What is the computational cost of automating brilliance or serendipity?
(Computational complexity and P vs NP question)

COS 116: 4/15/2008

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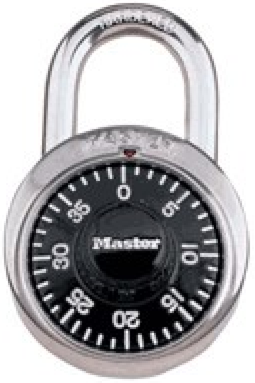
Combination lock

Why is it secure?

(Assume it cannot be picked)



Ans: Combination has 3 numbers 0-39…
thief must try $39^3 = 59{,}319$ combinations

Exponential running time

2^n time to solve instances of "size" n

Increase n by 1 → running time doubles!

Main fact to remember:

For n =300,
2^n > number of atoms in the visible universe.

Boolean satisfiability

$$(A + B + C) \cdot (\overline{D} + F + G) \cdot (\overline{A} + G + K) \cdot (\overline{B} + P + Z) \cdot (C + \overline{U} + \overline{X})$$

- Does it have a satisfying assignment?
- What if instead we had 100 variables?
- 1000 variables?
- How long will it take to determine the assignment?

Discussion

Is there an inherent difference between

being creative / brilliant

and

being able to appreciate creativity / brilliance?

What is a computational analogue of this phenomenon?

A Proposal

Brilliance = Ability to find
"needle in a haystack"

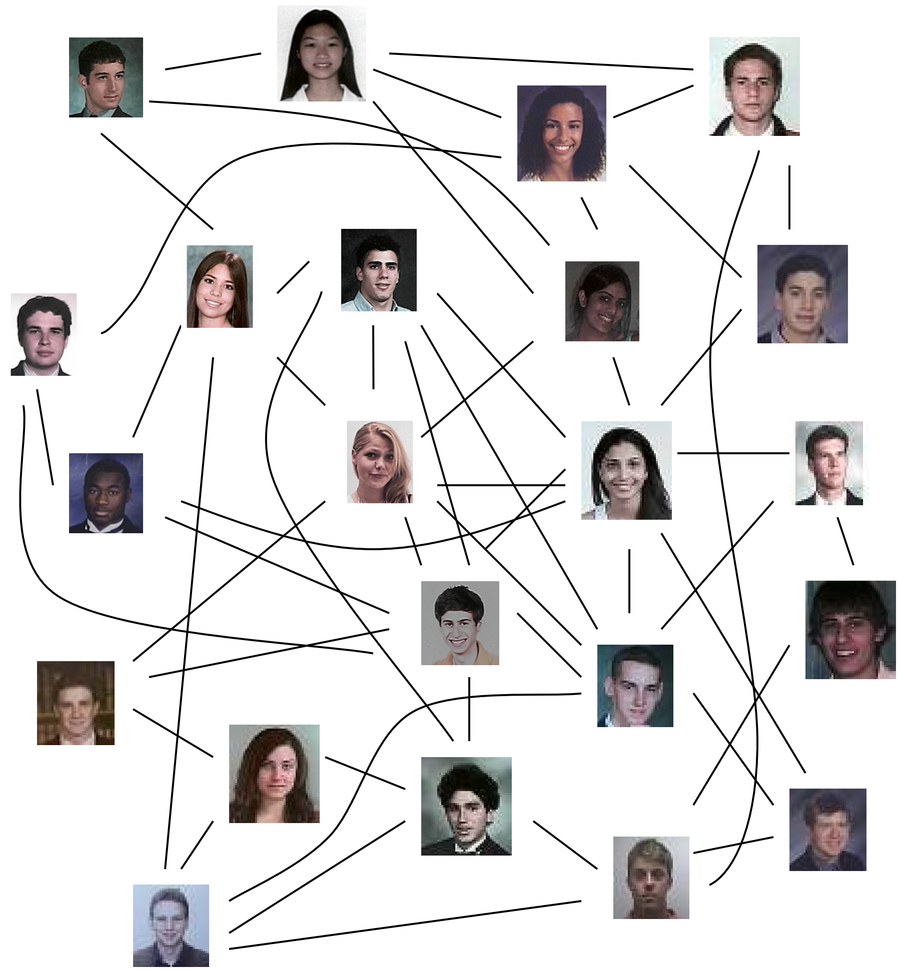
Beethoven found
"satisfying assignments"
to our neural circuits
for music appreciation

Comments??

