Programming Assignment Collaboration Policy

No sharing code.
Do not share code under any circumstances.
[OK to use code from course materials provided you cite source.]

Where to get help.
- Email.
- Office hours.
- Lab TAs in Friend 008/009.
- Bounce ideas (but not code) off of classmates.

Pairs programming. [`http://www.cs.princeton.edu/introcs/papers/pairs-kindergarten.pdf`]
- One driver, one navigator. On demand, programmers brainstorm. Switch roles every 30-40 minutes.
- One partner submits code; both submit `readme.txt`.

Note. Programming in groups except as above is a serious violation.

Exercise Collaboration Policy

Permitted collaboration. You are welcome and encouraged to work (or check your work) with classmates.

No copying solutions. Write up your own solutions.

Stacks and Queues

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications

Stacks and Queues

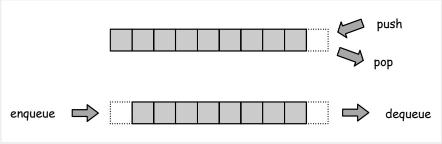
Fundamental data types.
- Values: sets of objects
- Operations: insert, remove, test if empty.
- Intent is clear when we insert.
- Which item do we remove?

LIFO = "last in first out"

Stack.
- Remove the item most recently added.
- Analogy: cafeteria trays, Web surfing.

FIFO = "first in first out"

Queue.
- Remove the item least recently added.
- Analogy: Registrar's line.



Client, Implementation, Interface

Separate interface and implementation so as to:

- Build layers of abstraction.
- Reuse software.
- Ex: stack, queue, symbol table.

> Interface: description of data type, basic operations.
> Client: program using operations defined in interface.
> Implementation: actual code implementing operations.

Client, Implementation, Interface

Benefits.

- Client can't know details of implementation ⇒ client has many implementation from which to choose.
- Implementation can't know details of client needs ⇒ many clients can re-use the same implementation.
- Design: creates modular, re-usable libraries.
- Performance: use optimized implementation where it matters.

> Interface: description of data type, basic operations.
> Client: program using operations defined in interface.
> Implementation: actual code implementing operations.

Stack

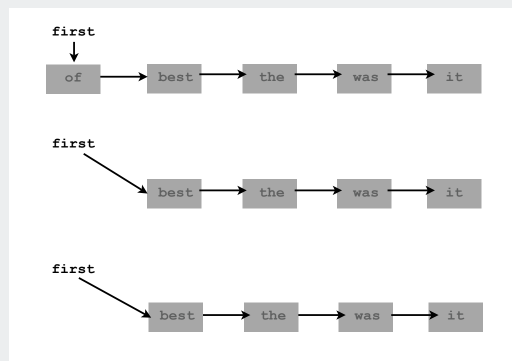
Stack operations.

- `push()` Insert a new item onto stack.
- `pop()` Remove and return the item most recently added.
- `isEmpty()` Is the stack empty?


```
public static void main(String[] args)
{
   StackOfStrings stack = new StackOfStrings();
   while(!StdIn.isEmpty())
   {
      String s = StdIn.readString();
      stack.push(s);
   }
   while(!stack.isEmpty())
   {
      String s = stack.pop();
      System.out.println(s);
   }
}
```

a sample stack client

Stack pop: Linked-list implementation

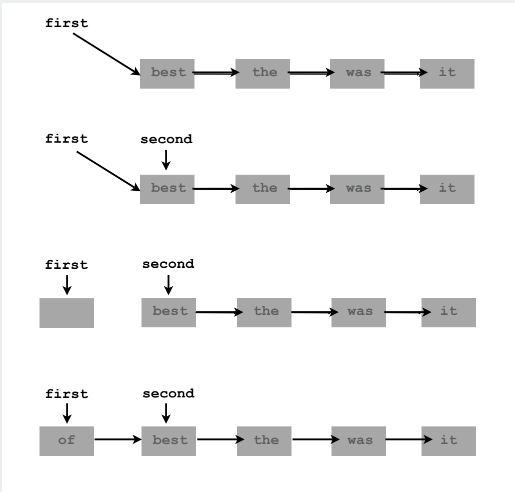


`item = first.item;`

`first = first.next;`

`return item;`

Stack push: Linked-list implementation




```
second = first;
```

```
first = new Node();
```

```
first.item = item;
first.next = second;
```


