

# Self-reproducing programs. And Introduction to logic.

COS 116: 3/6/2008

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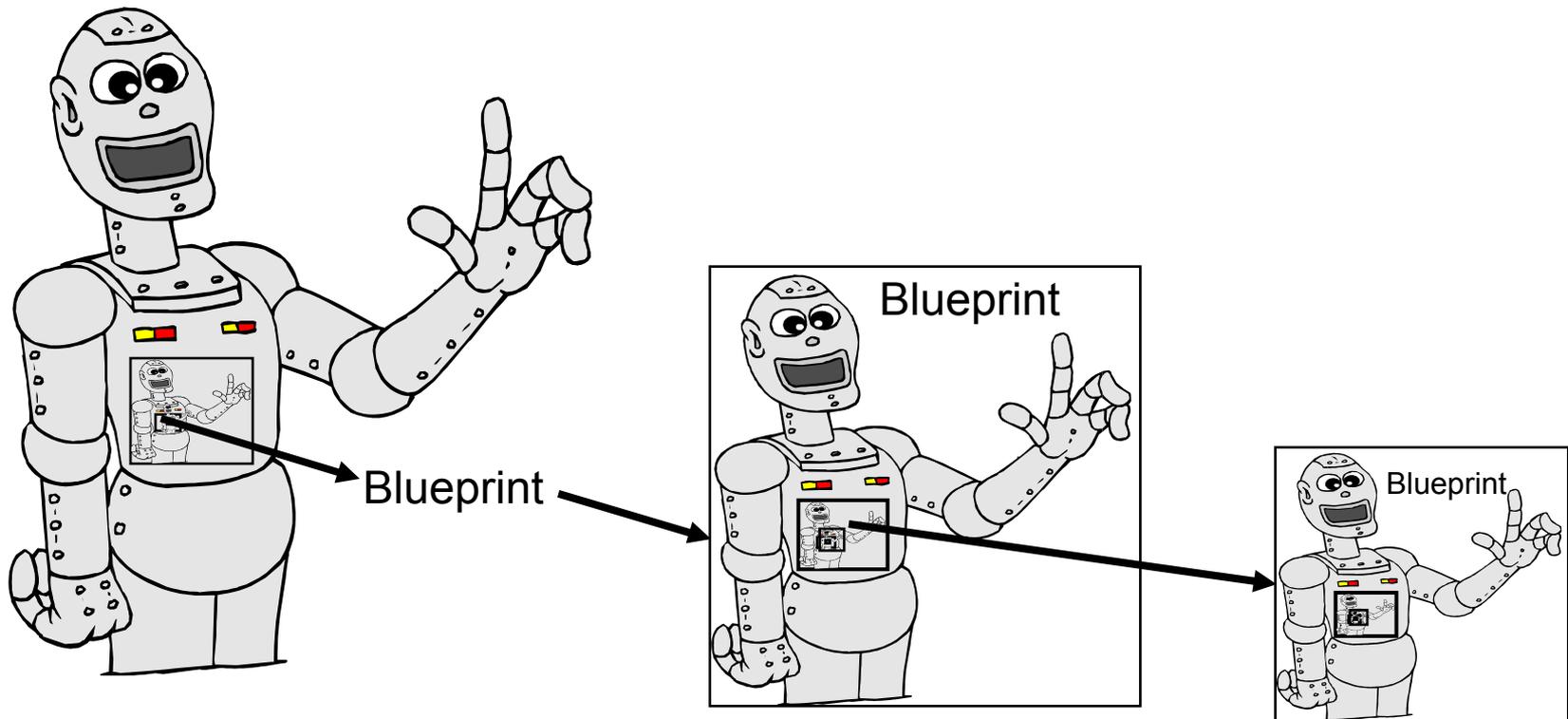