There are many computational problems where finding a solution involves "finding a needle in a haystack"….

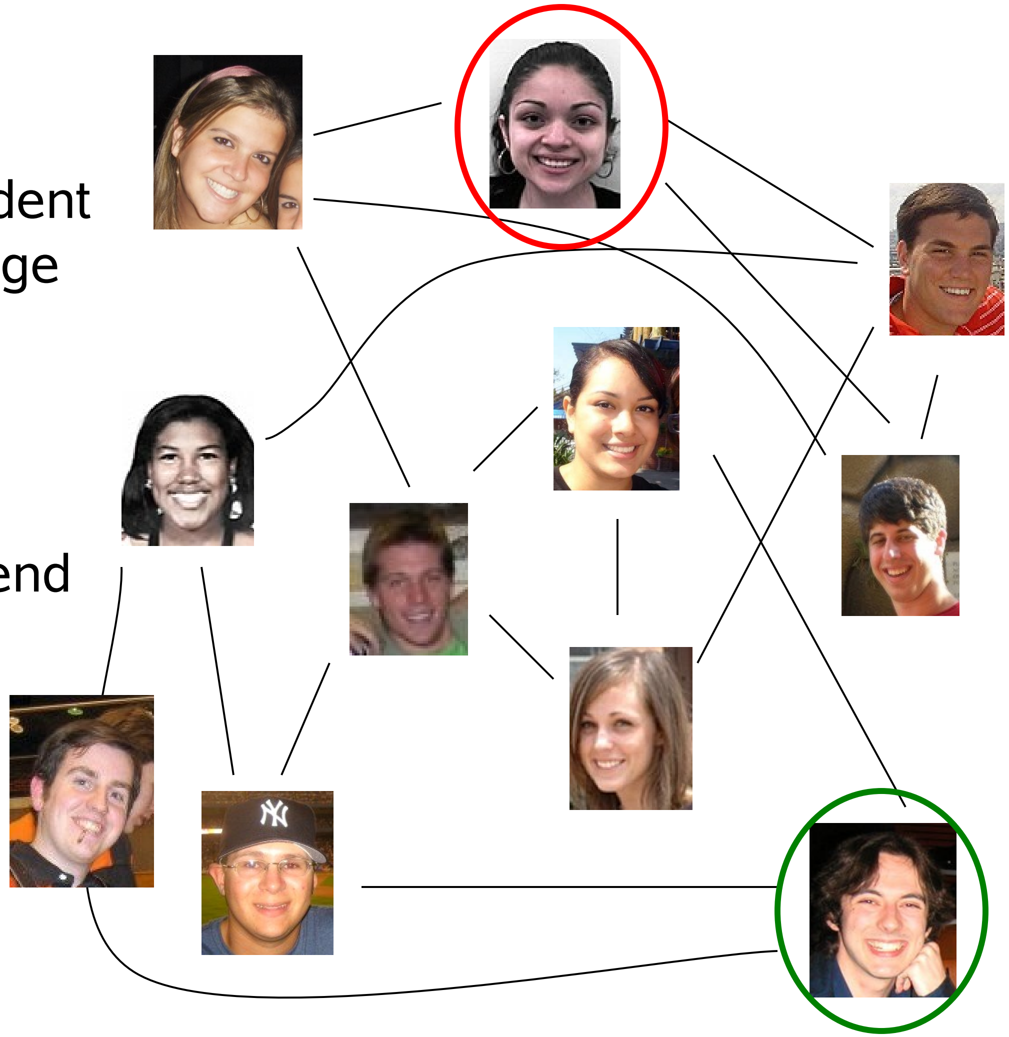
CLIQUE Problem

- In this social network, is there a CLIQUE with 5 or more students?
- CLIQUE: Group of students, every pair of whom are friends
- What is a good algorithm for detecting cliques?
- How does efficiency depend on network size and desired clique size?



Rumor mill problem

- Social network for COS 116
- Each node represents a student
- Two nodes connected by edge if those students are friends
- Johanna starts a rumor
- Will it reach Kieran?
- Suggest an algorithm
- How does running time depend on network size?
- Internet servers solve this problem all the time ("traceroute" in Lab 9).



