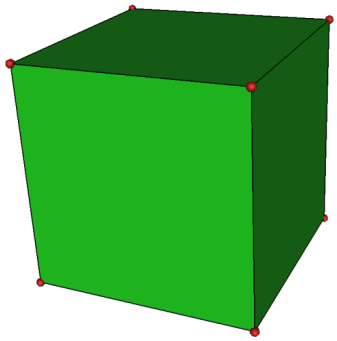


Repeated Averaging – Examples



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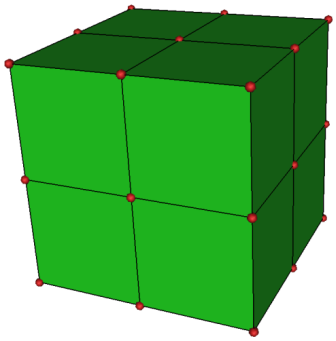

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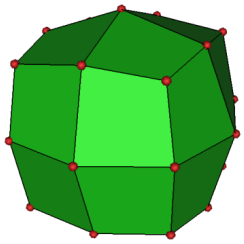

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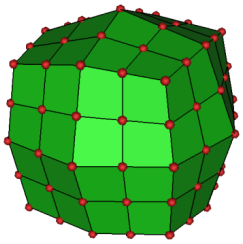

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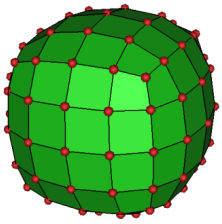

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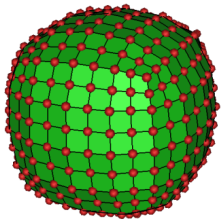

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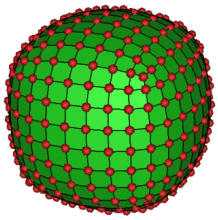

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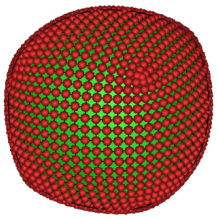

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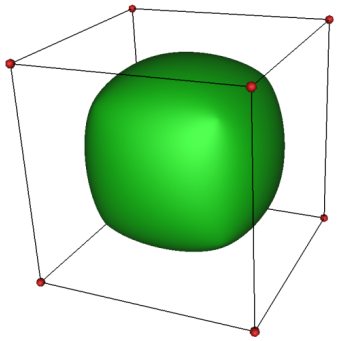

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Implementing Linear Subdivision

linearSub (F, V)
  $newV$ = V
  $newF$ = {}
  for each face F_i
    for j = 1 to 4
      e_j = getVert ($F_{i,j}$, $F_{i,j+1}$)
    add centroid to $newV$ and store index in c
    for j = 1 to 4
      add face ($F_{i,j}$, e_j, c, e_{j-1}) to $newF$
  return ($newF$, $newV$)

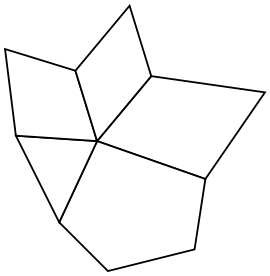
Implementing Linear Subdivision

getVert (i_1, i_2)
  if orderless key (i_1,i_2) not in $hash$
    add midpoint of $V[i_1]$, $V[i_2]$ to $newV$
    hash[(i_1,i_2)] = index of new point
  return hash[(i_1,i_2)]

Implementing Averaging

Average(F, V)
  $newV$ = 0 * V
  val = array of 0 whose size is number of vertices
  $newF$ = F
  for each face F_i
    $cent$ = centroid for F_i
    $newV[F_i]$ += $cent$ // syntax: repeat for all vtx indices in F_i
    $val[F_i]$ += 1 // syntax: repeat for all vtx indices in F_i
  for each vertex $newV[i]$
    $newV[i]$ /= $val[i]$
  return ($newF$, $newV$)

Catmull-Clark Subdivision



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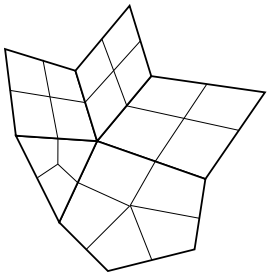

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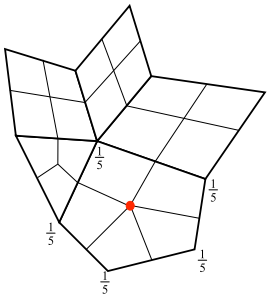

