Mesh Representations & Subdivision Surfaces

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Princeton University
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3D Object Representations

- Raw data
  - Voxels
  - Point cloud
  - Range image
  - Polygons
- Solids
  - Octree
  - BSP tree
  - CSG
  - Sweep
- Surfaces
  - Mesh
  - Subdivision
  - Parametric
  - Implicit
- High-level structures
  - Scene graph
  - Application specific

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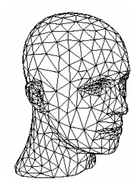
Polygon Meshes

- How should we represent a mesh in a computer?
  - Efficient traversal of topology
  - Efficient use of memory
  - Efficient updates
- Mesh Representations
  - Independent faces
  - Vertex and face tables
  - Adjacency lists
  - Winged-Edge
  - Half-Edge
  - etc.



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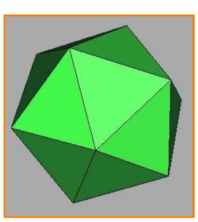
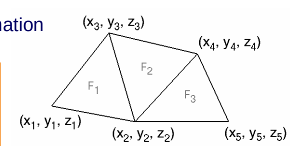
Independent Faces

- Each face lists vertex coordinates
  - Redundant vertices
  - No adjacency information

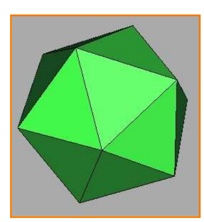


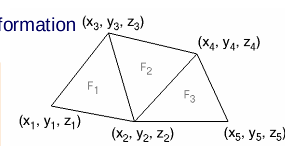


FACE TABLE			
F_1	(x_1, y_1, z_1)	(x_2, y_2, z_2)	(x_3, y_3, z_3)
F_2	(x_2, y_2, z_2)	(x_4, y_4, z_4)	(x_3, y_3, z_3)
F_3	(x_2, y_2, z_2)	(x_5, y_5, z_5)	(x_4, y_4, z_4)

Vertex and Face Tables

- Each face lists vertex references
  - Shared vertices
  - Still no adjacency information



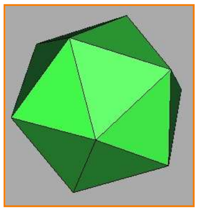


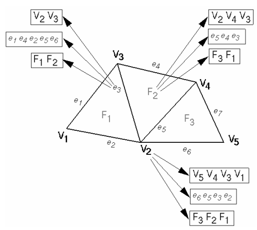
VERTEX TABLE			
V_1	X_1	Y_1	Z_1
V_2	X_2	Y_2	Z_2
V_3	X_3	Y_3	Z_3
V_4	X_4	Y_4	Z_4
V_5	X_5	Y_5	Z_5

FACE TABLE			
F_1	V_1	V_2	V_3
F_2	V_2	V_4	V_3
F_3	V_2	V_5	V_4

Adjacency Lists

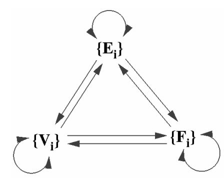
- Store all vertex, edge, and face adjacencies
  - Efficient adjacency traversal
  - Extra storage

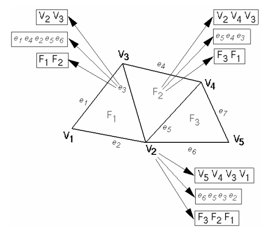




Partial Adjacency Lists

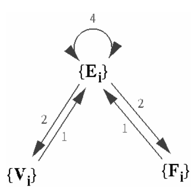
- Can we store only some adjacency relationships and derive others?