Stack: Linked-list implementation

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications


```
public class StringStack
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

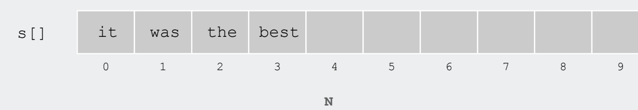
"inner class"

Stack: Array Implementation

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications

Array implementation of a stack.

- Use array `s[]` to store `N` items on stack.
- `push()` add new item at `s[N]`.
- `pop()` remove item from `s[N-1]`.



Stack: Array implementation

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications


```
public class StringStack
{
   private String[] s;
   private int N = 0;

   public StringStack(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   { return N == 0; }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {
      String item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }
}
```

avoid loitering
(garbage collector only reclaims memory if no outstanding references)

Stack Array Implementation: Resizing

How to grow array when capacity reached?
How to shrink array (else it stays big even when stack is small)?

First try:
- increase size of `s[]` by 1 if the array is full.
- decrease size of `s[]` by 1 if the array is full.

Too expensive.
- Increasing the size of an array involves copying all of the elements to a new array.
- Inserting N elements: time proportional to $1 + 2 + \ldots + N \approx N^2/2$.

infeasible for large N

Thrashing.
- Subtract by 1 on pop??
- push-pop-push-pop... sequence: time proportional to N for each op.

Need to guarantee that array resizing happens infrequently

Stack array implementation: Dynamic resizing

How to grow array?
Use repeated doubling:
if `s[]` too small, create a new array of twice the size, and copy items.

no-argument constructor

```
public StringStack()
{ this(8); }

public void push(String item)
{
   if (N >= s.length) resize();
   s[N++] = item;
}
```

create new array (twice the size) copy items to it

```
private void resize()
{
   String[] dup = new String[2*N];
   for (int i = 0; i < N; i++)
      dup[i] = s[i];
   s = dup;
}
```

Consequence. Inserting N items takes time proportional to N (not N^2).

Stack array implementation: Dynamic resizing

How to shrink array?
Use repeated halving:
if `s[]` too large, create a new array of half the size, and copy items.

```
public String pop(String item)
{
   String item = a[--N];
   a[N] = null;
   if (N == a.length/4)
      resize(a.length/2);
   return item;
}
```

Why not `a.length/2` ?

Consequences.
- Any sequence of N ops takes time proportional to N.
- Stack never overflows and is never less than 1/4 full

Stack Implementations: Array vs. Linked List

Stack implementation tradeoffs. Can implement with either array or linked list, and client can use interchangeably. Which is better?

Array.
- Most operations take constant time.
- Expensive re-doubling operation every once in a while.
- Any sequence of N operations (starting from empty stack) takes time proportional to N.

"amortized" bound

Linked list.
- Grows and shrinks gracefully.
- Every operation takes constant time.
- Every operation uses extra space and time to deal with references.

Bottom line: tossup for stacks
but differences are significant when other operations are added

Stack implementations: Array vs. Linked list

Which implementation is more convenient?

	array?	linked list?
return count of elements in stack		
remove the kth most recently added		
sample a random element		

Queue

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications

Queue operations.

- `enqueue()` Insert a new item onto queue.
- `dequeue()` Delete and return the item least recently added.
- `isEmpty()` Is the queue empty?

```
public static void main(String[] args)
{
   QueueOfStrings q = new QueueOfStrings();
   q.enqueue("Vertigo");
   q.enqueue("Just Lose It");
   q.enqueue("Pieces of Me");
   q.enqueue("Pieces of Me");
   System.out.println(q.dequeue());
   q.enqueue("Drop It Like It's Hot");

   while(!q.isEmpty()

