



Discussion

- What did you learn from the Boole/Clarke “proof” of the existence of God?
- How convincing was it?



Part1: Wrapup of Boolean logic and Combinational circuits

COS 116

3/16/2006

Instructor: Sanjeev Arora

Pickup: HW3, Blogging Assignment.

Combinational circuit for binary addition

$$\begin{array}{r} 25 \qquad 11001 \\ + 29 \qquad 11101 \\ \hline 54 \qquad 110110 \end{array}$$

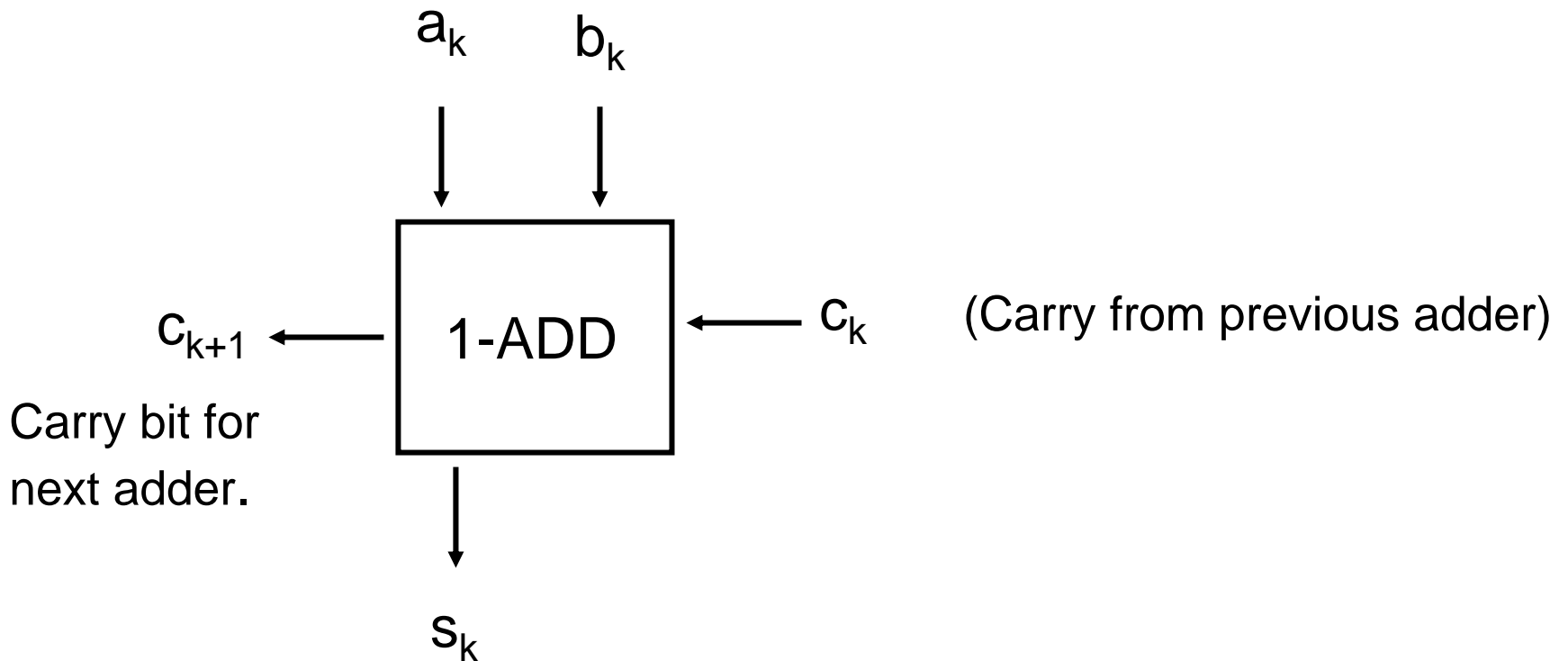
- Desired: circuit for adding any two N -bit integers

Modular design

$$\begin{array}{rcccccc} & & \mathbf{c}_{N-1} & \mathbf{c}_{N-2} & \dots & \mathbf{c}_1 & \mathbf{c}_0 & \text{Carry bits} \\ & & \mathbf{a}_{N-1} & \mathbf{a}_{N-2} & \dots & \mathbf{a}_1 & \mathbf{a}_0 & \\ + & & \mathbf{b}_{N-1} & \mathbf{b}_{N-2} & \dots & \mathbf{b}_1 & \mathbf{b}_0 & \\ \hline & \mathbf{s}_N & \mathbf{s}_{N-1} & \mathbf{s}_{N-2} & \dots & \mathbf{s}_1 & \mathbf{s}_0 & \end{array}$$

