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INDEPENDENT PRIVATE VALUE AUCTIONS: BIDDER BEHAVIOUR IN FIRST-, SECOND- AND THIRD-PRICE AUCTIONS WITH VARYING NUMBERS OF BIDDERS*

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A well known theoretical result for independent private value (IPV) auctions is the Revenue-Equivalence Theorem: English, Dutch, first- and second-price auctions yield the same expected revenue for risk neutral bidders. More fundamental than this, second-price and English auctions are strategically equivalent, as are first-price and Dutch auctions, so that these institutions yield the same expected revenue even in the absence of risk neutrality (Vickrey, 1961; Meyerson, 1981; Riley and Samuelson, 1981). Experimental comparisons show a systematic failure of the strategic equivalence of second-price and English (Kagel *et al.* 1987) and of first-price and Dutch auctions (Cox *et al.* 1982), with higher revenues in the sealed bid auctions in both cases. These results raise questions regarding the validity of Nash equilibrium bidding theory as a descriptive model of auction behaviour. Do differences in bidding between strategically equivalent auctions represent some uncontrolled element in the environment, or in bidder's utility functions, that induces higher bids in sealed bid auctions, or do they involve more fundamental breakdowns in Nash equilibrium bidding theory to the point that the comparative static predictions of the theory fail to hold?

One way to sort out between these alternatives is to hold this uncontrolled element constant by working within the context of sealed bid, or open outcry, auctions and examining the predictive properties of the theory. We do this by examining the comparative static effects of changing sealed bid price rules, comparing first-price (FPA), second-price (SPA) and third-price (TPA) auctions with varying numbers of bidders. The properties of FPA and SPA are well known. The novel theoretical part of this paper is the characterisation of TPA, a new and completely synthetic institution that does not exist outside the laboratory. One of the real strengths of laboratory experimental methods is the ability to construct and implement institutions explicitly designed to test theory and to understand behaviour. Although TPA may never be observed outside an experimental laboratory, it has a number of surprising and contrasting predictions relative to FPA and SPA, thereby providing a basis for thorough and demanding tests of a number of qualitative implications of Nash

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equilibrium bidding theory: In TPA (1) bids *exceed* private values, (2) the marginal effect on bids of an increase in private values is *greater* than one, (3) increasing the number of bidders *reduces* bids, and (4) risk averse bidders bid *below* the RNNE. These contrasting predictions provide an excellent vehicle for testing the descriptive validity of Nash bidding theory.

Under the changing bid price rules, the comparative static implications of Nash bidding theory are satisfied, at least directionally, as (1) around 90% of all bids lie below valuations in FPA, less than 4% of all bids lie below valuations in SPA, and over 80% of all bids are greater than valuations in TPA, (2) the slope of the bid function is less than 1 in FPA, equal to 1 in SPA and greater than 1 in TPA, and (3) increases in the number of bidders increase average bids in FPA, generate no change, or a modest reduction, in bids in SPA, and produce a more sizable reduction in bids in TPA. Finally, with respect to the question of risk aversion, bids are below the RNNE in TPA with 5 bidders, as risk aversion requires, but are above the RNNE in TPA with 10 bidders, which is inconsistent with the risk aversion hypothesis.

I. THEORETICAL PREDICTIONS

Consider an IPV auction in which private values, x, are drawn from a uniform distribution on $[0, \bar{x}]$. As is well known, in a FPA the risk neutral bid function is

$$B_1(x) = x(N-1)/N \tag{1}$$

and the SPA bid function is

$$B_2(x) = x. \tag{2}$$

In the FPA, bids are below x and increasing in N. In the SPA, bidding one's valuation is a dominant strategy.

