

Scene Graphs & Modeling Transformations

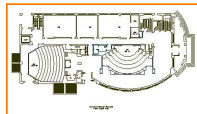
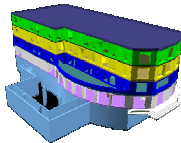
Tom Funkhouser
Princeton University
COS 426, Spring 2007

3D Object Representations

- Points
 - Range image
 - Point cloud
- Surfaces
 - Polygonal mesh
 - Subdivision
 - Parametric
 - Implicit
- Solids
 - Voxels
 - BSP tree
 - CSG
 - Sweep
- High-level structures
 - Scene graph
 - Application specific

3D Object Representations

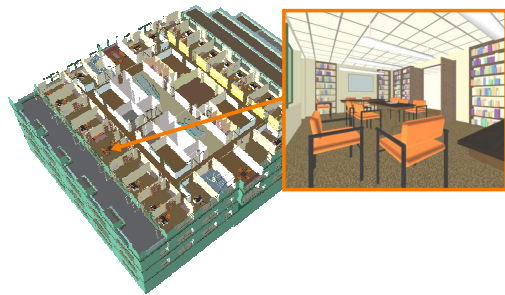
- Desirable properties of an object representation
 - Easy to acquire
 - Accurate
 - Concise
 - Intuitive editing
 - Efficient editing
 - Efficient display
 - Efficient intersections
 - Guaranteed validity
 - Guaranteed smoothness
 - etc.



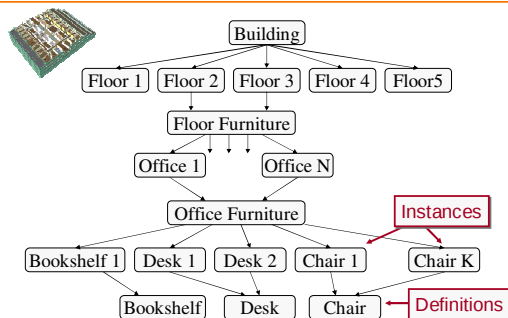
(CS Building, Princeton University)

3D Object Representations

- What object representation is best for this?

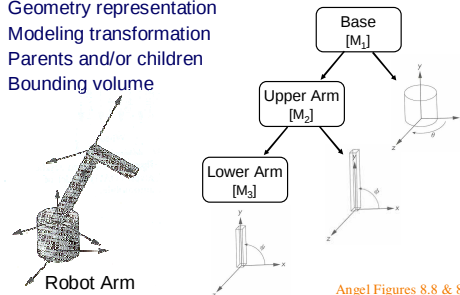


Scene Graphs



Scene Graphs

- Hierarchy of nodes, where each node may have:
 - Geometry representation
 - Modeling transformation
 - Parents and/or children
 - Bounding volume



Angel Figures 8.8 & 8.9

Scene Graphs



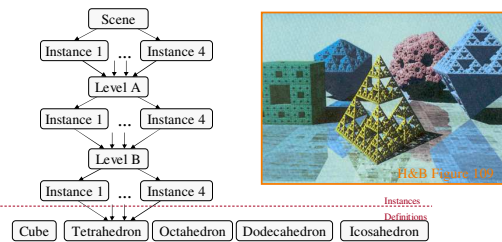
- Advantages
 - Allows definitions of objects in own coordinate systems
 - Allows use of object definition multiple times in a scene



Scene Graphs



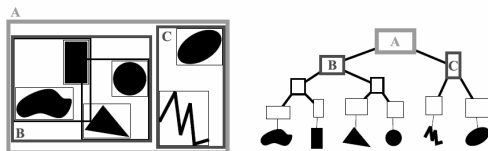
- Advantages
 - Allows definitions of objects in own coordinate systems
 - Allows use of object definition multiple times in a scene



Scene Graphs



- Advantages
 - Allows definitions of objects in own coordinate systems
 - Allows use of object definition multiple times in a scene
 - Allows hierarchical processing (e.g., intersections)

