



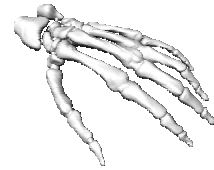
Solid Modeling

Tom Funkhouser
Princeton University
COS 426, Spring 2007



3D Object Representations

- Desirable properties of an object representation
 - Easy to acquire
 - Accurate
 - Concise
 - Intuitive editing
 - Efficient editing
 - Efficient display
 - Efficient intersections
 - Guaranteed validity
 - Guaranteed smoothness
 - etc.

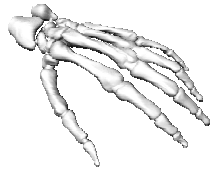


Large Geometric Model Repository
Georgia Tech



3D Object Representations

- Properties of surface representations
 - Easy to acquire
 - Accurate
 - Concise
 - Intuitive editing
 - Efficient editing
 - Efficient display
 - ~~Efficient intersections~~
 - ~~Guaranteed validity~~
 - Guaranteed smoothness
 - etc.



Large Geometric Model Repository
Georgia Tech



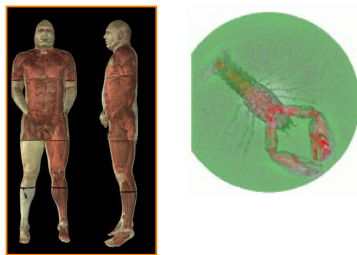
3D Object Representations

- Points
 - Range image
 - Point cloud
- Surfaces
 - Polygonal mesh
 - Subdivision
 - Parametric
 - Implicit
- Solids
 - Voxels
 - BSP tree
 - CSG
 - Sweep
- High-level structures
 - Scene graph
 - Application specific



Solid Modeling

- Represent solid interiors of objects
 - Surface may not be described explicitly

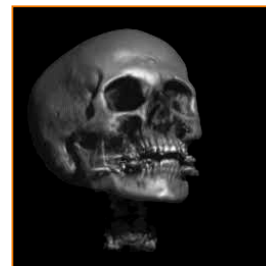


Visible Human
(National Library of Medicine)



Motivation 1

- Some acquisition methods generate solids
 - Example: CAT scan



Stanford University

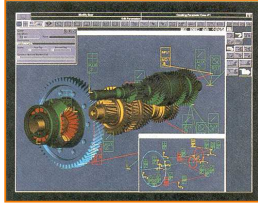
Motivation 2



- Some applications require solids
 - Examples: medicine, CAD/CAM



SUNY Stony Brook

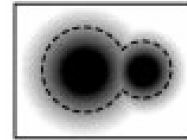


Intergraph Corporation

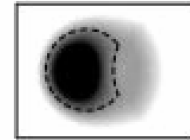
Motivation 3



- Some operations are easier with solids
 - Example: union, difference, intersection



Union



Difference

Bloomenthal

Solid Modeling Representations



- Voxels
- Quadtrees & Octrees
- KD and BSP Trees
- Constructive solid geometry

Solid Modeling Representations

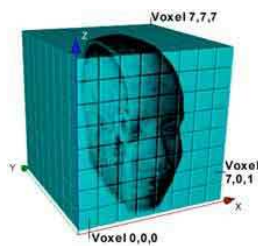


- Voxels
- Quadtrees & Octrees
- KD and BSP Trees
- Constructive solid geometry

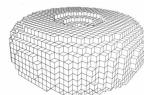
Voxels



- Regular array of 3D samples (like image)
 - Samples are called *voxels* (like pixels)



www.volumegraphics.com

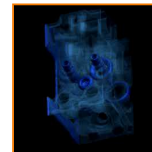


FvDFH Figure 12.20

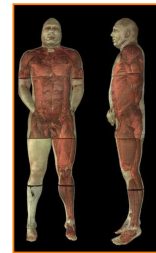
Voxels



- Store properties of solid object with each voxel
 - Occupancy
 - Color
 - Density
 - Temperature
 - etc.