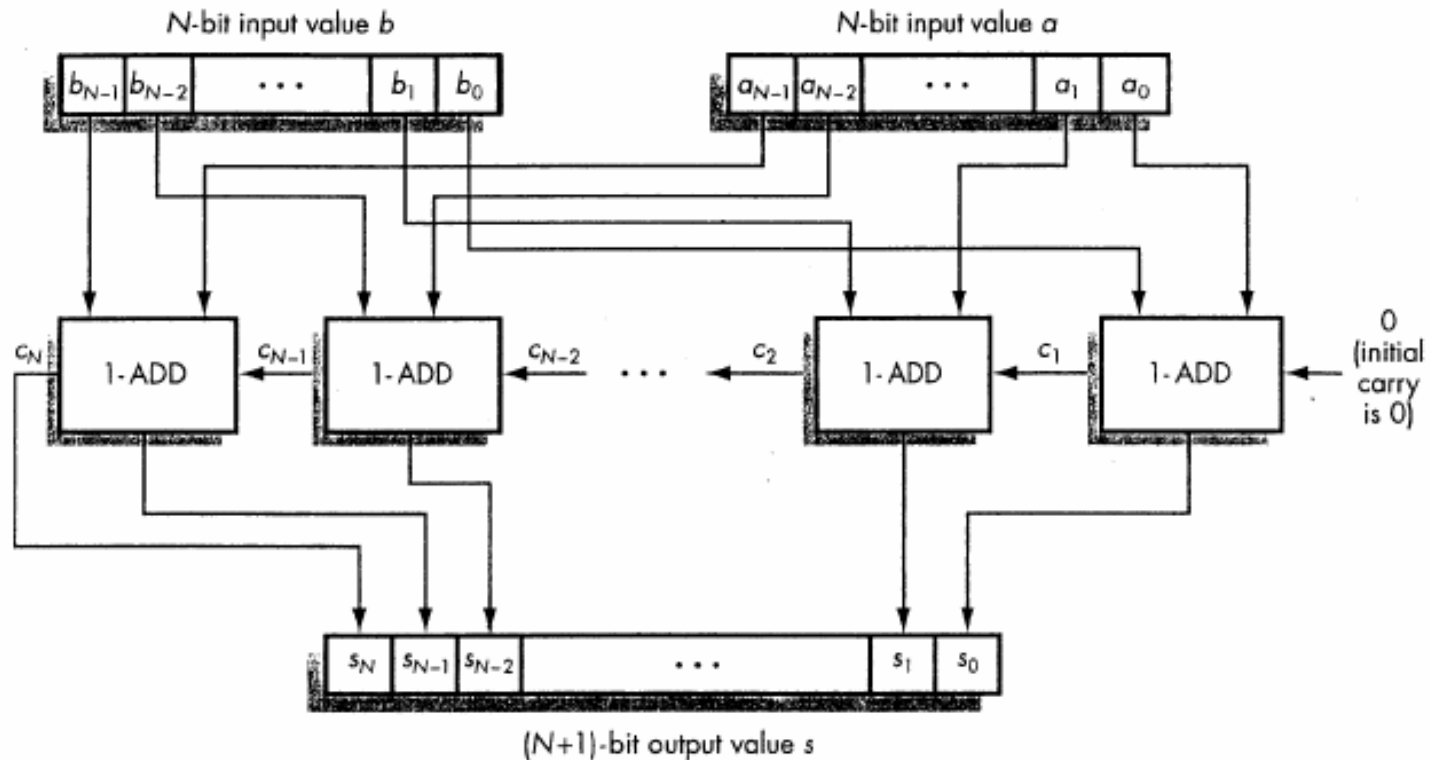
Need N 1-bit adders

1-bit adder



Do yourself: Write truth table, circuit.

A Full Adder (from handout)