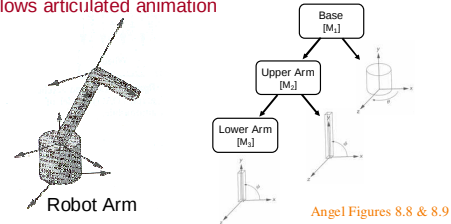


Haverkort

Scene Graphs

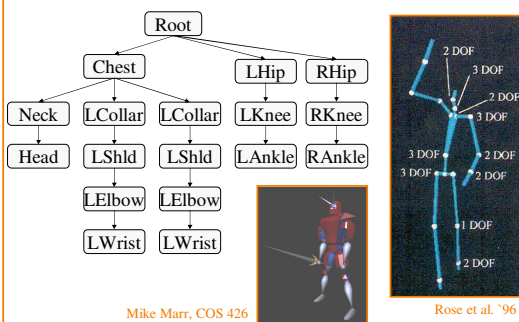


- Advantages
 - Allows definitions of objects in own coordinate systems
 - Allows use of object definition multiple times in a scene
 - Allows hierarchical processing (e.g., intersections)
 - Allows articulated animation



Angel Figures 8.8 & 8.9

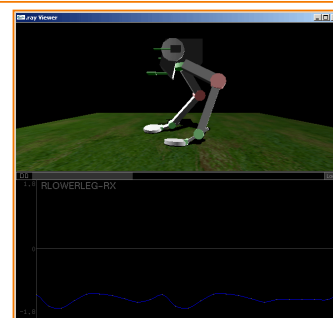
Scene Graph Example



Mike Marr, COS 426

Rose et al. '96

Scene Graph Example



Mihai Parparita, COS 426

Overview



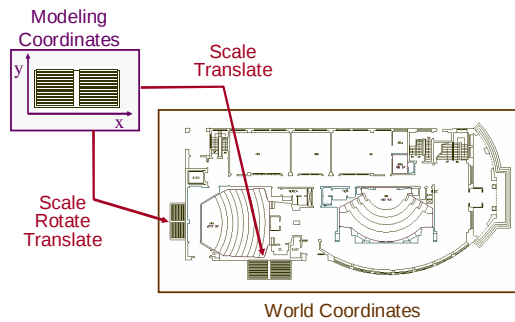
- Scene graphs
 - Geometry & attributes
 - Transformations
 - Bounding volumes
- Transformations
 - Basic 2D transformations
 - Matrix representation
 - Matrix composition
 - 3D transformations

Overview

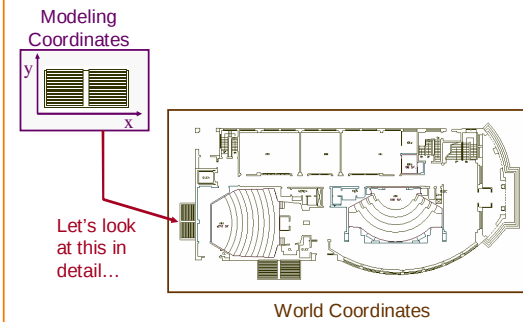


- Scene graphs
 - Geometry & attributes
 - Transformations
 - Bounding volumes
- Transformations
 - Basic 2D transformations
 - Matrix representation
 - Matrix composition
 - 3D transformations