Exhaustive Search / Combinatorial Explosion

Naïve algorithms for many “needle in a haystack” tasks involve checking all possible answers → exponential running time.

- Ubiquitous in the computational universe
- Can we design smarter algorithms (as for “Rumor Mill”)? Say, n^2 running time.

Harmonious Dorm Floor

Given: Social network involving n students.

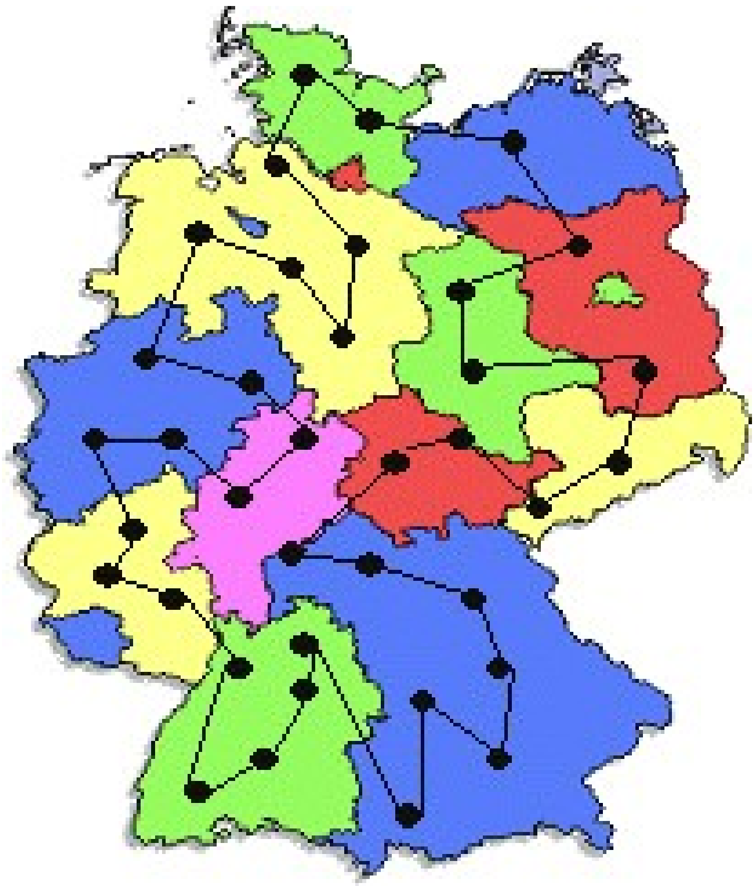
Edges correspond to pairs of students who <u>don't</u> get along.

Decide if there is a set of k students who would make a harmonious group (everybody gets along).

Just the Clique problem in disguise!

Traveling Salesman Problem (aka UPS Truck problem)

- Input: *n* points and all pairwise inter-point distances, and a distance *k*
- Decide: is there a path that visits all the points ("salesman tour") whose total length is at most *k*?



Finals scheduling

- Input: *n* students, *k* classes, enrollment lists, *m* time slots in which to schedule finals
- Define “conflict”: a student is in two classes that have finals in the same time slot
- Decide: if schedule with at most 100 conflicts exists?

The P vs NP Question

- P: problems for which solutions can be found in polynomial time (n^c where c is a fixed integer and n is "input size"). Example: Rumor Mill

- NP: problems where a *good solution* can be <u>checked</u> in n^c time. Examples: Boolean Satisfiability, Traveling Salesman, Clique, Finals Scheduling

- Question: Is P = NP?

  "Can we automate brilliance?"

  (Note: Choice of computational model ---
  Turing-Post, pseudocode, C++ etc. --- irrelevant.)

NP-complete Problems

Problems in NP that are "the hardest"

- If they are in P then so is ***every*** NP problem.

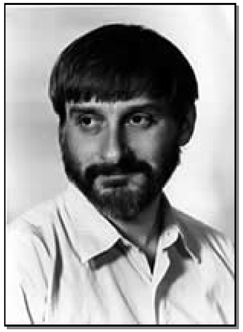
Examples: Boolean Satisfiability, Traveling Salesman, Clique, Finals Scheduling, 1000s of others

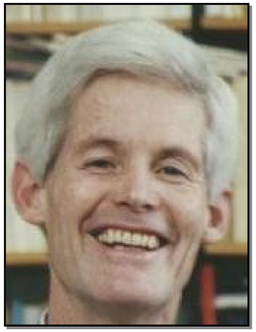
How could we possibly prove these problems are "the hardest"?