Memory in boolean circuits

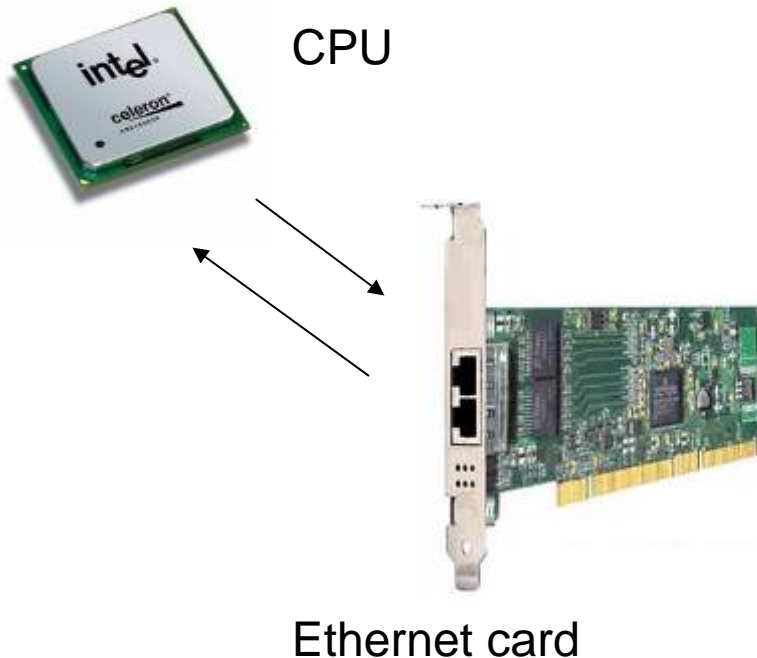
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Going beyond combinational circuits

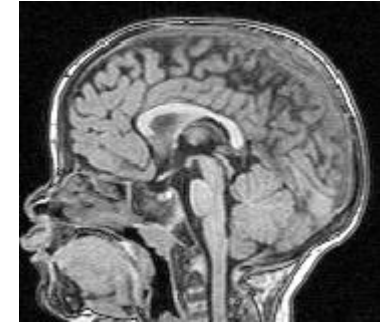
- Need 2-way communication between circuits (i.e. need cycles!)



- Need memory (scratchpad)



What do you understand by ‘memory’?”?



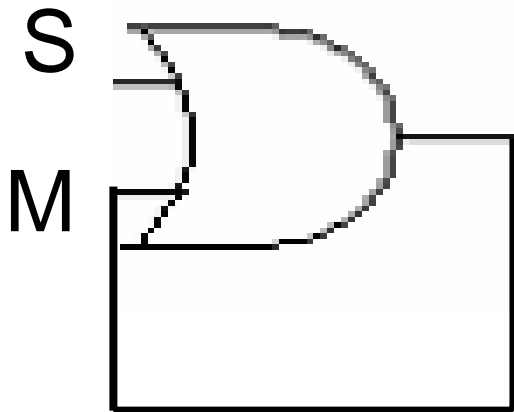
How can you tell that a 1-year old child has it?

Behaviorist's answer:
His/her actions depend upon past events.



Matt likes Sue but he doesn't like changing his mind

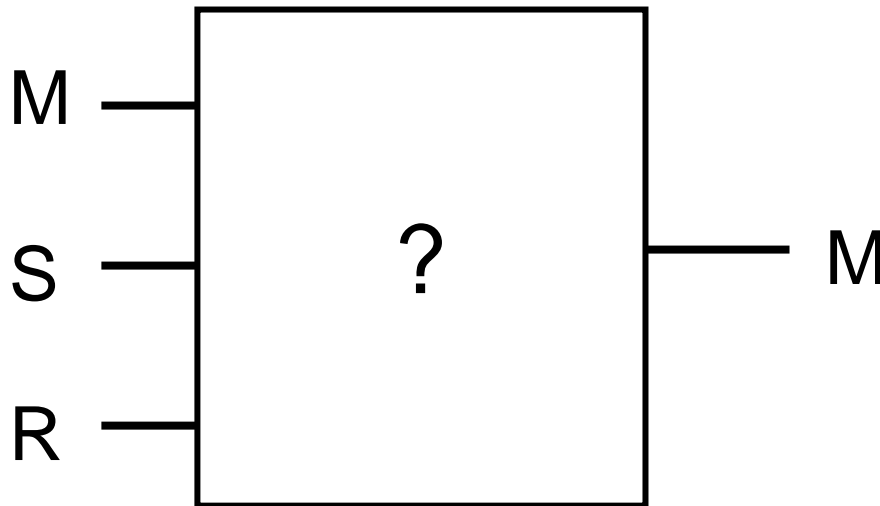
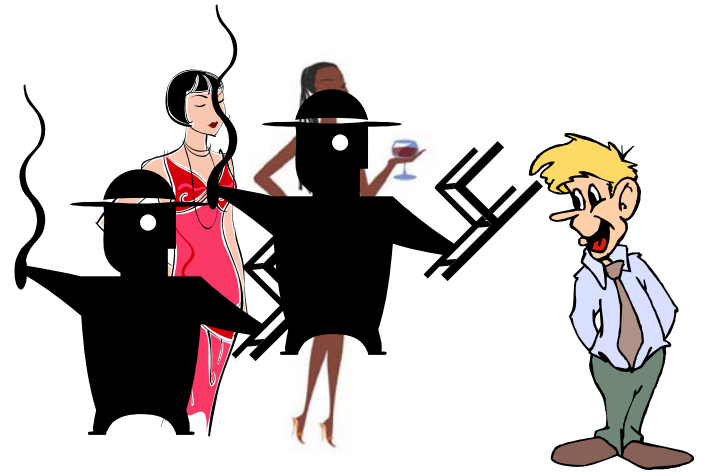
- Represent with a circuit:
Matt will go to the party if Sue goes or if he already wanted to go



Is this well-defined?