      System.out.println(q.dequeue());
}
```

Dequeue: Linked List Implementation

- stacks
- array implementation
- linked implementation
- dynamic resizing
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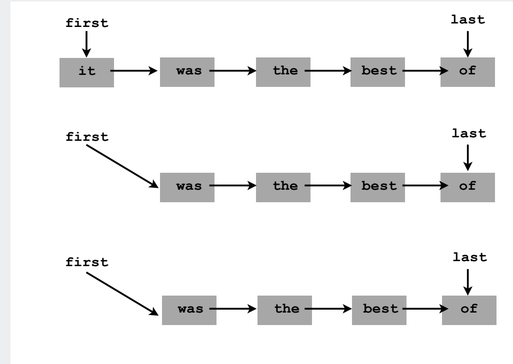

`item = first.item;`

`first = first.next;`

`return item;`

Aside:
dequeue (pronounced DQ) means "remove from a queue"
deque (pronounced "deck") is a data structure (see PA 1)

Enqueue: Linked List Implementation

- stacks
- array implementation
- linked implementation
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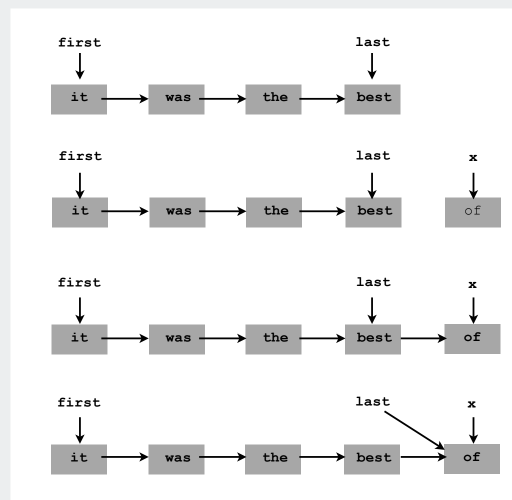


```
x = new Node();
x.item = item;
x.next = null;
```

`last.next = x;`

`last = x;`

Queue: Linked List Implementation


```java
public class QueueOfStrings
{
   private Node first;
   private Node last;

   private class Node
   { String item; Node next; }

   public boolean isEmpty()
   { return first == null; }

   public void enqueue(String item)
   {
      Node x = new Node();
      x.item = item;
      x.next = null;
      if (isEmpty()) { first      = x; last = x; }
      else           { last.next = x; last = x; }
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      return item;
   }
}
```


Queue: Array implementation

Array implementation of a queue.

- Use array `q[]` to store items on queue.
- `enqueue()`: add new object at `q[tail]`.
- `dequeue()`: remove object from `q[head]`.
- Update `head` and `tail` modulo the `capacity`.

q[]			the	best	of	times				
	0	1	2	3	4	5	6	7	8	9
			head				tail			

capacity = 10

[details: good exercise or exam question]

Generics (parameterized data types)

We implemented: `StackOfStrings`, `QueueOfStrings`.

We also want: `StackOfURLs`, `QueueOfCustomers`, etc?

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! only solution possible in Java until 1.5 [hence, used in AlgsJava]

Stack of Objects

We implemented: `StackOfStrings`, `QueueOfStrings`.

We also want: `StackOfURLs`, `QueueOfCustomers`, etc?

Attempt 2. Implement a stack with items of type `Object`.

- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```java
Stack  s = new Stack();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop()); // run-time error
```

Generics

Generics. Parameterize stack by a single type.

- Avoid casting in both client and implementation.
- Discover type mismatch errors at compile-time instead of run-time.

parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);      compile-time error
a = s.pop();
```

no cast needed in client

Guiding principles.

- Welcome compile-time errors
- Avoid run-time errors

Why?

Generic Stack: Linked List Implementation

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
- generics
- applications


```
public class StackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;   }

   public void push(String item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
}
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;   }

   public void push(Item item)
   {
      Node second = first;
      first = new Node();
      first.item = item;
      first.next = second;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
}
```

Generic type name

Generic stack: array implementation

- stacks
- array implementation
- linked implementation
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The way it should be.

```
public class Stack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int cap)
   {  s = new Item[cap];   }

   public boolean isEmpty()
   { return N == 0; }

   public void push(Item item)
   {  s[N++] = item;   }

   public String pop()
   {
      Item item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }
}
```

```
public class StackOfStrings
{
   private String[] s;
   private int N = 0;

   public StackOfStrings(int cap)
   {  s = new String[cap];   }

   public boolean isEmpty()
   { return N == 0; }

   public void push(String item)
   {  s[N++] = item;   }

   public String pop()
   {
      String item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }
}
```

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

- stacks
- array implementation
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The way it is: an ugly cast in the implementation.