In a TPA, the risk neutral bid function is

$$B_3(x) = x(N-1)/(N-2) \tag{3}$$

(derivation of this bid function is provided in the Appendix). Here, bids are above x and *decrease* in N. The intuition underlying bidding in TPA is as follows: As in SPA, bidding one's valuation dominates bidding below it. Further, there is an incentive to bid above x. Suppose all rivals are at the symmetric equilibrium and you consider raising your bid by a small $\epsilon > 0$ above your private value. This change matters only if the second highest bid falls between your private value and your new, higher, bid. We can partition the outcomes in this event into two distinct possibilities: when the third highest bid is below your private value (case 1) and when it is above your private value (case 2). Raising your bid above your private value earns you gains in case 1 and losses in case 2. For a small enough ϵ, the probability of case 2 is negligible, relative to the probability of case 1, and the potential loss is quite small, so that it is optimal to bid above your private value. Further, it can be shown that the relative probabilities shift towards case 2 when private values are more congested as a result of increasing numbers of bidders. This explains why, with risk neutrality, bids go down with increased competition.

The effects of risk aversion on the symmetric equilibrium are well established in FPA and SPA. In FPA risk aversion raises individual bids relative to (1) (Riley and Samuelson, 1981), with bids still below x and increasing in response to increased competition. In SPA Vickrey (1961) showed that (2) is a dominant strategy, insensitive to N and to bidders' risk preferences.

In bidding above their valuations in TPA, bidders are trading off potential gains against possible losses. With risk aversion, the attractiveness of the gains is reduced relative to the losses. This provides an incentive for lower equilibrium bids in the presence of risk aversion.

Although we have been unable to prove that the above intuition applies to all concave utility functions, we have been able to solve for the equilibrium bid function for the important case of *constant absolute* risk aversion (CARA).[1] The symmetric equilibrium bid function for our design with CARA bidders is

$$B_3(x; A) = x + (1/A) \ln [1 + Ax/(N-2)], \tag{4}$$

where $A > 0$ is the CARA measure. Under CARA, bids exceed private values, the bid function is decreasing in A, so that the more risk averse bidders are the more their bids lie below the RNNE, and the bid function decreases with N (derivation of this bid function and its properties is provided in the Appendix).

II. EXPERIMENTAL DESIGN AND PROCEDURES

Each auction series had several auction periods with 5 or 10 subjects bidding for a single unit of a commodity under a sealed-bid procedure. In each period the high bidder earned profit equal to his valuation less the price paid. Other bidders earned zero profit.

Private valuations, x, were randomly drawn in each auction period from a uniform distribution on the interval [\$0.00, \$28.30]. Bidders knew their own valuation, the distribution from which others' values were drawn, and the number of bidders. A new set of random draws preceded each auction period. After each auction period all bids were reported, in descending order, along with the corresponding valuations (subject identification numbers were suppressed). The highest bid and the price were highlighted, and profits were reported to all bidders.

Subjects were given starting balances of \$10.00 to cover the possibility of losses. Although rational (Nash equilibrium) bidders will not suffer losses in FPA or SPA, they may suffer losses in TPA. Given the need for starting cash balances in these auctions, providing cash balances in the other auctions minimises potential procedural confounds.

Profits (or losses) were added to starting cash balances. If a subject's balance went negative, they were no longer permitted to bid, were paid their \$4.00

[1] No effort has been made to provide more general results regarding risk aversion since (1) the intuition underlying the result reported suggests that it is quite general, (2) given the limited amounts of money at stake in the experiment, CARA is likely to be a useful approximation to bidder behaviour, and (3) for risk averse bidders, CARA permits treating the utility of earnings from each auction period in a series of auctions as independent, as is commonly done in experiments.

Table 1
Experimental Treatment Conditions

Bid-price rule	Auction series	Number bidders	Auction periods	Experience
First price	1	5	1–12	Common value auctions and some private value
		5 and 10	13–24	
Second price	2.1	5	1–12	Common value auctions
		5 and 10	13–24	
	2.2	5	1–13	None
		10	14–25	
		5	26–35	
Third price	3.1	5	1–12	Common value auctions
		5 and 10	13–21	
	3.2	10	1–12	Common value auctions and some private value
		5	13–21	
		5 and 10	22–33	
	3.3	5	1–11	None
		10	12–23	
		5	24–35	
	3.4	10	1–11	Private value auctions
		5	12–21	
		10	22–31	
		5 and 10	32–41	
	3.5	9	1–13	Private value auctions
		4 or 5 and 9	14–25	
		9	26–31	

participation fee, and were free to leave the room. End of experiment balances were paid in cash, along with the participation fee.[2]

All auctions employed several substitute bidders who observed the outcomes and were used to replace bankrupt bidders. Bankruptcies occurred in three of the eight auction series.[3] Substitutes' cash balances were $10.00. Subjects were MBA and/or upper-level economics majors at the University of Houston.

