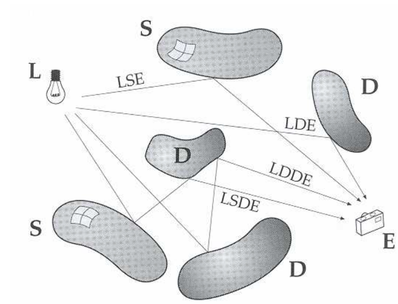
Global Illumination

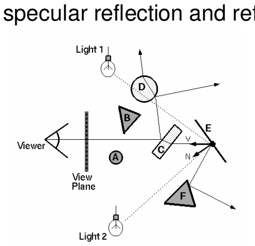
Tom Funkhouser
Princeton University
COS 426, Spring 2007

Global Illuminaton



Ray Tracing

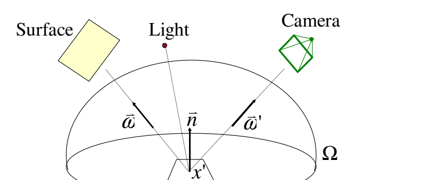
- Trace secondary rays from hit surfaces in directions of specular reflection and refraction



$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Ray Tracing

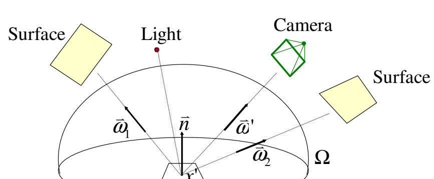
- Assume only significant indirect illumination is from specular reflection and refraction



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \sum_{i=1}^{lights,spec,refr} f_r(x',\vec{\omega},\vec{\omega}')(\vec{\omega} \bullet \vec{n}) L_i(x',\vec{\omega})$$

Rendering Equation

- Compute radiance in outgoing direction by integrating reflections over all incoming directions



$$L_o(x',\vec{\omega}') = L_e(x',\vec{\omega}') + \int_{\Omega} f_r(x',\vec{\omega},\vec{\omega}') L_i(x',\vec{\omega})(\vec{\omega} \bullet \vec{n}) d\vec{\omega}$$

Rendering Equation

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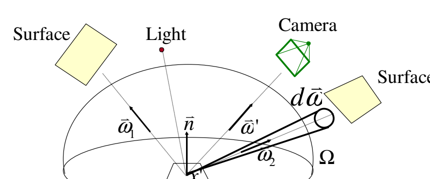

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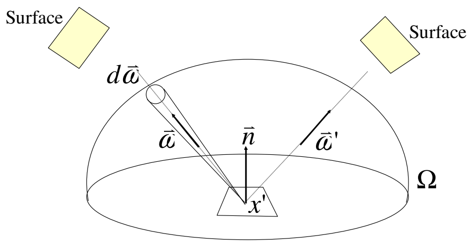
Rendering Equation



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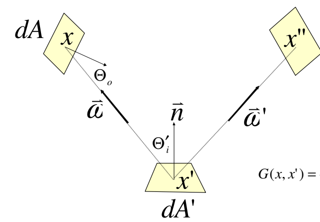
Kajiya 1986

Rendering Equation (2)



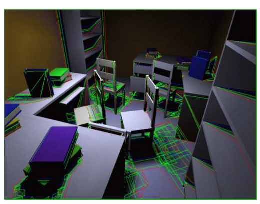
$$G(x,x') = \frac{\cos\Theta_i'\cos\Theta_o}{\|x-x'\|^2}$$

$$L(x'\to x'') = L_e(x'\to x'') + \int_S f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$

Kajiya 1986

Challenge

- Rendering integrals are difficult to evaluate
  - Multiple dimensions
  - Discontinuities
    - Partial occluders
    - Highlights
    - Caustics



Drettakis

$$L(x,\vec{w}) = L_e(x,x\to e) + \int_S f_r(x,x'\to x,x\to e)L(x'\to x)V(x,x')G(x,x')dA$$

Challenge

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  - Multiple dimensions
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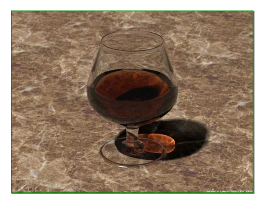

Jensen

$$L(x,\vec{w}) = L_e(x,x\to e) + \int_S f_r(x,x'\to x,x\to e)L(x'\to x)V(x,x')G(x,x')dA$$