```
public class Stack<Item>
{
   private Item[] s;
   private int N = 0;

   public Stack(int cap)
   { s = (Item[]) new Object[cap]; }      <-- the ugly cast

   public boolean isEmpty()
   { return N == 0; }

   public void push(Item item)
   {  s[N++] = item;   }

   public String pop()
   {
      Item item = s[N-1];
      s[N-1] = null;
      N--;
      return item;
   }
}
```

Number of casts in good code: 0 or 1

Generic data types: autoboxing

Generic stack implementation is object-based.

What to do about primitive types?.

Wrapper type.
- Each primitive type has a wrapper object type.
- Ex: `Integer` is wrapper type for `int`.

Autoboxing. Automatic cast between a primitive type and its wrapper.

Syntactic sugar. Behind-the-scenes casting.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);        // s.push(new Integer(17));
int a = s.pop();   // int a = ((Integer) s.pop()).intValue();
```

Bottom line: Client code can use generic stack for any type of data

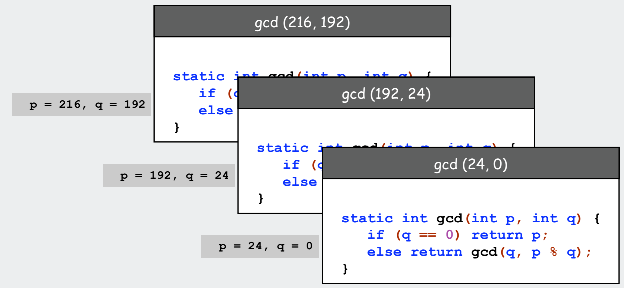
Stack Applications

Real world applications.
- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.

Function Calls

How a compiler implements functions.
- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.
Note. Can always use an explicit stack to remove recursion.



Arithmetic Expression Evaluation

Goal. Evaluate infix expressions.

(1 + ((2 + 3) * (4 * 5)))

operand   operator

Two stack algorithm. [E. W. Dijkstra]
- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parens: ignore.
- Right parens: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!

Arithmetic Expression Evaluation


```
public class Evaluate {
   public static void main(String[] args) {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
         if      (s.equals("("))                ;
         else if (s.equals("+"))    ops.push(s);
         else if (s.equals("*"))    ops.push(s);
         else if (s.equals(")")) {
            String op = ops.pop();
            if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
            else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
         }
         else vals.push(Double.parseDouble(s));
      }
      StdOut.println(vals.pop());
   }
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Note: Old books have two-pass algorithm because generics were not available!

Correctness

Why correct?

When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity.

```
1 + (2 - 3 - 4) * 5 * sqrt(6 + 7)
```


Stack-based programming languages

Observation 1.

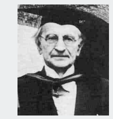
Remarkably, the 2-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Observation 2.

All of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```



Jan Lukasiewicz

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, …

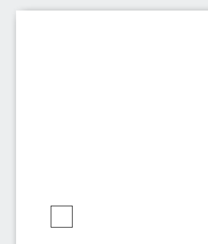
Stack-based programming languages: PostScript

Page description language

- explicit stack
- full computational model
- graphics engine

Basics

- %!: "I am a PostScript program"
- literal: "push me on the stack"
- function calls take args from stack
- turtle graphics built in



a PostScript program

```
%!
72 72 moveto
0 72 rlineto
72 0 rlineto
0 -72 rlineto
-72 0 rlineto
2 setlinewidth
stroke
```

Stack-based programming languages: PostScript

Variables (and functions)

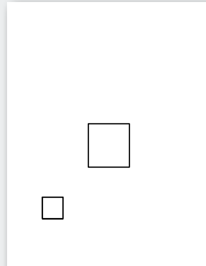
- identifiers start with /
- def operator associates id with value
- braces

function definition → `/box`

function calls → `72 box`, `144 box`

```
%!
/box
{
  /sz exch def
  0 sz rlineto
  sz 0 rlineto
  0 sz neg rlineto
  sz neg 0 rlineto
} def

72 144 moveto
72 box
288 288 moveto
144 box
2 setlinewidth
stroke
```



Stack-based programming languages: PostScript

- stacks
- array implementation
- linked implementation
- dynamic resizing
- queues
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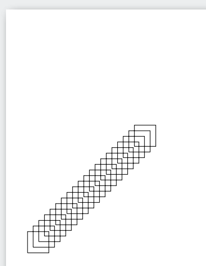
- applications

for loop

- "from, increment, to" on stack
- loop body in braces
- `for` operator