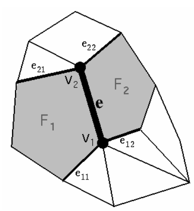




Winged Edge

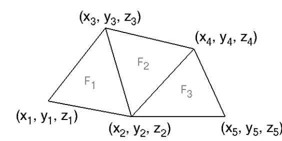
- Adjacency encoded in edges
  - All adjacencies in O(1) time
  - Little extra storage (fixed records)
  - Arbitrary polygons





Winged Edge

- Example:



VERTEX TABLE

V_1	X_1	Y_1	Z_1	e_1
V_2	X_2	Y_2	Z_2	e_6
V_3	X_3	Y_3	Z_3	e_3
V_4	X_4	Y_4	Z_4	e_5
V_5	X_5	Y_5	Z_5	e_6

EDGE TABLE

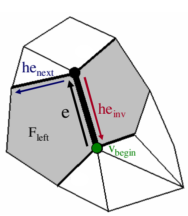
					11	12	21	22
e_1	V_1	V_3		F_1	e_2	e_2	e_4	e_3
e_2	V_1	V_2	F_1		e_1	e_1	e_3	e_6
e_3	V_2	V_3	F_1	F_2	e_2	e_5	e_1	e_4
e_4	V3	V_4		F_2	e_1	e_3	e_7	e_5
e_5	V_2	V_4	F_2	F_3	e_3	e_6	e_4	e_7
e_6	V_2	V_5	F_3		e_5	e_2	e_7	e_7
e_7	V_4	V_5		F_3	e_4	e_5	e_6	e_6

FACE TABLE

F_1	e_1
F_2	e_3
F_3	e_5

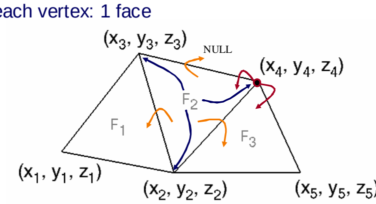
Half Edge

- Adjacency encoded in edges
  - All adjacencies in O(1) time
  - Little extra storage (fixed records)
  - Arbitrary polygons
- Similar to winged-edge, except adjacency encoded in half-edges



Simple Triangle Mesh

- Do not store edges at all
  - All faces have 3 vertices and 3 neighbors
- Store adjacency in vertices and faces
  - For each face: 3 vertices and 3 faces
  - For each vertex: 1 face



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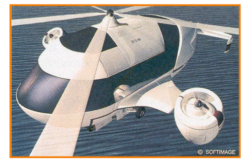
Surfaces

- What makes a good surface representation?
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections



H&B Figure 10.46

Surfaces

- What makes a good surface representation?
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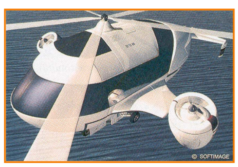

H&B Figure 10.46

Subdivision

- How do you make a smooth curve?

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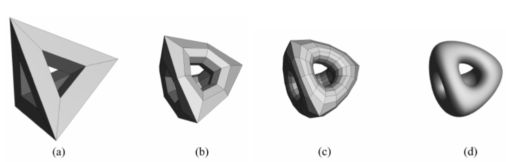
Subdivision Surfaces

- Coarse mesh & subdivision rule
  - Define smooth surface as limit of sequence of refinements



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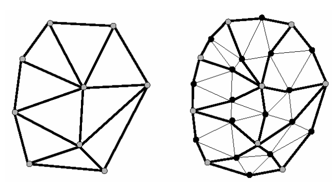
Key Questions

- How refine mesh?
  - Aim for properties like smoothness
- How store mesh?
  - Aim for efficiency for implementing subdivision rules



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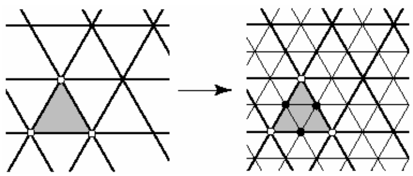
Loop Subdivision Scheme

- How refine mesh?
  - Refine each triangle into 4 triangles by splitting each edge and connecting new vertices
  - Need rules for “even / odd” (white / black) vertices



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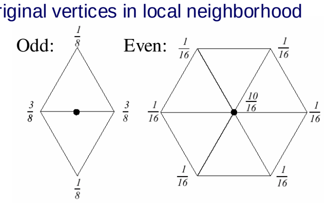
Loop Subdivision Scheme

- How position new vertices?
  - Choose locations for new vertices as weighted average of original vertices in local neighborhood



What if odd vertex only touches one triangle?