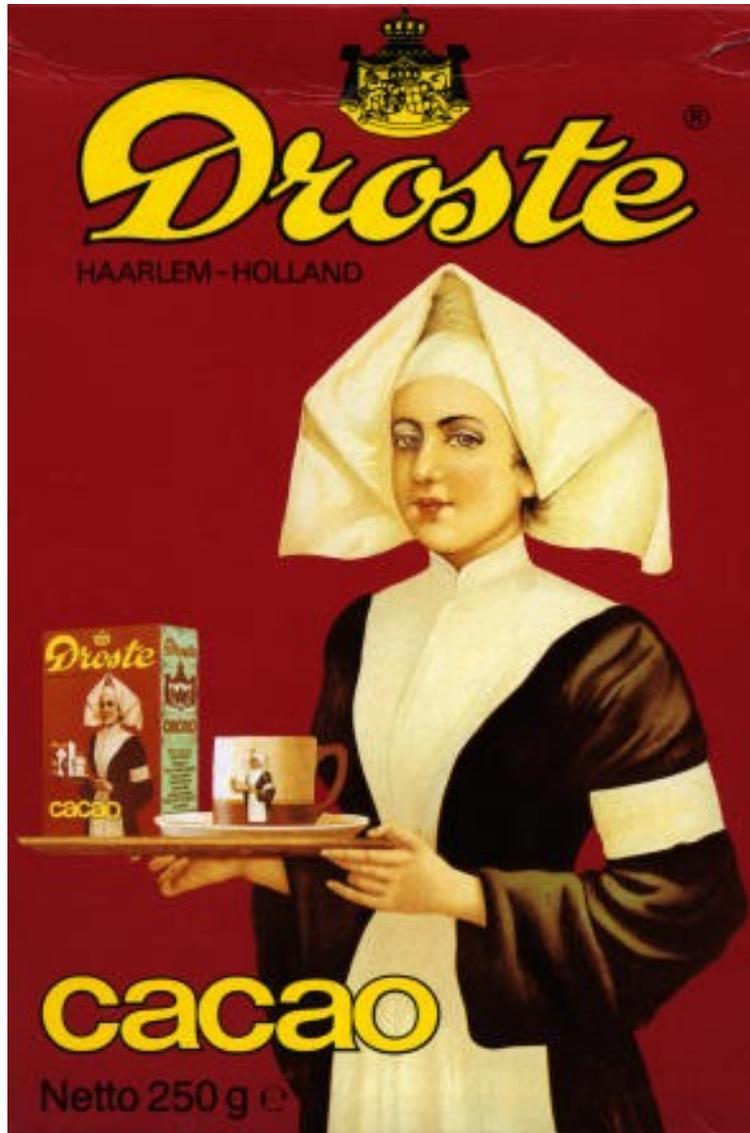
## Discussion Time

- Did “Theory of Everything” article make you look at something in a new way?
- How does it connect to Tuesday’s lecture?

# Self-Reproduction

Fallacious argument for impossibility:





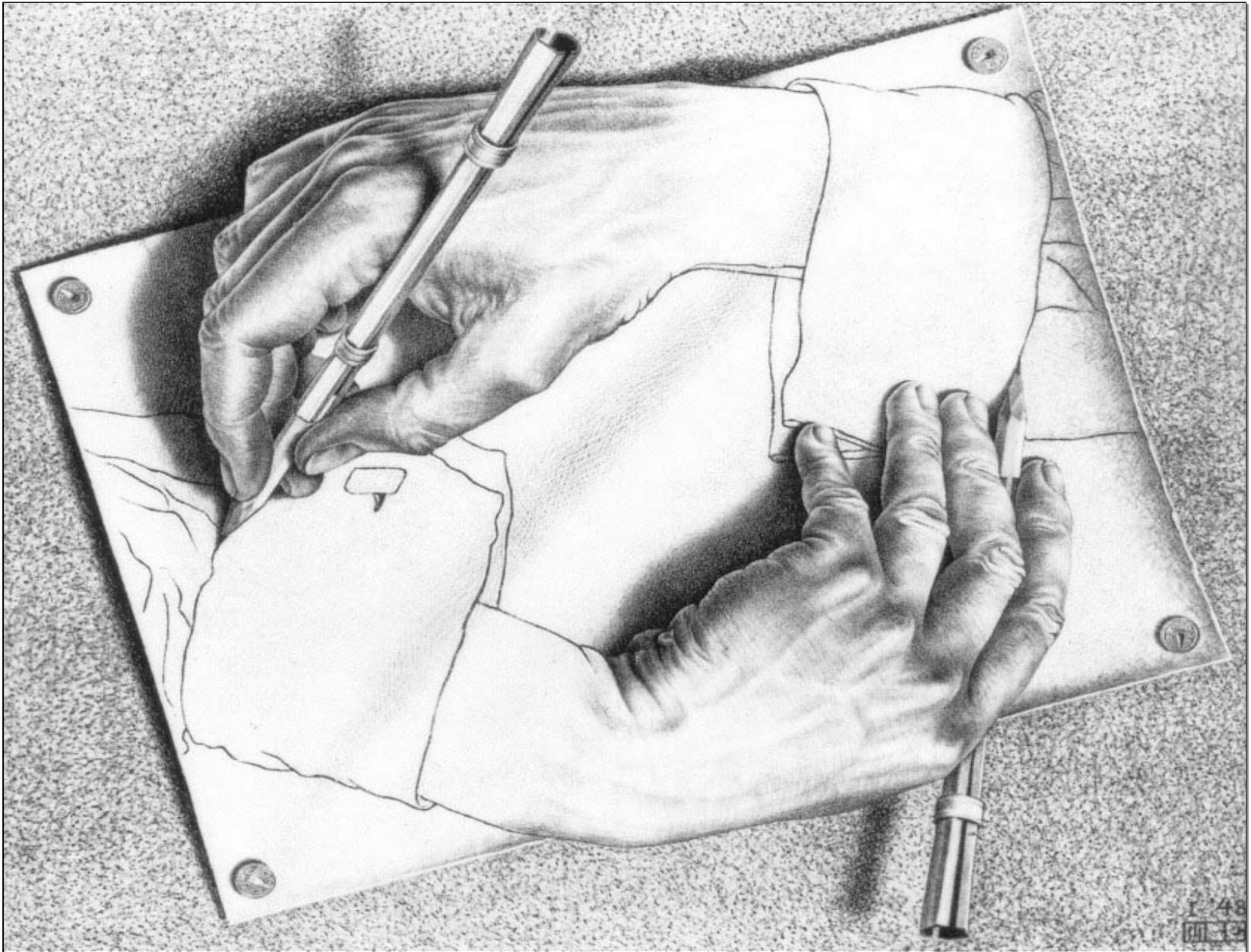
“Droste Effect”



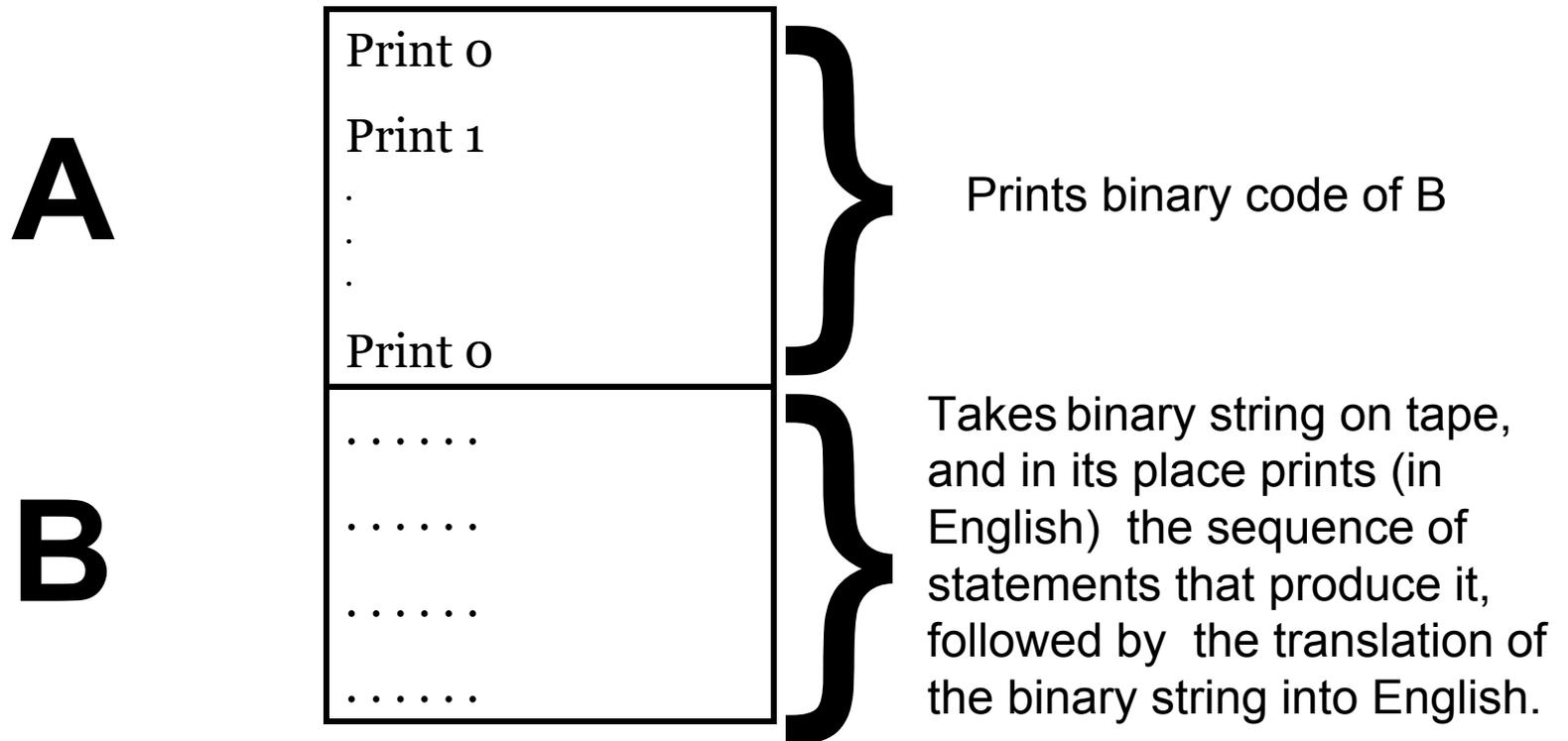
Fallacy Resolved:

“Blueprint” can involve *computation*;  
need not be an exact copy!

*Print the following sentence twice,  
the second time in quotes. “Print the  
following sentence twice, the second  
time in quotes.”*

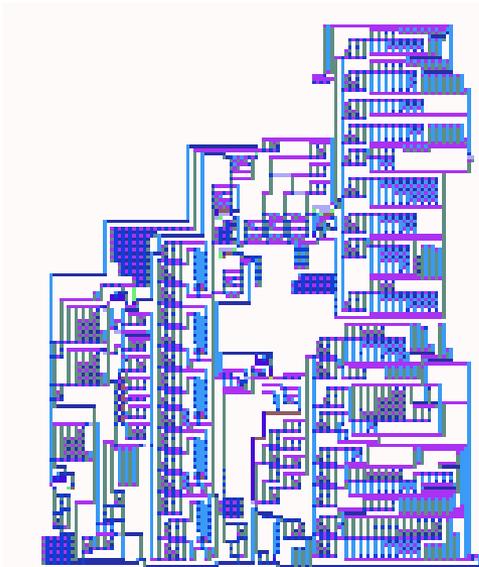


# High-level view of self-reproducing program



# Self-reproducing machines

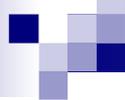
[John von Neumann, 1940s]



2-D and 3-D cellular automata (with a “moving arm” controlled by the automaton itself) that makes a precise copy of itself.

“Accidental changes” during copying --> mutations, evolution

This and related ideas of Pauli motivated discovery of the molecular basis of life on earth (DNA, RNA etc.)



# Upcoming lectures:

## Computational Hardware

- Boolean logic and Boolean circuits
- Sequential circuits (circuits with memory)
- Clocked circuits and Finite State Machines
- CPUs
- Operating System
- Networks, Internet



## Discussion Time

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

It is raining today, Ben overslept, and there's an exam. Will Ben bike today?

“Logical reasoning”, “*Propositional logic*.”