In auctions with 10 bidders, all subjects participated in a single auction market. In auctions with 5 bidders, subjects were randomly assigned to one of two small markets in each auction period, with a high bidder declared, and profits paid in both markets. The effect of varying the number of bidders was studied using both cross-over procedures – subjects started out bidding in a market of size 5 (10), after which they were 'crossed over' to bid in auctions of size 10 (5) – and dual market procedures – subjects bid in both a large and a small market simultaneously, using the same private valuation, with profits paid in either the large market or both small markets (decided randomly after bids had been submitted). Dual market bidding permits direct evaluation of the effects of changing numbers of rivals without having to specify and estimate individual subject bid functions.

Table 1 specifies the experimental treatment conditions.

[2] Average profit per auction was $4.72 with 5 bidders and $2.57 with 10 bidders under the RNNE. Earnings averaged over $24.00 per subject for auction sessions lasting a little under 2 hours.

[3] One subject went bankrupt in auction series 2.2 and 3.4; two subjects went bankrupt in series 3.3.

III. EXPERIMENTAL RESULTS

Table 2 reports our primary results. The first column shows the relationship of bids to valuations where differences of \$0.05 or less are classified as no difference. This somewhat arbitrary classification captures the fact that subjects tend to round off their bids to the nearest nickel or dime, and that 'really small' differences ought to be placed in a separate category from bigger differences.

Results from FPA show the overwhelming majority of individual bids were below valuations as any rational theory will predict. In SPA a sizable minority (30%) of all bids were effectively equal to valuations, in contrast to results from both FPA and TPA. In TPA, close to 90% of all bids were above valuations in markets with 5 bidders, with close to 85% of all bids above valuations with 10 bidders, well above the frequencies reported in FPA and SPA. Chi-square tests make it clear that the bid distributions are different from each other, and confirm the visual impression that the SPA bid distribution is shifted to the right of the FPA distribution ($\chi^2 = 528$ and 225 with $N = 5$ and 10, respectively), and that the TPA distribution is shifted to the right of the SPA distribution ($\chi^2 = 128$ and 90 with $N = 5$ and 10 respectively).

Bidding above x in SPA has to be labelled a mistake, since bidding x is a dominant strategy irrespective of risk attitudes.[4] We explain overbidding in SPA on the grounds that: (1) The dominant bidding strategy is not transparent. Rather it requires answering the question of what is to be gained from bidding above x and winning, as compared to bidding x. Apparently, it is not natural for subjects to pose this question, and (2) Learning feedback mechanisms that would correct for this overbidding are weak under sealed bid procedures. For example, an *ex post* analysis of our SPA shows that the probability of losing money conditional on winning the auction averaged 25%, with $N = 5$, with the overall probability of losing money averaging 5% (with $N = 10$ these probabilities were 25·5 and 2·6% respectively).[5] These punishment probabilities are weak, particularly if bidders start with the illusion that bidding above x improves their chances of winning the auction without materially affecting the prices paid; and the majority of the time the auction outcomes reinforce this supposition.[6]

[4] Kagel *et al.* (1987) report similar results. Earlier reports of convergence to the dominant bidding strategy in SPA (Cox *et al.* 1982) employed procedures which prohibited bidding above valuations. See Kagel (1992) for reviews of these and other earlier studies of SPA.

[5] As the regression analysis reported below shows, bidding above the dominant strategy in SPA is independent of the private valuation drawn, so that it cannot be attributed to rivalistic behaviour on the part of players with low valuations. Learning to play the dominant strategy is quite limited as only 2 out of 21 subjects play it precisely for 5 or more consecutive periods starting from the end of an auction series. Further, there is no evidence of one shot learning with bidders adopting the dominant strategy (or bidding within \$0.05 of it) for the remainder of an auction session following losses.