Enter Rita

- Matt will go to the party if Sue goes OR if the following holds: if Rita does not go and he already wanted to go



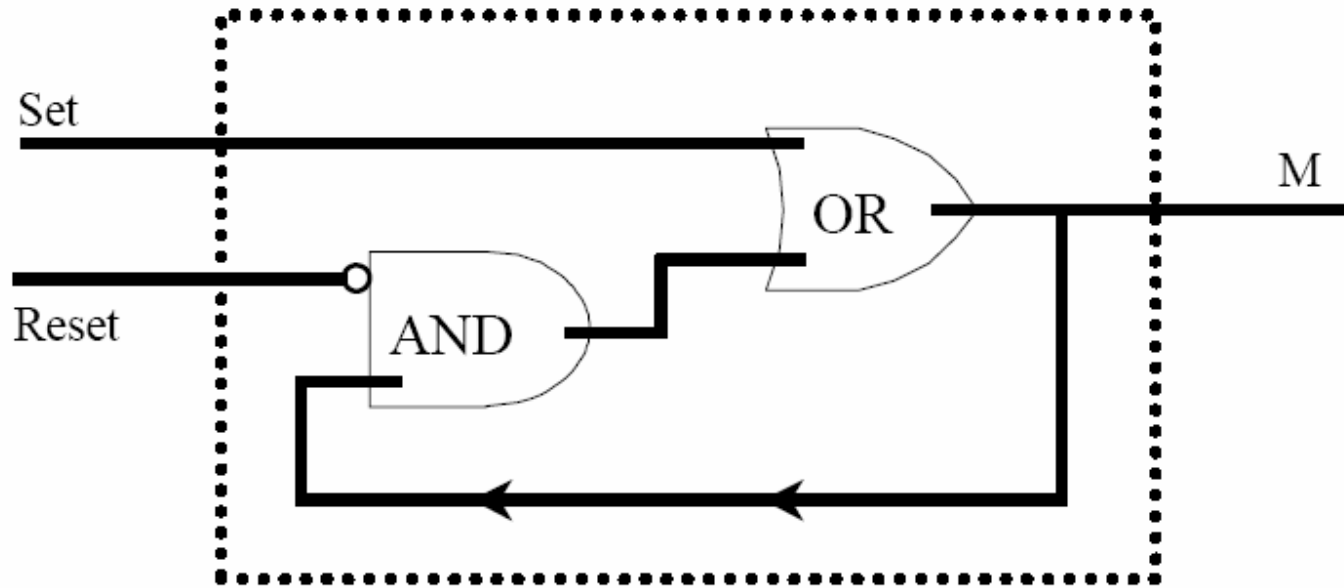
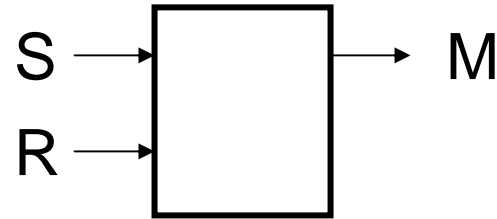
R, S: **“control”** inputs

What combination of R, S changes M?

Flip-Flop

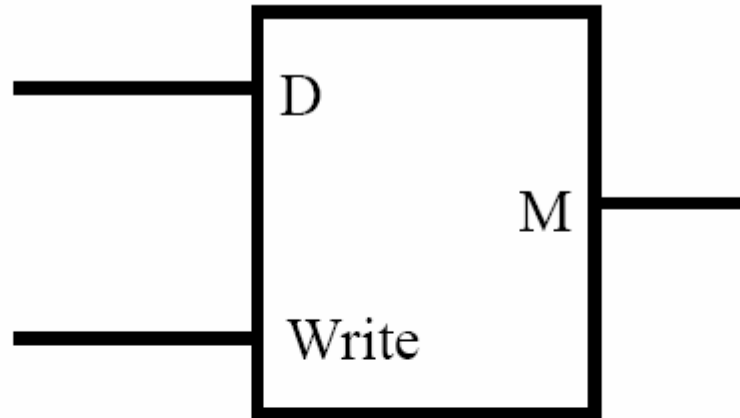


Flip-Flop



- M becomes 1 if Set is turned on
- M becomes 0 if Reset is turned on
- Otherwise (if both are 0), M just remembers its value