Overview

- Global illumination
  - Rendering equation
- Solution methods
  - Sampling
    - Distribution ray tracing
    - Monte Carlo path tracing
    - Bidirectional path tracing
  - Discretization
    - Radiosity

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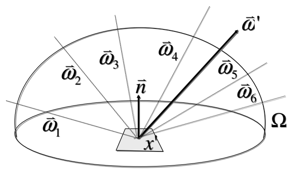
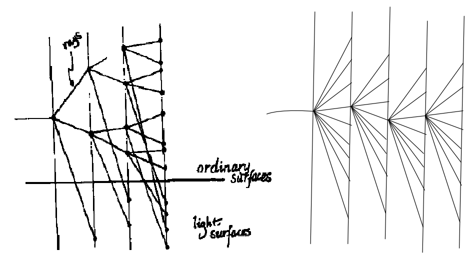
Distribution Ray Tracing

- Estimate integral for each reflection by sampling incoming directions



$$L_o(x',\vec\omega') = L_e(x',\vec\omega') + \sum_{nsamples} f_r(x',\vec\omega,\vec\omega')L_i(x',\vec\omega)(\vec\omega \bullet \vec n)d\vec\omega$$

Ray Tracing vs. Distribution Ray Tracing

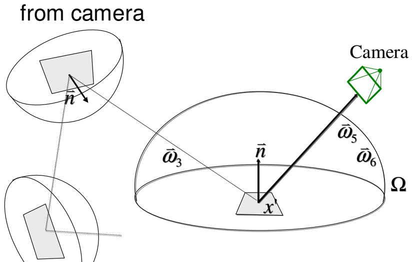


Ray tracing

Distribution ray tracing

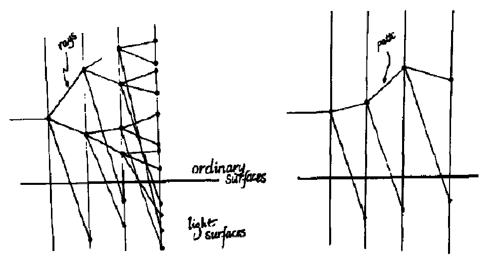
Monte Carlo Path Tracing

- Estimate integral for each pixel by sampling paths from camera



$L = L_e + TL$ where $T(x,\vec\omega') = \int_\Omega f_r(x',\vec\omega,\vec\omega')g(x,\vec\omega)(\vec\omega \bullet \vec n)d\vec\omega$

Ray Tracing vs. Path Tracing



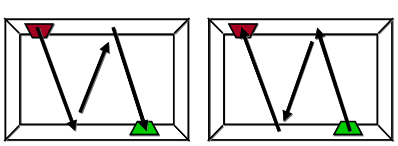
Ray tracing

Path tracing

Kajiya

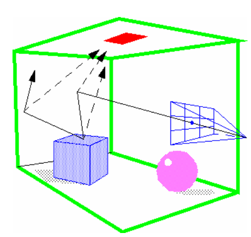
Bidirectional Path Tracing

- Role of source and receiver can be switched



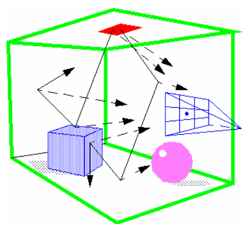
Dutré

Tracing From Eye



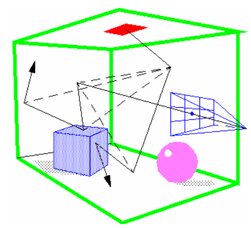
Dutré

Tracing from Lights



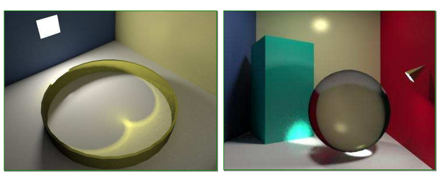
Dutré

Bidirectional Path Tracing



Dutré

Bidirectional Path Tracing



(RenderPark 98)