[6] In English auctions, which are isomorphic to SPA, bidders lock into the dominant bidding strategy after a few periods of overbidding (Kagel *et al.* 1987). We attribute these differences in bidding between the two auctions to what psychologists refer to as response mode effects and the fact that bidding above valuation, and winning, in the English auction, necessarily involves losses (unlike the SPA) (Kagel, 1992). These results also rule out utility of winning as an explanation for bidding above the dominant strategy in SPA, unless one argues that there is greater utility of winning in sealed bid auctions.

Table 2
Auction outcomes
(Theoretical predictions provided in brackets)

Auction series	Bidding frequencies relative to X (percent of bids) $N=5$	Bidding frequencies relative to X (percent of bids) $N=10$	Regression results (standard errors) Intercept	x_{it}	Dx_{it}	R^2	Bidding frequencies relative to RNNE§ (percent of bids) $N=5$	Bidding frequencies relative to RNNE§ (percent of bids) $N=10$
	[< x]	[< x]		[< 1·0]	[> 0·0]		[> RNNE]	[> RNNE]
	< 92·1	89·2	1·14**	0·92 + +	0·03*	0·93	< 20·4	35·8
	= 7·5‡	10·8	(0·24)	(0·02)	(0·02)		= 1·7‡	3·3
1	> 0·4	0·0					> 77·9	60·8
	[= x]	[= x]		[= 1·0]	[= 0·0]		[= RNNE]	[= RNNE]
2·1	< 3·8	3·3	0·72**	1·00	−0·01	0·98	< 3·8	3·3
	= 32·5‡	45·0	(0·12)	(0·01)	(0·01)		= 32·5‡	45·0
	> 63·8	51·7					> 63·8	51·7
2·2	< 7·8	15·8	1·38**	1·04	−0·06*	0·86	< 7·8	15·8
	= 21·3‡	20·0	(0·37)	(0·02)	(0·02)		= 21·3‡	20·0
	> 70·9	64·2					> 70·9	64·2
2	< 5·7	9·6	1·08**	1·02	−0·03**	0·92	< 5·7	9·6
(pooled)	= 27·0‡	32·5	(0·19)	(0·01)	(0·01)		= 27·0‡	32·5
	> 67·2	57·9					> 67·2	57·9
	[> x]	[> x]		[> 1·0]	[< 0·0]		[< RNNE]	[< RNNE]
3·1	< 6·2	7·8	0·54	1·13 + +	−0·02	0·91	< 78·6	44·4
	= 11·4‡	14·4	(0·35)	(0·02)	(0·02)		= 0·0‡	3·3
	> 82·4	77·8					> 21·4	52·2
3·2	< 5·2	8·3	−1·08*	1·35 + +	−0·10**	0·81	< 69·5	57·5
	= 4·3‡	7·5	(0·50)	(0·04)	(0·03)		= 2·4‡	1·7
	> 90·5	84·2					> 28·1	40·8
3·3	< 4·3	4·2	4·95**	1·16 + +	−0·03	0·79	< 38·3	14·2
	= 0·4‡	0·8	(0·54)	(0·03)	(0·04)		= 1·3‡	0·8
	> 95·2	95·0					> 60·4	85·0
3·4	< 3·0	5·8	3·25**	1·09 + +	−0·05*	0·87	< 54·0	18·7
	= 2·0‡	1·9	(0·31)	(0·02)	(0·02)		= 0·5‡	1·6
	> 95·0	92·3					> 45·5	79·7
3·5†	< 0·0	0·7	4·10**	0·96	−0·02	0·90	< 72·2	51·7
	= 18·7‡	24·7	(0·28)	(0·02)	(0·02)		= 0·0‡	2·4
	> 81·3	74·7					> 27·8	45·8
3	< 4·2	5·0	2·44**	1·15 + +	−0·06**	0·84	< 61·1	38·4
(pooled)	= 6·1‡	10·4	(0·19)	(0·01)	(0·01)		= 0·9‡	1·9
	> 89·8	84·6					> 38·0	59·7

† Small markets with $N = 4$ and $N = 5$ and large market with $N = 9$.
‡ $|B(x) - x| \leqslant \$0.05$.
§ Theoretical predictions are for risk averse bidders.
* Significantly different from 0 at the 5 % level; ** Significantly different from 0 at the 1 % level.
\+ Significantly different from 1·0 at the 5 % level; + + Significantly different from 1·0 at the 1 % level.