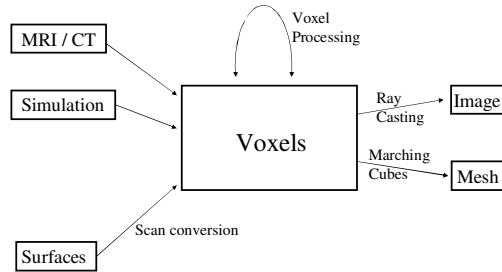


Engine Block
Stanford University



Visible Human
(National Library of Medicine)

Voxels

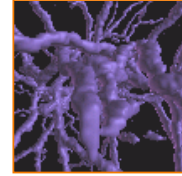


Voxel Acquisition

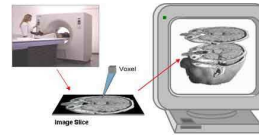


- Scanning devices

- MRI
- fMRI
- CAT
- etc.



SUNY Stony Brook



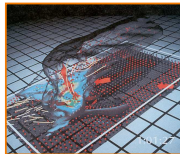
www.volumegraphics.com

Voxel Acquisition

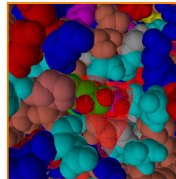


- Simulation

- Models
- FEM
- etc.



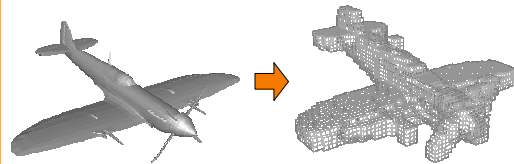
Airflow Inside a Thunderstorm
(Bob Williamson, University of Illinois at Urbana-Champaign)



Voxel Acquisition



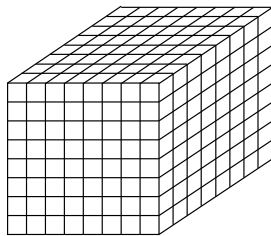
- Voxelization of surfaces



Voxel Storage



- $O(n^3)$ storage for $n \times n \times n$ grid
 - 1 billion voxels for $1000 \times 1000 \times 1000$



Voxel Processing

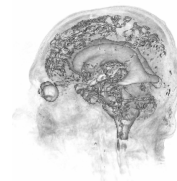


- Signal processing (just like images)

- Reconstruction
- Resampling

- Typical operations

- Blur
- Edge detect
- Warp
- etc.

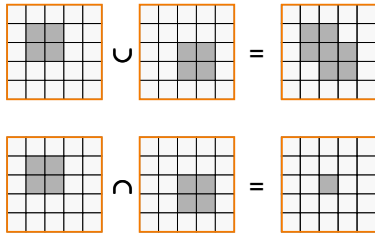


www.volumegraphics.com

Voxel Boolean Operations



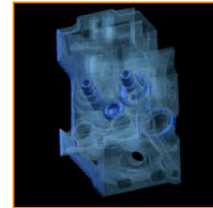
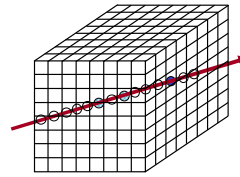
- Compare objects voxel by voxel
 - Trivial



Voxel Display



- Ray casting
 - Integrate density along rays through pixels

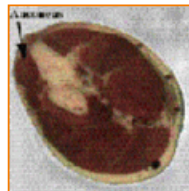
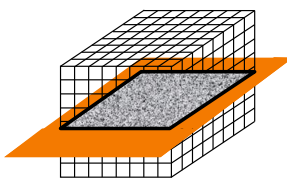


Engine Block
Stanford University

Voxel Display



- Slicing
 - Draw 2D image resulting from intersecting voxels with a plane



Visible Human
(National Library of Medicine)

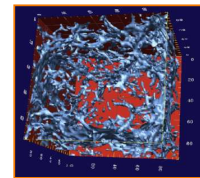
Voxel Display



- Isosurface rendering
 - Render surfaces bounding volumetric regions of constant value (e.g., density)



SUNY Stony Brook

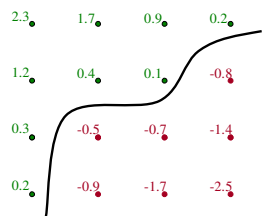


Princeton University

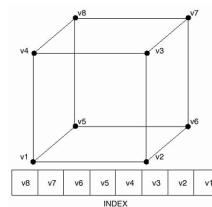
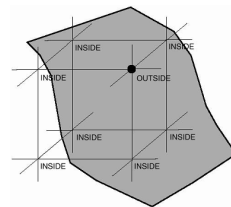
Voxel Display