```
1 1 20
{ 19 mul dup 2 add moveto 72 box }
for
```



if-else

- boolean on stack
- alternatives in braces
- `if` operator

Stack-based programming languages: PostScript

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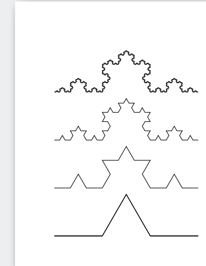
An application: all figures in Algorithms in Java

```
%!
72 72 translate

/kochR
  {
    2 copy ge { dup 0 rlineto }
      {
        3 div
        2 copy kochR 60 rotate
        2 copy kochR -120 rotate
        2 copy kochR 60 rotate
        2 copy kochR
      } ifelse
    pop pop
  } def

0   0 moveto     81 243 kochR
0  81 moveto     27 243 kochR
0 162 moveto      9 243 kochR
0 243 moveto      1 243 kochR
stroke
```



See page 218

Queue Applications

- stacks
- array implementation
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- dynamic resizing
- queues
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Some applications.

- iTunes playlist.
- Data buffers (iPod, TiVo).
- Asynchronous data transfer (file IO, pipes, sockets).
- Dispensing requests on a shared resource (printer, processor).

Simulations of the real world.

- Traffic analysis.
- Waiting times of customers at call center.
- Determining number of cashiers to have at a supermarket.

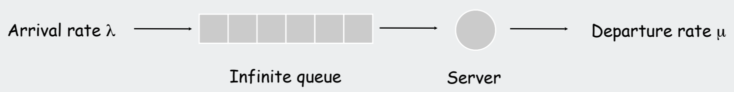
M/D/1 Queuing Model

M/D/1 queue.

- Customers are serviced at fixed rate of μ per minute.
- Customers arrive according to Poisson process at rate of λ per minute.

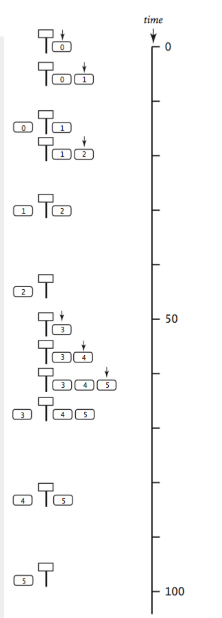
inter-arrival time has exponential distribution

$$\Pr[X \le x] \; = \; 1 - e^{-\lambda x}$$



Q. What is average wait time W of a customer?

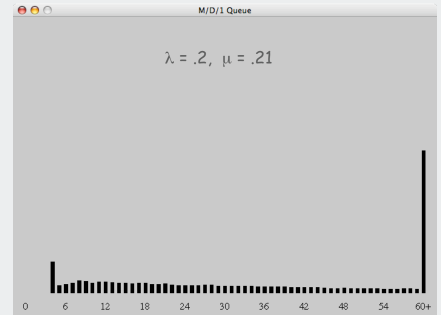
Q. What is average number of customers L in system?



	arrival time	departure time	wait
0	0	15	15
1	6	30	24
2	20	45	25
3	52	67	15
4	57	83	26
5	62	98	36

M/D/1 Queue Analysis

Observation. As service rate approaches arrival rate, service goes to h***.



see ORFE 309

Queueing theory. $W \; = \; \dfrac{\lambda}{2\,\mu(\mu-\lambda)} + \dfrac{1}{\mu}\,, \qquad L \; = \; \lambda \; W$

Little's law

Event-Based Simulation


```
public class MD1Queue
{
   public static void main(String[] args)
   {
      double lambda = Double.parseDouble(args[0]);
      double mu     = Double.parseDouble(args[1]);
      Queue<Double> q = new Queue<Double>();
      double nextArrival = StdRandom.exp(lambda);
      double nextService = nextArrival + 1/mu;
      while(true)
      {
         if (nextArrival < nextService)
         {
            q.enqueue(nextArrival);
            nextArrival += StdRandom.exp(lambda);
         }
         else
         {
            double wait = nextService - q.dequeue();
            // add waiting time to histogram
            if (q.isEmpty()) nextService = nextArrival + 1/mu;
            else             nextService = nextService + 1/mu;
         }
      }
   }
}
```