The second column in Table 2 reports the results of fixed effect regression models

$$B_{it}(x) = a_1 + a_2 x_{it} + a_3 Dx_{it} + s_i + e_{it}, \tag{5}$$

where $B_{it}(x)$ and x_{it} are the bid and private value of subject i in auction period t, Dx_{it} is a slope dummy variable which is equal to x_{it} when $N = 10$ and zero otherwise, s_i is a subject specific dummy variable (the sum of the individual subject dummy variables was restricted to sum to zero), and e_{it} is an error term with the usual properties. In estimating these regression equations our primary concern is with the relationship of the slope coefficients (a_2 and a_3) to the predictions of the different bid-price rules.[7]

The slope coefficients (a_2) line up as predicted across bid-price rules, with a_2 significantly less than 1 in the FPA and significantly greater than 1 in four of the five TPA (the exception is auction series 3·5). Further, we are unable to reject, at conventional significance levels, the hypothesis that $a_2 = 1$ in SPA. The fact that the slope coefficient is not significantly different from one in SPA indicates that bidding above the equilibrium (dominant) strategy is largely accounted for by bidders adding a fixed markup to their valuations. F-tests of the null hypothesis that $a_2 + a_3 = 1$ show that the slope restrictions of the different bid-price rules are satisfied in auctions with 10 bidders as well.

The sign and statistical significance of coefficient a_3 in the regressions provides one measure of the impact of increasing numbers of bidders. In FPA, as anticipated, a_3 is positive, and significantly different from zero, meaning that subjects bid more when faced with more rivals. In SPA, there is a statistically significant decrease in bids in auction series 2·2 that carries over to the pooled data analysis. This response is not anticipated *in equilibrium* in SPA. However, given that subjects are *out of equilibrium* and bidding above x, increasing the number of bidders initiates forces similar to those resulting from increasing N in TPA. Finally, as predicted, coefficient values for a_3 are negative in all five TPA series, achieving statistical significance in series 3·2, 3·4 and in the pooled data regression.

The dual market technique provides an alternative means of examining the effect of increasing numbers of bidders under the different bid-price rules, in this case through looking at simple differences in bids as N varies. Table 3 provides the relevant data, with the first half of the table reporting mean differences computed over all valuations, and the second half restricting these calculations to valuations lying in the top half of the distribution (while Nash equilibrium bidding theory is relevant to all valuations, previous experiments find a tendency for bidders with low valuations to be less than fully responsive to obvious comparative static manipulations). In the FPA series, all 10 bidders increased their bids in auctions with 10 compared to 5 bidders. The appropriate response here is clear and relatively unambiguous, and has been replicated in a number of other FPA experiments as well (reviewed in Kagel, 1992).

In auction series 2·1, a majority of subjects did not change their bids by more than \$0·05, on average, in response to more rivals, with a few subjects (2–3) mistakenly increasing their bids. χ^2 tests show the differences in individual subject response frequencies between FPA and SPA to be significant at better than the 1 % level, although cell sizes are too thin to have full confidence in the χ^2 distribution employed.[8]

[7] Restricting the sum of the subject dummy variables to 0·0 results in an intercept term that can be interpreted as the average fixed markup (markdown) in bids relative to value. However, estimates of these markup effects are quite sensitive to the data employed. Restricting the analysis to private values drawn form the top half of the value distribution, the intercept term loses its significance in the FPA and in all TPA except series 3.2. In contrast, the slope coefficients are robust to the data employed. F tests for differences in subject intercept terms invariably prove highly significant.

[8] $\chi^2 = 13·3$ (full data set) and 10·8 (restricted data set) with 2 degrees of freedom.