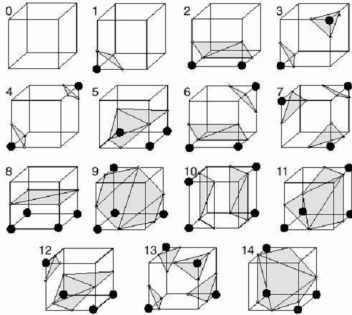
- Isosurface rendering
 - Interpolate samples stored on regular grid
 - Isosurface at $f(x,y,z) = 0$ defines surface



Marching Cubes



Marching Cubes (15 Cases)



Voxels

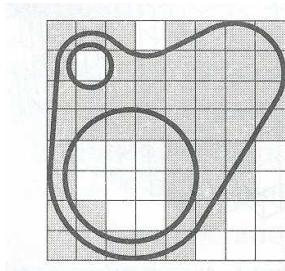


- Advantages
 - Simple, intuitive, unambiguous
 - Same complexity for all objects
 - Natural acquisition for some applications
 - Trivial boolean operations
- Disadvantages
 - Approximate
 - Not affine invariant
 - Expensive display
 - Large storage requirements

Voxels



- What resolution should be used?



FvDFH Figure 12.21

Solid Modeling Representations

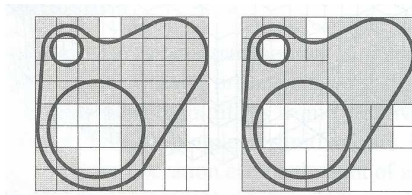


- Voxels
- Quadrees & Octrees
- KD and BSP Trees
- Constructive solid geometry

Quadrees & Octrees



- Refine resolution of voxels hierarchically
 - More concise and efficient for non-uniform objects



Uniform Voxels

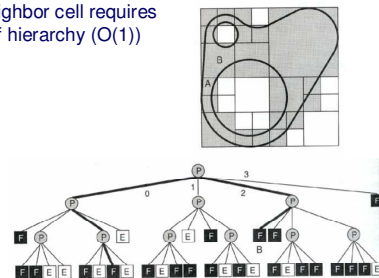
Quadtree

FvDFH Figure 12.21

Quadtree Processing

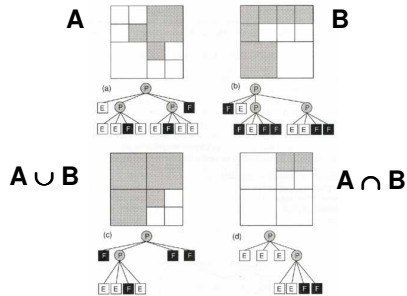


- Hierarchical versions of voxel methods
 - Finding neighbor cell requires traversal of hierarchy ($O(1)$)



FvDFH Figure 12.25

Quadtree Boolean Operations

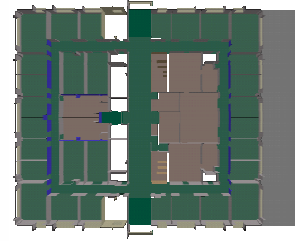


FvDFH Figure 12.24

Quadtrees



- How concise/accurate are they?



Solid Modeling Representations

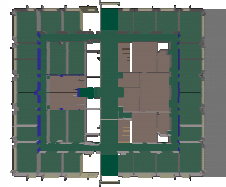


- Voxels
- Quadtrees & Octrees
- KD and BSP Trees**
- Constructive solid geometry

KD Trees



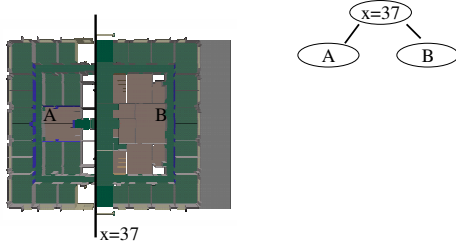
- Recursive binary partition of space by axis aligned planes



KD Trees



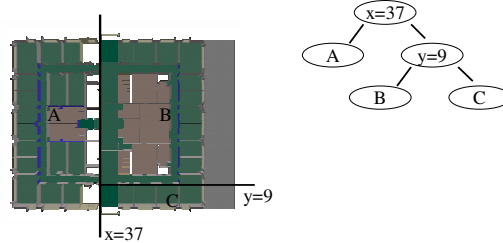
- Recursive binary partition of space by axis aligned planes



KD Trees



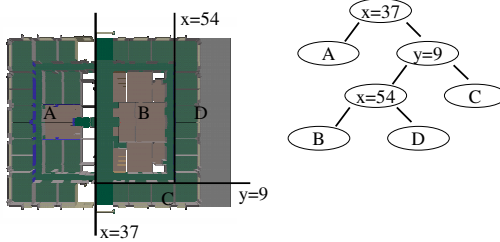
- Recursive binary partition of space by axis aligned planes