Dutré

Overview

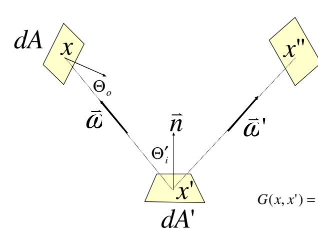
- Global illumination
  - Rendering equation
- Solution methods
  - Sampling
    - § Distribution ray tracing
    - § Monte Carlo path tracing
    - § Bidirectional path tracing
  - Discretization
    - § Radiosity

Radiosity

- Indirect diffuse illumination – LD*E

Rendering Equation (2)

$$L(x'\rightarrow x'') = L_e(x'\rightarrow x'') + \int_S f_r(x\rightarrow x'\rightarrow x'')L(x\rightarrow x')V(x,x')G(x,x')dA$$



$$G(x,x') = \frac{\cos\Theta_i' \cos\Theta_o}{\|x-x'\|^2}$$

Kajiya 1986

Radiosity Equation

$$L(x'\to x'') = L_e(x'\to x'') + \int_S f_r(x\to x'\to x'')L(x\to x')V(x,x')G(x,x')dA$$

Assume everything is Lambertian

$$\boxed{\rho(x') = f_r(x\to x'\to x'')\pi}$$

$$L(x') = L_e(x') + \frac{\rho(x')}{\pi}\int_S L(x)V(x,x')G(x,x')dA$$

Convert to Radiosities

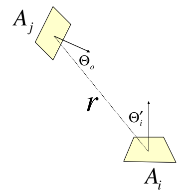
$$\boxed{B = \int_\Omega L_o \cos\theta d\omega} \quad \boxed{L = \frac{B}{\pi}}$$

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

Radiosity Approximation

$$B(x') = B_e(x') + \frac{\rho(x')}{\pi}\int_S B(x)V(x,x')G(x,x')dA$$

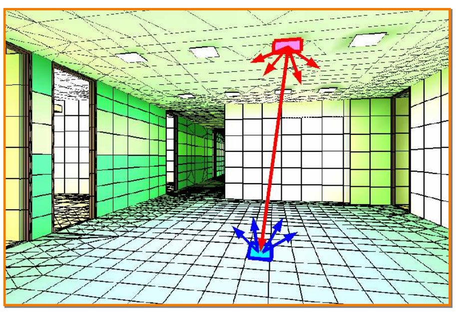
Discretize the surfaces into "elements"



$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

where $F_{ij} = \frac{1}{A_i}\int_{A_i}\int_{A_j} \frac{V_{ij}\cos\Theta_i'\cos\Theta_o}{\pi r^2} dA_j dA_i$

Radiosity Approximation



System of Equations

$$B_i = E_i + \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$E_i = B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij}$$

$$B_i - \rho_i \sum_{j=1}^{N} B_j F_{ij} = E_i$$

$$\begin{bmatrix} 1-\rho_1F_{1,1} & . & . \; . & -\rho_1F_{1,n} \\ -\rho_2F_{2,1} & 1-\rho_2F_{2,2} & . \; . & -\rho_2F_{2,n} \\ . & . & . \; . & . \\ . & . & . \; . & . \\ -\rho_{n-1}F_{n-1,1} & . & . \; . & -\rho_{n-1}F_{n-1,n} \\ -\rho_nF_{n,1} & . & . \; . & 1-\rho_nF_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

$$(1-\rho_i\sum_{j=1}^{N} F_{ii})B_i - \rho_i \sum_{j=1}^{N} F_{ij}B_j = E_i$$

$$B_iA_i = E_iA_i + \rho_i \sum_{j=1}^{N} F_{ji}B_jA_j \longleftarrow$$

This is an energy balance equation

Radiosity

- Issues
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Radiosity

- Issues
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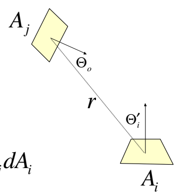
Form Factor

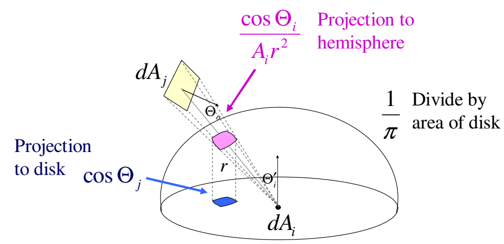
- Fraction of energy leaving element i that arrives at element j

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{V_{ij} \cos\Theta_i' \cos\Theta_o}{\pi r^2} dA_j dA_i$$