# Propositional Logic: History

- Aristotle – Law of excluded middle, Law of contradiction.
- Stoic Philosophers (3<sup>rd</sup> century BC) – Basic inference rules (*modus ponens* etc.)
- Some work by medieval philosophers
- De Morgan and Boole (19<sup>th</sup> century): Symbolic logic – “automated”, “mechanical”
- C. Shannon (1930s) – Proposal to use digital hardware

# Example

*Ed goes to the party if*

*Dan does not and Stella does.*

Choose “Boolean variables” for 3 events:

$\left\{ \begin{array}{l} \mathbf{E: Ed goes to party} \\ \mathbf{D: Dan goes to party} \\ \mathbf{S: Stella goes to party} \end{array} \right\}$  Each is either  
TRUE or FALSE

$$\mathbf{E = S \ AND \ (NOT \ D)}$$

$$\text{Alternately: } \mathbf{E = S \ AND \ \bar{D}}$$

# Logical “OR”

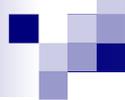
Ed goes to the party if Dan goes **or** Stella goes

$E = D \text{ OR } S$

E is TRUE if one or both of D and S are TRUE

Note: In everyday language OR has another meaning too!

Example: You can eat an orange or an apple



# Boolean expressions

Composed of boolean variables, **AND**, **OR**, and **NOT**

Examples:

**D AND ( P OR (NOT Q))**

**C OR D OR E**

# Truth table

Lists the truth value of the Boolean expression for all combinations of values for the variables.

**Boolean Expression**       $E = S \text{ AND } \bar{D}$

## Truth table

0 = FALSE

1 = TRUE

Write E for all possible values of D, S.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0

# Let's work an example...

**Boolean Expression**

$$E = D \text{ OR } \bar{S}$$

What are x and y ?!?

Possibilities:

x =0, y=0

x=0, y =1

x=1, y=0,

X=1, y=1

D	S	E
0	0	1
0	1	x
1	0	y
1	1	1

# Ben Revisited

Ben only rides to class if he overslept, but even then if it is raining he'll walk and show up late (he hates to bike in the rain). But if there's an exam that day he'll bike if he overslept, even in the rain.

**B:** Ben Bikes

**R:** It is raining

**E:** There is an exam today

**O:** Ben overslept

Break up in groups of three and come up with Boolean expression for B in terms of R, E and O.

# Boolean “algebra”

A **AND** B written as  $A \cdot B$

$$0 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

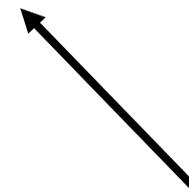
$$\begin{array}{r} 1 \cdot 1 = \\ 1 \end{array}$$

A **OR** B written as  $A + B$

$$0 + 0 = 0$$

$$1 + 0 = 1$$

$$1 + 1 = 1$$

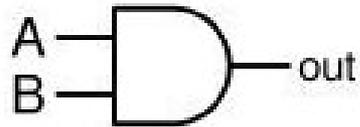


Funny arithmetic

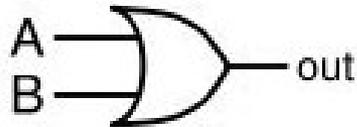
Will provide readings on this...

# Boolean circuit

Pictorial representation of Boolean expression using Special symbols for AND, OR and NOT



A AND B



A OR B



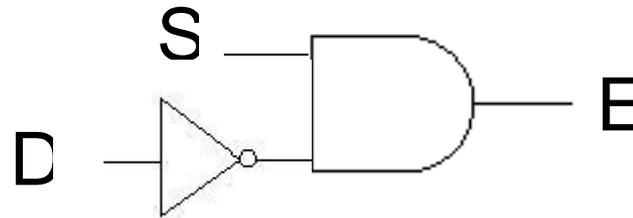
$\overline{A}$

# Three Equivalent Representations

**Boolean Expression**

$$E = S \text{ AND } \bar{D}$$

**Boolean Circuit**



**Truth table:**

Value of E for every possible D, S.

TRUE=1; FALSE=0.

D	S	E
0	0	0
0	1	1
1	0	0
1	1	0



Next time: Boolean circuits, the  
basic components of the digital world

Midterm will have a question on boolean logic.

Ed goes to the party if

Dan doesn't AND Stella doesn't

$$E = \bar{D} \text{ AND } \bar{S}$$

Is this equivalent to:

*Ed goes to the party if*

*NOT (Dan goes OR Stella goes)*

.....?

(De Morgan's Laws)