KD Trees



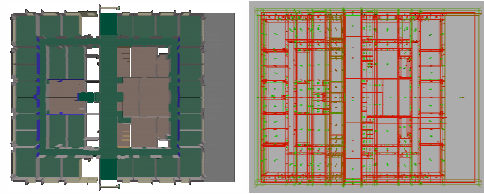
- Recursive binary partition of space by axis aligned planes



KD Trees



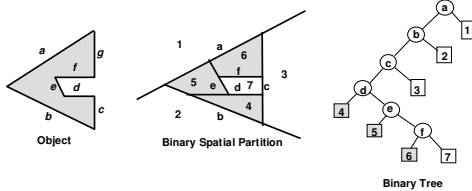
- Recursive binary partition of space by axis aligned planes



BSP Trees



- Recursive binary partition of space by arbitrary planes

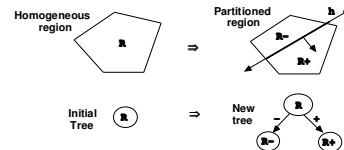


Naylor

BSP Trees



- Single geometric operation
 - Partition a convex region by a hyperplane
- Single combinatorial operation
 - Two child nodes added as leaf nodes

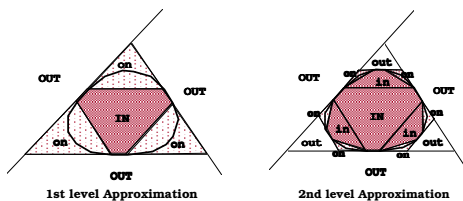


Naylor

BSP Trees



- Processing
 - Exploit hierarchy of convex regions
 - Exploit visibility ordering (later)

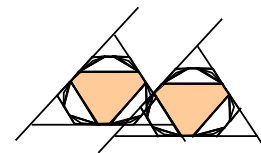


Naylor

BSP Trees



- Processing
 - Exploit hierarchy of convex regions
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Naylor

Solid Modeling Representations

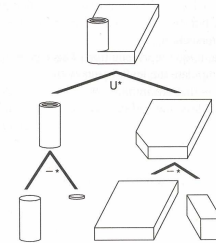


- Voxels
- Quadtrees & Octrees
- KD and BSP Trees
- **Constructive solid geometry**

Constructive Solid Geometry (CSG)



- Represent solid object as hierarchy of boolean operations
 - Union
 - Intersection
 - Difference

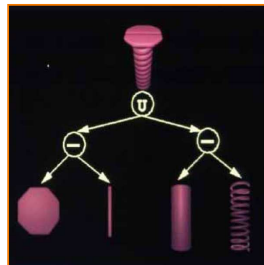


FvDFH Figure 12.27

CSG Acquisition



- Interactive modeling programs
 - Intuitive way to design objects

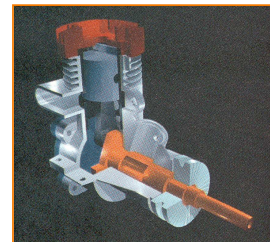


SUNY Stony Brook

CSG Acquisition



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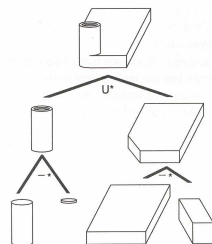


H&B Figure 9.9

CSG Boolean Operations



- Create a new CSG node joining subtrees
 - Union
 - Intersection
 - Difference

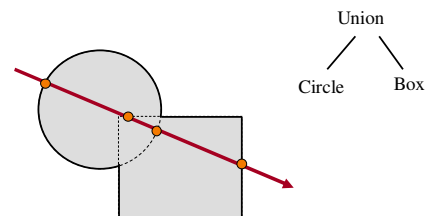


FvDFH Figure 12.27

CSG Display & Analysis



- Ray casting



Summary



	Voxels	Octree	BSP	CSG
Accurate	No	No	Some	Some
Concise	No	No	No	Yes
Affine invariant	No	No	Yes	Yes
Easy acquisition	Some	Some	No	Some
Guaranteed validity	Yes	Yes	Yes	No
Efficient boolean operations	Yes	Yes	Yes	Yes
Efficient display	No	No	Yes	No