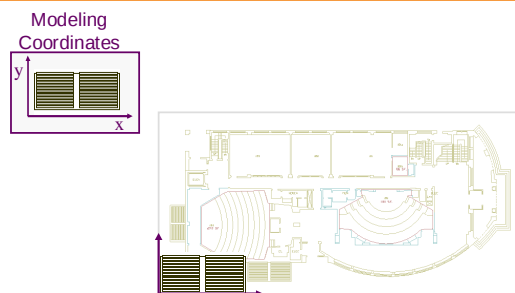
2D Modeling Transformations



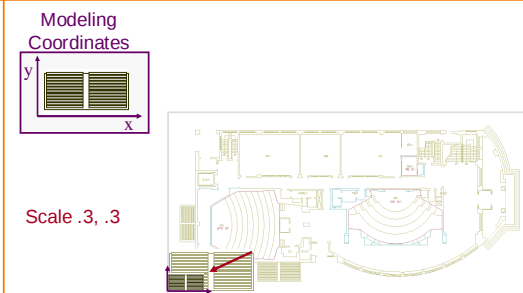
2D Modeling Transformations



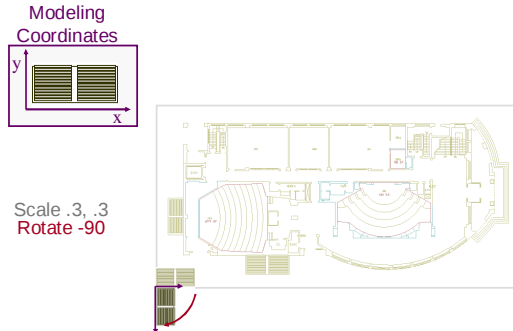
2D Modeling Transformations



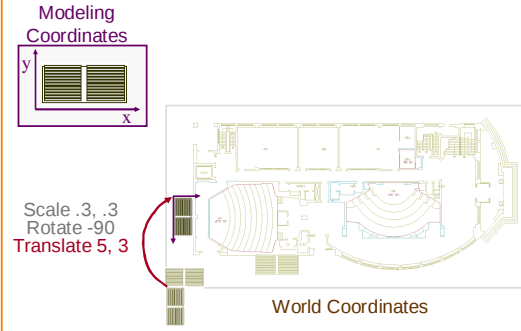
2D Modeling Transformations



2D Modeling Transformations



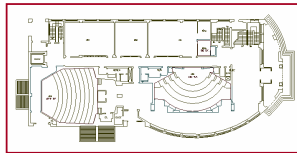
2D Modeling Transformations



Basic 2D Transformations



- Translation:
 - $x' = x + tx$
 - $y' = y + ty$
- Scale:
 - $x' = x * sx$
 - $y' = y * sy$
- Shear:
 - $x' = x + hx*y$
 - $y' = y + hy*x$
- Rotation:
 - $x' = x*cos\theta - y*sin\theta$
 - $y' = x*sin\theta + y*cos\theta$



Transformations
can be combined
(with simple algebra)

Basic 2D Transformations



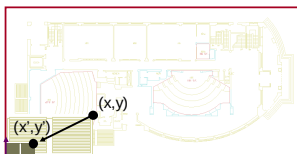
- Translation:
 - $x' = x + tx$
 - $y' = y + ty$
- Scale:
 - $x' = x * sx$
 - $y' = y * sy$
- Shear:
 - $x' = x + hx*y$
 - $y' = y + hy*x$
- Rotation:
 - $x' = x*cos\theta - y*sin\theta$
 - $y' = x*sin\theta + y*cos\theta$



Basic 2D Transformations



- Translation:
 - $x' = x + tx$
 - $y' = y + ty$
- Scale:
 - $x' = x * sx$
 - $y' = y * sy$
- Shear:
 - $x' = x + hx*y$
 - $y' = y + hy*x$
- Rotation:
 - $x' = x*cos\theta - y*sin\theta$
 - $y' = x*sin\theta + y*cos\theta$