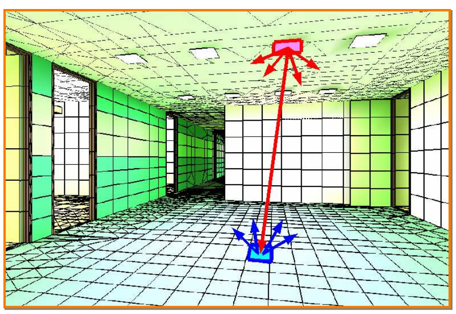


Form Factor Intuition

$$F_{di-dj} = \frac{1}{A_i} \frac{V_{ij} \cos\Theta_i \cos\Theta_j}{\pi r^2}$$



Beyond Diffuse Reflection?



Radiosity

- Issues
  - Computing form factors
  - Selecting basis functions for radiosities
  - Solving linear system of equations
  - Meshing surfaces into elements
  - Rendering images

Solving the System of Equations

- Challenges:
  - Size of matrix
  - Cost of computing form factors
  - Computational complexity

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}}_{\mathbf{b}}$$

Solving the System of Equations

- Solution methods:
  - ~~Invert the matrix – O(n³)~~
  - Iterative methods – O(n²)
  - Hierarchical methods – O(n)

$$\underbrace{\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}}_{\mathbf{b}}$$

Gauss-Seidel Iteration

for all i
  $B_i = E_i$
while not converged
  for each i in turn
    $B_i = E_i + \rho_i \sum_{j \neq i} B_j F_{ij}$
display the image using B_i as the intensity of patch i.

Gauss-Seidel Iteration

- Two interpretations:
  - Iteratively relax rows of linear system
  - Iteratively gather radiosity to elements

$$\begin{bmatrix} 1-\rho_1 F_{1,1} & . & . & . & -\rho_1 F_{1,n} \\ -\rho_2 F_{2,1} & 1-\rho_2 F_{2,2} & . & . & -\rho_2 F_{2,n} \\ . & . & . & . & . \\ . & . & . & . & . \\ -\rho_{n-1} F_{n-1,1} & . & . & . & -\rho_{n-1} F_{n-1,n} \\ -\rho_n F_{n,1} & . & . & . & 1-\rho_n F_{n,n} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ . \\ . \\ . \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ . \\ E_n \end{bmatrix}$$

Gauss-Seidel Iteration

- Two interpretations:
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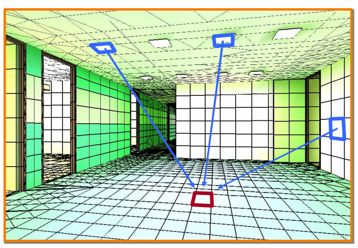
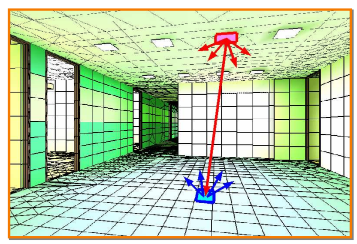


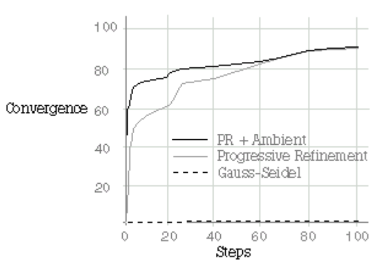
Progressive Radiosity

- Interpretation:
  - Iteratively shoot “unshot” radiosity from elements
  - Select shooters in order of unshot radiosity



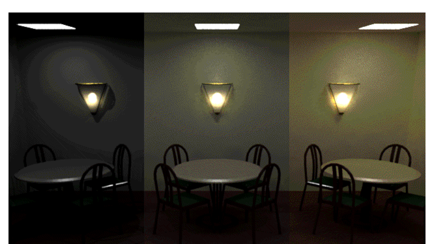
Progressive Radiosity

- Adaptive refinement



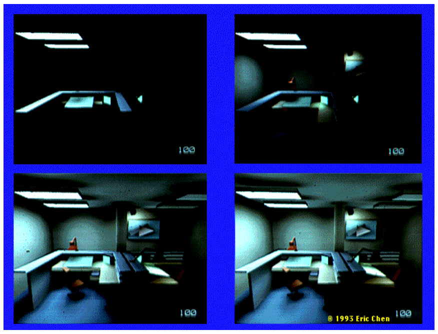
Yeap

Progressive Radiosity



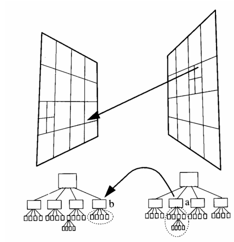
PROGRESSIVE SOLUTION
The above images show increasing levels of global diffuse illumination. From left to right: 0 bounces, 1 bounce, 3 bounces.