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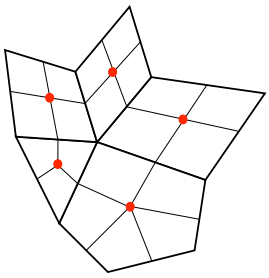

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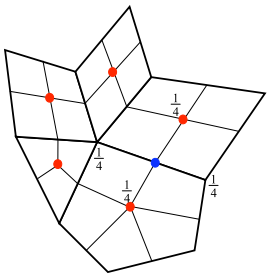

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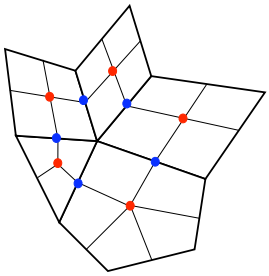

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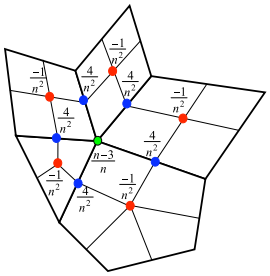

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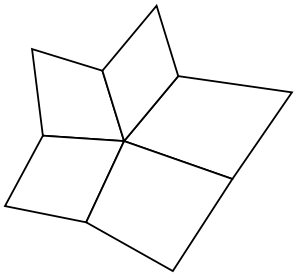

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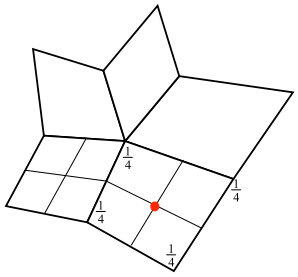

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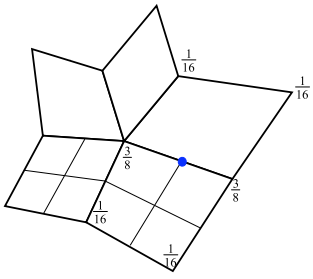

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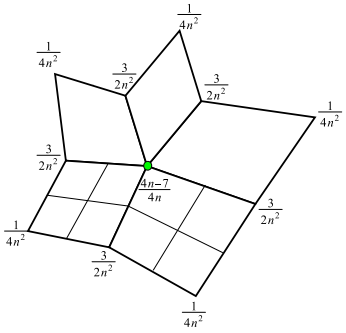

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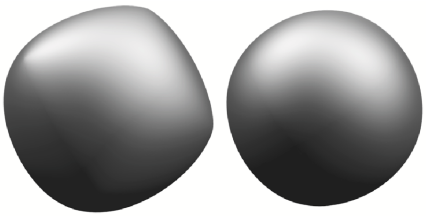

Repeated Averaging Catmull-Clark

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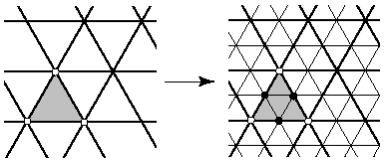
Catmull-Clark Subdivision

- One round of subdivision produces all quads
- C^2 almost everywhere
- C^1 at vertices with valence ≠ 4
- Most commonly used subdivision scheme in existence

Pixar

Loop Subdivision

- Operates on pure triangle meshes
- Subdivision rules
  - o Linear subdivision
  - o Averaging rules for “even / odd” (white / black) vertices



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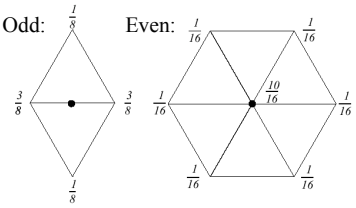
Loop Subdivision

- Averaging rules
  - o Weights for “odd” and “even” vertices



What if odd vertex only touches one triangle (boundary)?

What if even vertex does not have degree 6?

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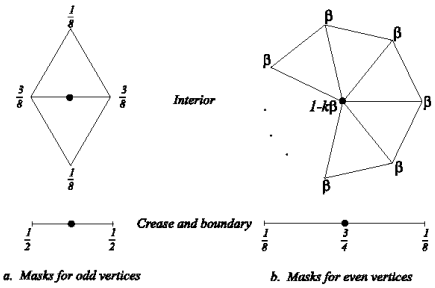
Loop Subdivision

- Rules for *extraordinary vertices* and *boundaries*:



a. Masks for odd vertices b. Masks for even vertices

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Loop Subdivision

- How to choose β?
  - o Analyze properties of limit surface
  - o Interested in continuity of surface and smoothness
  - o Involves calculating eigenvalues of matrices
    - » Original Loop