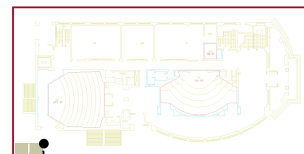


$$\begin{aligned} x' &= x*sx \\ y' &= y*sy \end{aligned}$$

Basic 2D Transformations



- Translation:
 - $x' = x + tx$
 - $y' = y + ty$
- Scale:
 - $x' = x * sx$
 - $y' = y * sy$
- Shear:
 - $x' = x + hx*y$
 - $y' = y + hy*x$
- Rotation:
 - $x' = x*cos\theta - y*sin\theta$
 - $y' = x*sin\theta + y*cos\theta$



$$\begin{aligned} x' &= (x*sx)*cos\theta - (y*sy)*sin\theta \\ y' &= (x*sx)*sin\theta + (y*sy)*cos\theta \end{aligned}$$

Basic 2D Transformations



- Translation:

- $x' = x + tx$
- $y' = y + ty$

- Scale:

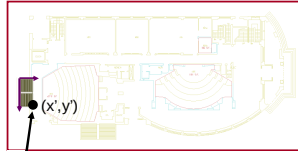
- $x' = x * sx$
- $y' = y * sy$

- Shear:

- $x' = x + hx*y$
- $y' = y + hy*x$

- Rotation:

- $x' = x*cos\theta - y*sin\theta$
- $y' = x*sin\theta + y*cos\theta$



$$\begin{aligned} x' &= ((x*sx)*cos\theta - (y*sy)*sin\theta) + tx \\ y' &= ((x*sx)*sin\theta + (y*sy)*cos\theta) + ty \end{aligned}$$

Basic 2D Transformations



- Translation:

- $x' = x + tx$
- $y' = y + ty$

- Scale:

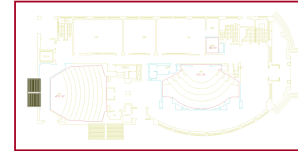
- $x' = x * sx$
- $y' = y * sy$

- Shear:

- $x' = x + hx*y$
- $y' = y + hy*x$

- Rotation:

- $x' = x*cos\theta - y*sin\theta$
- $y' = x*sin\theta + y*cos\theta$



$$\begin{aligned} x' &= ((x*sx)*cos\theta - (y*sy)*sin\theta) + tx \\ y' &= ((x*sx)*sin\theta + (y*sy)*cos\theta) + ty \end{aligned}$$

Overview



- Scene graphs

- Geometry & attributes
- Transformations
- Bounding volumes

- Transformations

- Basic 2D transformations
- Matrix representation
- Matrix composition
- 3D transformations

Matrix Representation



- Represent 2D transformation by a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- Multiply matrix by column vector

⇔ apply transformation to point

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{aligned} x' &= ax + by \\ y' &= cx + dy \end{aligned}$$

Matrix Representation



- Transformations combined by multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Matrices are a convenient and efficient way to represent a sequence of transformations!

2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

2D Identity?

$$\begin{aligned} x' &= x \\ y' &= y \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Scale around (0,0)?

$$\begin{aligned} x' &= sx * x \\ y' &= sy * y \end{aligned}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

2D Rotate around (0,0)?

$$\begin{aligned} x' &= \cos \Theta * x - \sin \Theta * y \\ y' &= \sin \Theta * x + \cos \Theta * y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta \\ \sin \Theta & \cos \Theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Shear?

$$\begin{aligned} x' &= x + shx * y \\ y' &= shy * x + y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & shx \\ shy & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

2D Mirror over Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2D Mirror over (0,0)?

$$\begin{aligned} x' &= -x \\ y' &= -y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

2D Translation?

$$\begin{aligned} x' &= x + tx \\ y' &= y + ty \end{aligned} \quad \text{NO!}$$

Only linear 2D transformations
can be represented with a 2x2 matrix

Linear Transformations



- Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:

- Satisfies: $T(s_1 \mathbf{p}_1 + s_2 \mathbf{p}_2) = s_1 T(\mathbf{p}_1) + s_2 T(\mathbf{p}_2)$
- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

2D Translation



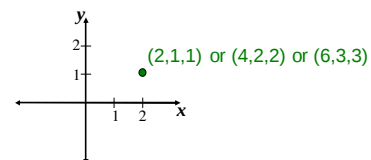
- 2D translation represented by a 3x3 matrix
 - Point represented with *homogeneous coordinates*

$$\begin{aligned} x' &= x + tx \\ y' &= y + ty \end{aligned} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous Coordinates