What if even vertex does not have degree 6?

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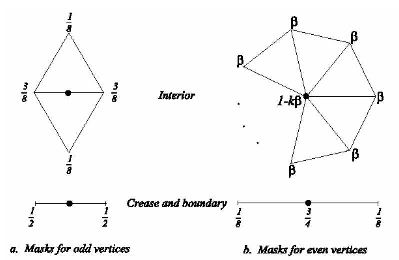
Loop Subdivision Scheme

- Rules for *extraordinary vertices* and *boundaries*:



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Loop

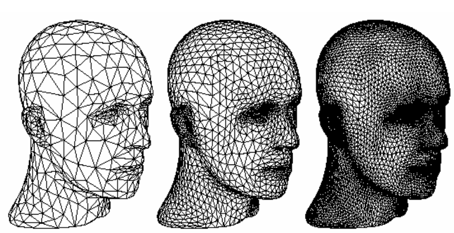
- How to choose β?
  - Analyze properties of limit surface
  - Interested in continuity of surface and smoothness
  - Involves calculating eigenvalues of matrices
    - » Original Loop

$$\beta = \frac{1}{n}\left(\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right)$$

    - » Warren

$$\beta = \begin{cases} \frac{3}{8n} & n > 3 \\ \frac{3}{16} & n = 3 \end{cases}$$

Loop Subdivision Scheme



Limit surface has provable smoothness properties!

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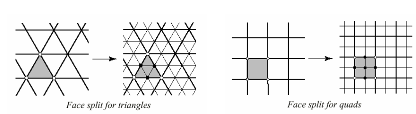
Subdivision Schemes

- There are different subdivision schemes
  - Different methods for refining topology
  - Different rules for positioning vertices
    - » Interpolating versus approximating



	Face split	
	Triangular meshes	*Quad. meshes*
Approximating	Loop (C^2)	Catmull-Clark (C^2)
Interpolating	Mod. Butterfly (C^1)	Kobbelt (C^1)

Vertex split
Doo-Sabin, Midedge (C^1)
Biquartic (C^2)

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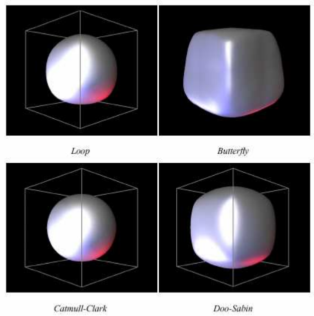
Subdivision Schemes



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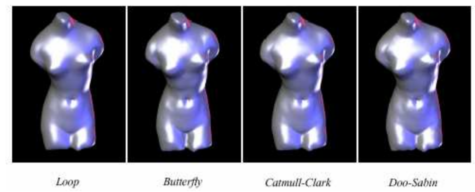
Subdivision Schemes



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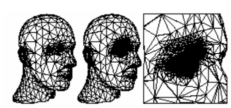
Subdivision Surfaces

- Properties:
  - Accurate
  - Concise
  - Intuitive specification
  - Local support
  - Affine invariant
  - Arbitrary topology
  - Guaranteed continuity
  - Natural parameterization
  - Efficient display
  - Efficient intersections

Pixar

Subdivision Surfaces

- Advantages:
  - Simple method for describing complex surfaces
  - Relatively easy to implement
  - Arbitrary topology
  - Local support
  - Guaranteed continuity
  - Multiresolution



- Difficulties:
  - Intuitive specification
  - Parameterization
  - Intersections

Summary

Feature	Polygonal Mesh	Subdivision Surface
Accurate	No	Yes
Concise	No	Yes
Intuitive specification	No	No
Local support	Yes	Yes
Affine invariant	Yes	Yes
Arbitrary topology	Yes	Yes
Guaranteed continuity	No	Yes
Natural parameterization	No	No
Efficient display	Yes	Yes
Efficient intersections	No	No