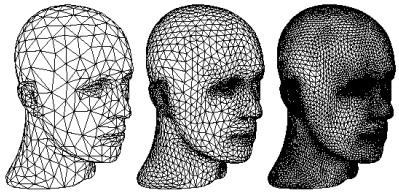
$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

    - » Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision

- Operates only on triangle meshes
- C^2 almost everywhere
- C^1 at vertices with valence $\neq 6$



Zorin & Schroeder
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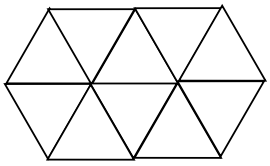
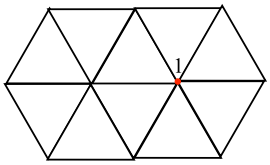
Butterfly Subdivision



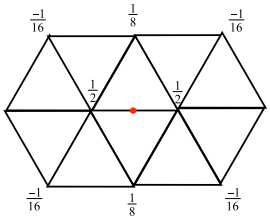
Scott Schaefer

Butterfly Subdivision



Scott Schaefer

Butterfly Subdivision



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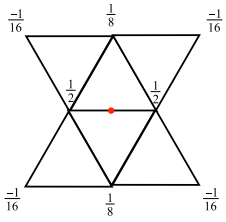
Butterfly Subdivision



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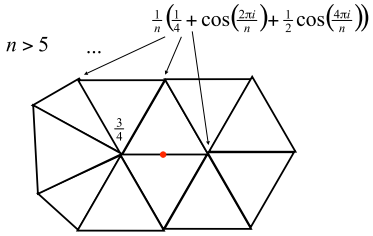
Butterfly Subdivision



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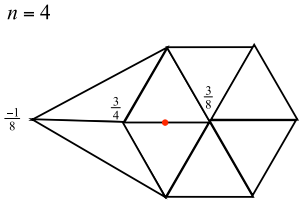
Butterfly Subdivision



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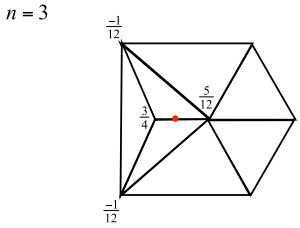
Butterfly Subdivision



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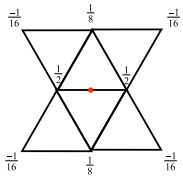
Butterfly Subdivision

- Interpolating subdivision scheme!
- Operates on triangle meshes
- Produces surfaces that are C^1 everywhere



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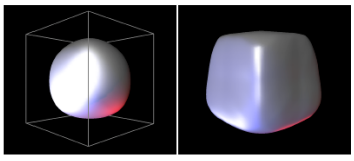
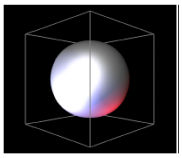
Subdivision Schemes



Loop *Butterfly*



Catmull-Clark

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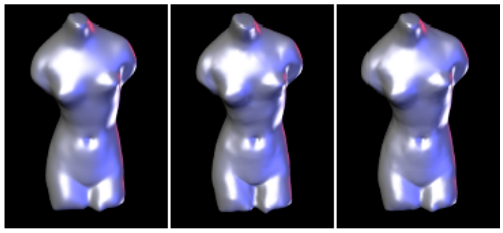
Subdivision Schemes



Loop *Butterfly* *Catmull-Clark*

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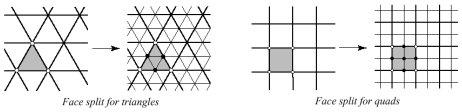
Subdivision Schemes

- There are different subdivision schemes
  - Different methods for refining topology
  - Different rules for positioning vertices
    - » Interpolating versus approximating



Face split for triangles *Face split for quads*

Face split		
	Triangular meshes	*Quad. meshes*
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

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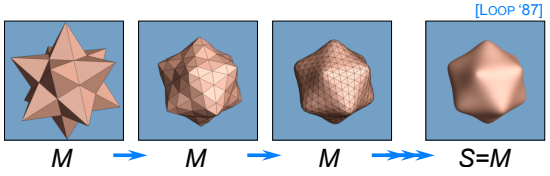
Extensions

- Common schemes assume *stationary* rules
- Locally altering subdivision rules allows for additional effects
- Examples for non-stationary rules:
  - Edge preservation
  - Adaptive subdivision