Table 3

Effects of Increasing Numbers of Bidders: Dual Market Results

	All private valuations				Private valuations in the interval [$14.15, $28·30]			
	Changes in average bids by individual subjects			Average dollar change in bids	Changes in average bids by individual subjects			Average dollar change in bids
Auction series	Increased	Decreased	No change†	(standard error mean)	Increased	Decreased	No change†	(standard error)
1	10	0	0	0·65** (0·11)	10	0	0	1·22** (0·12)
2·1	2	3	5	−0·04 (0·08)	3	1	6	0·02 (0·04)
3·1	1	6	3	−0·49 (0·25)	0	7	3	−0·98* (0·51)
3·2	4	6	0	−0·89* (0·42)	5	5	0	−1·15 (0·65)
3·4	6	3	1	−0·14 (0·28)	4	5	1	−0·91* (0·38)
3·5	5	3	1	−0·01 (0·35)	2	6	1	−0·38 (0·32)
3 (pooled)	16	18	5	−0·40* (0·17)	11	23	5	−0·86** (0·26)

† Average absolute difference is less than or equal to $0·05.
* Significantly different from 0 at 5% level; ** Significantly different from 0 at 1% level.

In TPA, 46% of all subjects decreased their bids, on average, when N increased, for means calculated over all valuations. A clear majority, 59% decreased their bids when the analysis is restricted to valuations drawn from the upper half of the distribution. In both cases, a sizable minority of bidders mistakenly increased their bids when N increased. Pooling over the TPA and conducting χ^2 tests show that we can reject the null hypothesis of no difference from FPA and SPA at better than the 5% level, although here, too, cell sizes are thinner than desirable.[9] Thus, we conclude that, taken as a whole, the comparative static predictions regarding the effect of increasing numbers of bidders are satisfied in the data.

The last column of Table 2 compares bids relative to the RNNE prediction. In the FPA bids were generally above the RNNE prediction, the common result for FPA, although there is an increased tendency for bids to lie below the RNNE with 10 bidders. Bidding above the RNNE in FPA is consistent with risk averse Nash equilibrium bidding. In TPA, the majority of bids lie below the RNNE in auctions with 5 bidders, again consistent with the predictions of risk averse Nash equilibrium bidding. However, for auctions with 10 bidders, a clear majority of bids (close to 60%) lie above the RNNE prediction,

[9] $\chi^2 = 11{\cdot}1$ (full data set) and 16·8 (restricted data set) for FPA compared to TPA, and $\chi^2 = 6{\cdot}8$ (full data set) and 11·8 (restricted data set) for SPA compared to TPA (2 degrees of freedom in each case).

behaviour which is inconsistent with risk aversion.[10] This, along with bidding above the dominant strategy in SPA, suggests that risk aversion alone, as specified in conventional expected utility theory, cannot explain the persistent deviations from RNNE bidding reported in FPA.

The simplest explanation for the breakdown of risk aversion with $N = 10$ is that the overbidding in SPA is at work in TPA, as well. To test for this possibility we computed the average overbid in SPA with 10 bidders, subtracted this from bids in TPA, and checked to see if this adjustment would push bids below the RNNE prediction. Results from this exercise show that 51 % of all bids continue to exceed the RNNE. However, the results still stand in marked contrast to auctions with 5 bidders, where a comparable adjustment shows 74% of all bids below the RNNE prediction. This indicates that something more than the overbidding found in SPA is responsible for bidding above the RNNE in TPA with 10 bidders. Looking at equations (3) and (4), it is clear that the differences between risk neutral and risk averse bidding decrease with increases in N in TPA. Thus, assuming some bidding errors like those found in SPA, and some genuine risk aversion on bidders' part, we would expect to find greater frequencies of bidding above the RNNE prediction with 10 bidders, since for any given degree of absolute risk aversion, the difference between risk neutral and risk averse bids is decreasing in N.[11]

The IPV model makes market level predictions about expected revenue: (1) with more bidders expected revenue increases, (2) if bidders are risk neutral expected revenue is the same under the different bid price rules (the Revenue Equivalence Theorem), and (3) if bidders are risk averse the expected revenue ordering is $R_1 > R_2 > R_3$. The first of these predictions is readily satisfied under all three bid-price rules, with average revenue 12–30 % higher in auctions with 10 bidders. In auctions with 5 bidders, average revenues were significantly greater in the FPA than in SPA, and significantly greater in SPA compared to TPA, consistent with risk aversion. In contrast, in auctions with 10 bidders there are no significant differences in average revenue under the different bid price rules, consistent with risk neutrality.