“Reduction”

“If you give me a place to
stand, I will move the earth.”
– Archimedes (~ 250BC)

“If you give me a polynomial-time algorithm for Boolean Satisfiability, I will give you a polynomial-time algorithm for every NP problem.” --- Cook, Levin (1971)





“Every NP problem is a satisfiability problem in disguise.”

Dealing with NP-complete problems

1. Heuristics (algorithms that produce reasonable solutions in practice)

2. Approximation algorithms (compute provably near-optimal solutions)

Computational Complexity Theory:

Study of Computationally Difficult problems.

A new lens on the world?

- Study matter $\rightarrow$ look at mass, charge, etc.
- Study processes $\rightarrow$ look at computational difficulty

Example 1: Economics

General equilibrium theory:

- Input: *n* agents, each has some initial endowment (goods, money, etc.) and preference function
- General equilibrium: system of prices such that for every good, demand = supply.
- Equilibrium exists [Arrow-Debreu, 1954]. Economists assume markets find it ("invisible hand")
- But, *no known* efficient algorithm to compute it. How does the market compute it?

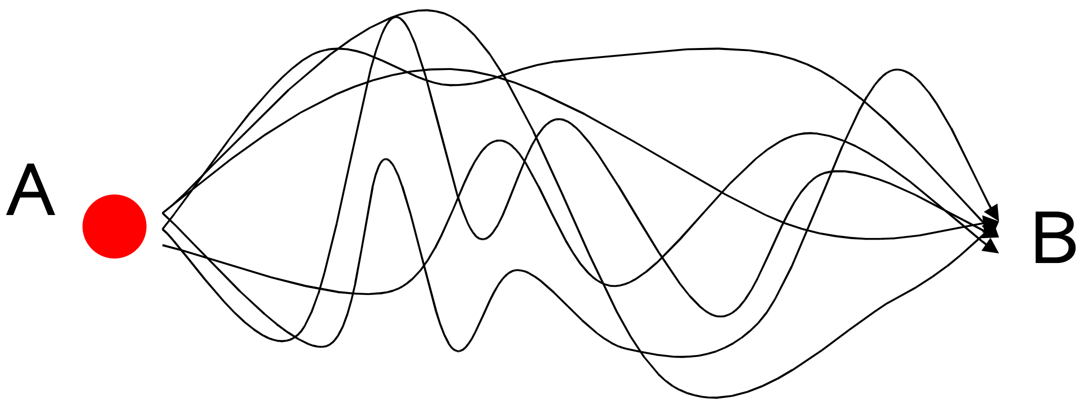
Example 2: Factoring problem

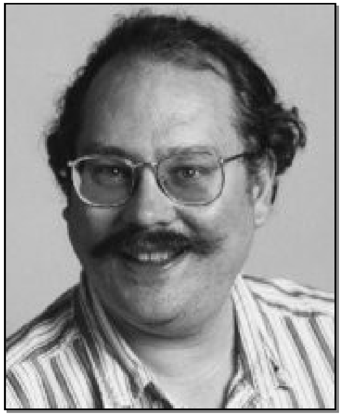
Given a number n, find two numbers p, q (neither of which is 1) such that n = p x q.

Any suggestions how to solve it?

Fact: This problem is believed to be hard.
It is the basis of much of cryptography.
(More next time.)

Example 3: Quantum Computation





Peter Shor

- Central tenet of quantum mechanics: when a particle goes from A to B, it takes *all possible paths all at the same time*
- [Shor'97] Can use quantum behavior to efficiently factor integers (and break cryptosystems!)
- Can quantum computers be built, or is quantum mechanics not a correct description of the world?

Example 4: Artificial Intelligence

What is computational complexity of language recognition?

Chess playing?

Etc. etc.

Potential way to show the brain is not a computer:
Show it routinely solves some problem that provably takes exponential time on computers.

(Will talk more about AI in a couple weeks)

Why is P vs NP a Million-dollar open problem?

- If $P = NP$ then Brilliance becomes routine (best schedule, best route, best design, best math proof, etc…)

- If $P \neq NP$ then we know something *new and fundamental* not just about computers but about the world (akin to "Nothing travels faster than light").

Next time: Cryptography (practical use of computational complexity)