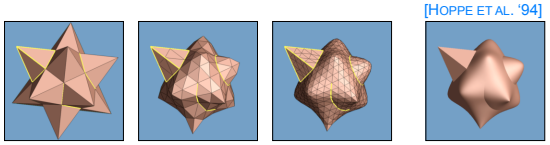
Edge Preservation

- Treat selected control mesh edges as boundary:

[LOOP '87]



[HOPPE ET AL. '94]



tagged mesh

Adaptive Subdivision

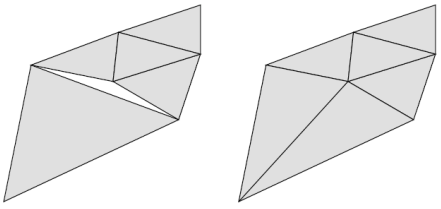
- Goal:
  - Best possible approximation of smooth limit surface
  - With limited triangle budget
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature

Adaptive Subdivision

- Goal:
  - Best possible approximation of smooth limit surface
  - With limited triangle budget
- Quality of approximation can be defined by
  - Projected (screen) area of final triangles
  - Local surface curvature
- Solution:
  - Stop subdivision at different levels across the surface
  - Stop-criterion depending on quality measure
  - Project each vertex onto limit surface

Adaptive Subdivision

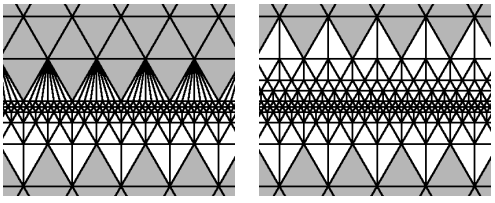
- Problem:
  - Different levels of subdivision may lead to gaps in the surface



[Kobbelt 2000]

Adaptive Subdivision

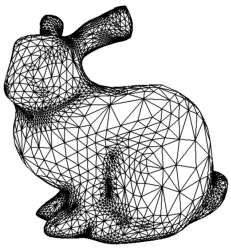
- Solution:
  - Replacing incompatible coarse triangles by *triangle fan*
  - Balanced subdivision: neighboring subdivision levels must not differ by more than one



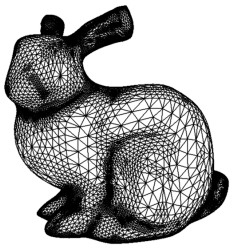
Unbalanced Balanced

[Kobbelt 2000]

Adaptive Subdivision



10072 Triangles

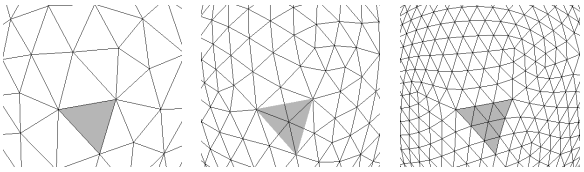


228654 Triangles

[Kobbelt 2000]

Subdivision Schemes

- More subdivision schemes:
  - Sqrt(3) subdivision

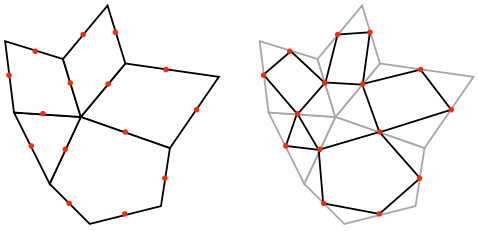


Rotating grid of sqrt(3) subdivision

[Kobbelt 2000]

Subdivision Schemes

- More subdivision schemes:
  - Sqrt(3) subdivision
  - Vertex-split subdivision (Doo-Sabin, Midedge, Biquartic)



One step of Midegde subdivision

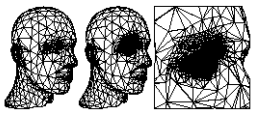
Subdivision Surfaces

- Properties:
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections

Pixar

Subdivision Surfaces

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Local support
  - Guaranteed continuity
  - Multiresolution
- Difficulties:
  - Intuitive specification
  - Parameterization
  - Intersections



Summary

Feature	Polygonal Mesh	Subdivision Surface
Accurate	No	Yes
Concise	No	Yes
Intuitive specification	No	No
Local support	Yes	Yes
Affine invariant	Yes	Yes
Arbitrary topology	Yes	Yes
Guaranteed continuity	No	Yes
Natural parameterization	No	No
Efficient display	Yes	Yes
Efficient intersections	No	No