Progressive Radiosity



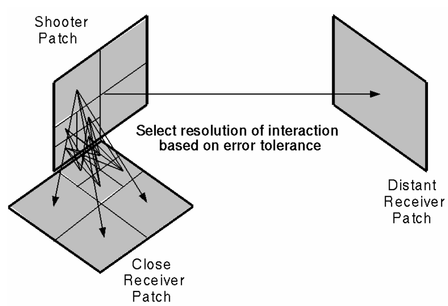
Hierarchical Radiosity

- Estimate errors, refine elements if too large



Cohen & Wallace

Hierarchical Radiosity



Radiosity

- Issues
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  - Meshing surfaces into elements
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Surface Meshing

- Store radiosity across surface
  - Few elements
  - Represents function well
  - Few visible artifacts

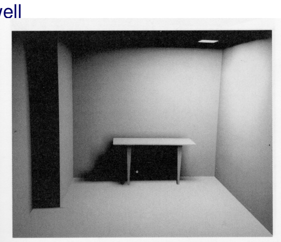
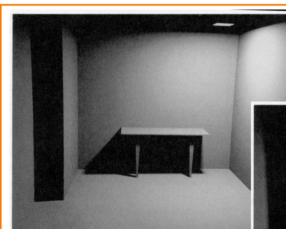


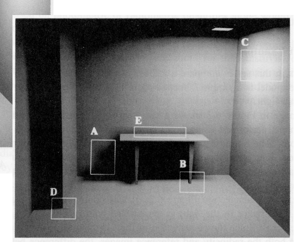
Figure 6.2: *A radiosity image computed using a uniform mesh.*

Cohen & Wallace

Artifacts of Bad Surface Meshing



(a) *Reference image.*

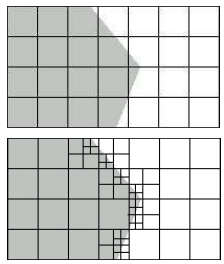


(b) *Artifacts introduced by the approximation.*

Cohen & Wallace

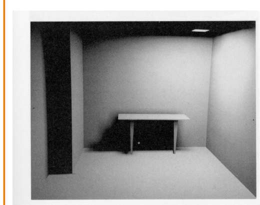
Adaptive Meshing

- Refine mesh in areas of high residual

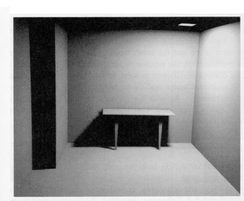


Yeap

Adaptive Meshing



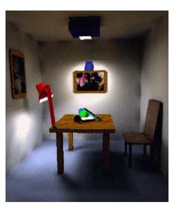
Uniform mesh

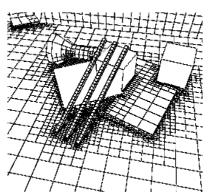


Adaptive mesh

Cohen & Wallace

Adaptive Meshing

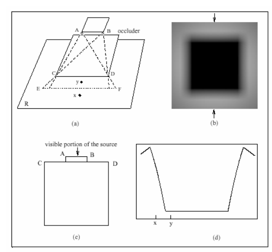




(table top from different angle)

Discontinuity Meshing

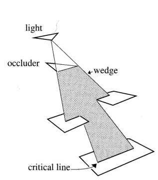
- Capture discontinuities in radiosity across a surface with explicit mesh boundaries



Lischinski et al.

Discontinuity Meshing

- Capture discontinuities in radiosity across a surface with explicit mesh boundaries



Discontinuity Mesh

Lischinski et al.

Summary

- Global illumination
  - Rendering equation
- Solution methods
  - Sampling
    - Distribution ray tracing
    - Monte Carlo path tracing
    - Bidirectional path tracing
  - Discretization
    - Radiosity

Photorealistic rendering with global illumination is an integration problem