- Add a 3rd coordinate to every 2D point
 - (x, y, w) represents a point at location $(x/w, y/w)$
 - $(x, y, 0)$ represents a point at infinity
 - $(0, 0, 0)$ is not allowed



Convenient coordinate system to
represent many useful transformations

Basic 2D Transformations



- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rotate

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & shx & 0 \\ shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Shear

Affine Transformations



- Affine transformations are combinations of ...

- Linear transformations, and
- Translations

$$\begin{bmatrix} x' \\ y' \\ w \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

Projective Transformations



- Projective transformations ...

- Affine transformations, and
- Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of projective transformations:

- Origin does not necessarily map to origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Ratios are not preserved (but "cross-ratios" are)
- Closed under composition

Overview



- Scene graphs

- Geometry & attributes
- Transformations
- Bounding volumes

- Transformations

- Basic 2D transformations
- Matrix representation
- Matrix composition
- 3D transformations

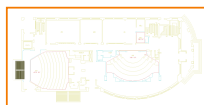
Matrix Composition



- Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{T}(tx, ty) \mathbf{R}(\Theta) \mathbf{S}(sx, sy) \mathbf{p}$$



Matrix Composition



- Matrices are a convenient and efficient way to represent a sequence of transformations

- General purpose representation
- Hardware matrix multiply
- Efficiency with premultiplication
 - » Matrix multiplication is associative

$$\mathbf{p}' = (\mathbf{T} * (\mathbf{R} * (\mathbf{S} * \mathbf{p})))$$

$$\mathbf{p}' = (\mathbf{T} * \mathbf{R} * \mathbf{S}) * \mathbf{p}$$



Matrix Composition



- Be aware: order of transformations matters
» Matrix multiplication is not commutative

$$\mathbf{p}' = \mathbf{T} * \mathbf{R} * \mathbf{S} * \mathbf{p}$$

← "Global"
"Local" →



Matrix Composition

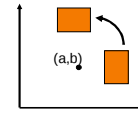


- Rotate by Θ around arbitrary point (a,b)

$$\mathbf{M} = \mathbf{T}(\mathbf{a}, \mathbf{b}) * \mathbf{R}(\Theta) * \mathbf{T}(-\mathbf{a}, -\mathbf{b})$$

The trick:

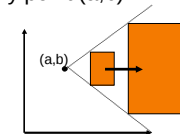
First, translate (a,b) to the origin.
Next, do the rotation about origin.
Finally, translate back.



- Scale by s_x, s_y around arbitrary point (a,b)

$$\mathbf{M} = \mathbf{T}(\mathbf{a}, \mathbf{b}) * \mathbf{S}(s_x, s_y) * \mathbf{T}(-\mathbf{a}, -\mathbf{b})$$

(Use the same trick.)



Overview



- Scene graphs
 - Geometry & attributes
 - Transformations
 - Bounding volumes
- Transformations
 - Basic 2D transformations
 - Matrix representation
 - Matrix composition
 - 3D transformations

3D Transformations



- Same idea as 2D transformations
 - Homogeneous coordinates: (x,y,z,w)
 - 4x4 transformation matrices

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Basic 3D Transformations



$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror over X axis

Basic 3D Transformations



Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 & 0 \\ \sin \Theta & \cos \Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \Theta & 0 & -\sin \Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \Theta & 0 & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \Theta & -\sin \Theta & 0 \\ 0 & \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Summary



- Scene graphs
 - Hierarchical
 - Modeling transformations
 - Bounding volumes
- Coordinate systems
 - World coordinates
 - Modeling coordinates
- 3D modeling transformations
 - Represent most transformations by 4x4 matrices
 - Composite with matrix multiplication (order matters)