IV. SUMMARY AND CONCLUSIONS

We have looked at individual bidder behaviour and revenue in auctions with 5 and 10 bidders under FPA, SPA and TPA bid rules. Comparing the data with the theory, there is no question regarding the existence of discrepancies between the two. These include persistent deviations from the dominant bidding strategy in SPA and the minority of subjects who increased bids with increased competition in SPA and TPA. Nevertheless, Nash equilibrium bidding theory seems to capture the main strategic forces underlying behaviour,

[10] Similar results are obtained when restricting the analysis to private values in the top half of the distribution.

[11] Similar factors may underlie the reduced frequency of bidding above the RNNE in FPA with 10 bidders. In this case subjects' valuations provide an upper bound on the amount of any overbid, with this restriction becoming more relevant as N goes from 5 to 10 bidders.

as the comparative static predictions of the theory regarding the effects of changes in bid-price rules and numbers of bidders, are satisfied.

These successful comparative static predictions of private value auction theory stand in marked contrast to results from common value auction experiments which exhibit strong traces of the winner's curse, which in turn leads to breakdowns in several key comparative static predictions (Dyer *et al.* 1989 and references cited therein). In common value auctions, bidders do not know the value of the item at the time they bid. Instead they receive private information signals which are affiliated with the item value, which introduces a potential adverse selection problem in addition to the strategic problems inherent in auctions. Statistical inference problems are minimised in private value auctions, leaving bidders to cope primarily with strategic considerations. The fact that they do so, even under the completely novel and counterintuitive conditions associated with TPA, suggests that bidders are indeed responding correctly to the strategic forces inherent in private value auctions rather than following some simple, *ad hoc*, bidding rule that coincides with the game theoretic predictions in FPA.

Bidding above the equilibrium price in SPA provides clear evidence of a bidding error in private value auctions, but the amount of overbidding is limited in scope. As a consequence, it has a limited impact on bidders' profits and predicted differences across bid-price rules. Further, even though the risk aversion hypothesis is not capable of organising all of the data from TPA, the nature of the deviations suggest that it captures some elements of behaviour, as bidding errors like those promoting overbidding in SPA, in conjunction with some genuine risk aversion on bidders' part, will produce greater frequencies of bidding above the RNNE prediction with more bidders.

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Appendix

We assume that there are N bidders, each bidder has a private value x_i, and the private values are affiliated (see Milgrom and Weber, 1982 for details). (Note that independent private values are a special case of affiliation.) Each bidder has the same (weakly) concave utility function $U(\cdot)$ with $U(0) = 0$.

Let X_1 be the highest private value among N bidders and let Y_j be the jth highest private value among $N-1$ rivals.

Assumption. There exists a continuous, monotonic function $B_k(z)$, $2 \leqslant k \leqslant N$, such that for every $x \in \operatorname{supp} X$ it solves

$$\mathrm{E}\{U[x - B_k(Y_{k-1})] \mid X_1 = x, Y_1 = x\} = 0. \tag{A 1}$$

Note that for $k = 2$, $Y_{k-1} = Y_1$. Thus, $B_2(x) = x$

THEOREM. *$B_k(x)$ is a symmetric kth price Nash equilibrium.*

Proof. Bidder 1 observes x_0 and considers $\bar{B} > B_k(x_0)$. Bidders $2, \ldots, N$ follow $B_k(x)$. $\bar{B} > B_k(x_0)$ only matters in the event $\Theta := [\bar{B} \geqslant B_k(Y_1) \geqslant B_k(x_0)]$ with one strict inequality. Θ implies that $B_k(Y_1) > B_k(x_0)$ (we ignore the possibility of a tie), and by monotonicity $Y_1 > x_0$. In Θ, using $B_k(x_0)$ bidder 1 does not win, and attains $U(0) = 0$, while with $\bar{B}$ he wins and attains

$$\mathrm{E}\{U[x_0 - B_k(Y_{k-1})] \mid X_1 = x_0, Y_1 > x_0\} < \mathrm{E}\{U[x_0 - B_k(Y_{k-1})] \mid X_1 = x_0, Y_1 = x_0\} = 0.$$

The strict inequality is due to monotonicity of B_k in conjunction with affiliation. Under independent private values the strict inequality holds as well since as Y_1 goes down, the sup (supp $Y_{k-1} \mid Y_1$) goes down as well, even though the densities are not affected. In the same way we can show that bidder 1 does not want to bid $\bar{\bar{B}} < B_k(x_0)$ Q.E.D.

Let
$$g_k(x) := \mathrm{E}(Y_{k-1} \mid X_1 = x, Y_1 = x).$$

LEMMA. *If $g_k(x)$ is linear, then $B_k(x) = g_k^{-1}(\cdot)$ for risk neutral bidders.*

Proof.

$$\begin{aligned}\mathrm{E}\{[x - g_k^{-1}(Y_{k-1})] \mid X_1 = x, Y_1 = x\} &= x - \mathrm{E}[g_k^{-1}(Y_{k-1}) \mid X_1 = x, Y_1 = x] \\ &= x - g_k^{-1}[\mathrm{E}(Y_{k-1} \mid X_1 = x, Y_1 = x)] = x - g_k^{-1}[g_k(x)] = 0.\end{aligned}$$

So $g_k^{-1}(x)$ solves (A 1) in this case and is monotonic.

Applications

Case 1. In the Vickrey set up, with x_i iid from a uniform distribution on $[\underline{x}, \bar{x}]$, one can show that

$$g_k(x) = \mathrm{E}(Y_{k-1} \mid X_1 = x, Y_1 = x) = \underline{x} + \frac{N+1-k}{N-1}(x - \underline{x})$$

which is linear. Thus the risk neutral equilibrium for the kth price auction is

$$B_k(x) = \underline{x} + \frac{N-1}{N+1-k}(x - \underline{x}), \quad 2 \leqslant k \leqslant N.$$

Use $\underline{x} = 0$ and $k = 3$ to derive (3) in the text.

Case 2. Third-price auctions with CARA and x distributed iid uniformly on $[0, \bar{x}]$. With CARA in (A 1),
$$U(\ ) = 1 - e^{-A[x - B_3(Y_2)]},$$
and with
$$x_i \text{ iid on } [0, \bar{x}], f(Y_2 = t \mid X_1 = x, Y_1 = x) = (N-2)\, t^{N-3}/x^{N-2}$$
so that (A 1) becomes:

$$\int_0^x \{1 - e^{-A[x - B_3(t)]}\}\, t^{N-3}\, dt = 0 \quad \text{for all} \quad x \in [0, \bar{x}]. \tag{A 2}$$

With a simple manipulation, condition (A 2) can be rewritten as:

$$\frac{x^{N-2} e^{Ax}}{N-2} = \int_0^x e^{AB_3(t)}\, t^{N-3} dt \quad \text{for all} \quad x \in [0, \bar{x}]. \tag{A 3}$$

For $B_3(t) = t + (1/A) \ln [1 + At/(N-2)]$,

$$\int_0^x e^{AB_3(t)}\, t^{N-3}\, dt = \left.\frac{t^{N-2} e^{At}}{N-2}\right|_0^x = \frac{x^{N-2} e^{Ax}}{N-2}, \quad \text{satisfying (A 3)}$$

for all $x \in [0, \bar{x}]$. Thus $B_3(t) = t + (1/A) \ln [1 + At/(N-2)]$ is the CARA bid function for third-price auctions and is equation (6) in the text.

Now, we show that equation (5) in the text $B_3(x;A) = x+(1/A)\ln[1+Ax/(N-2)]$ is declining in A for all $A > 0$.

$$\partial B_3/\partial A = (1/A^2)\{[Ax/(N-2+Ax)]-\ln[1+Ax/(N-2)]\}.$$

However, the concavity of $\ln(1+z)$ implies

$$\ln[1+Ax/(N-2)] > \ln(1)+[(N-2)/(N-2+Ax)][Ax/(N-2)]$$
$$= Ax/(N-2+Ax)\ (x>0).$$

Thus, we have $\partial B_3/\partial A < 0$ for all $A > 0$.

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