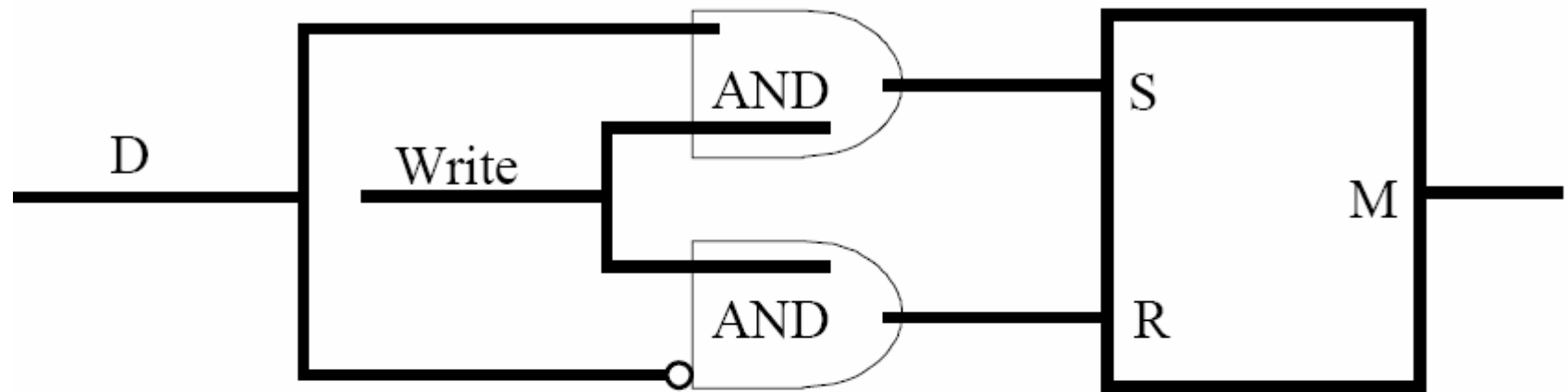
Desired: more convenient form of memory



- If $\text{Write} = 0$, M just keeps its value. (It ignores D .)
- If $\text{Write} = 1$, then M becomes set to D

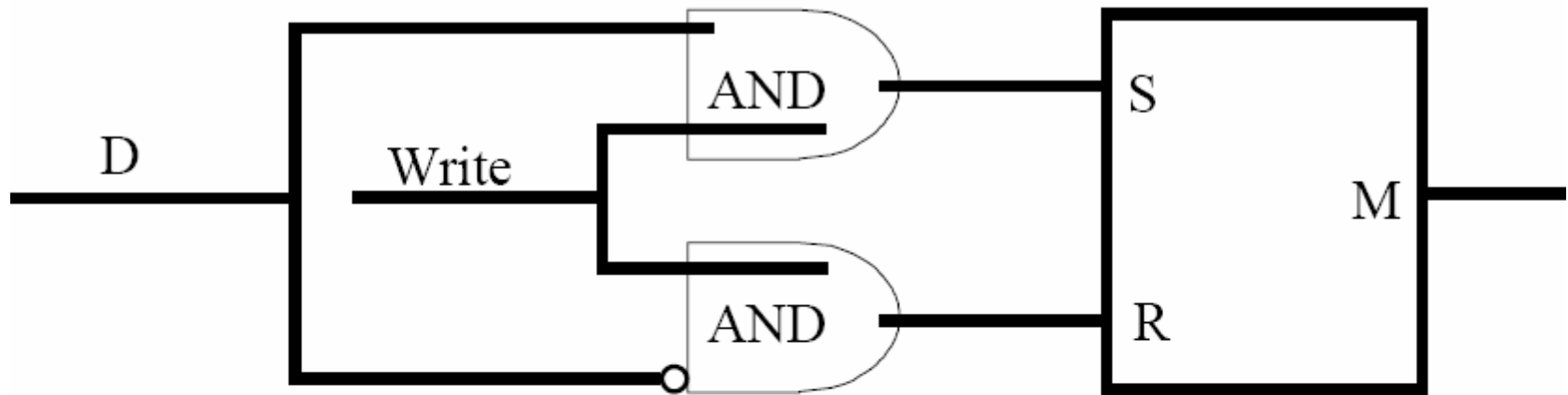
“Data Flip-Flop” or “D flip flop.”

Design of D Flip Flop



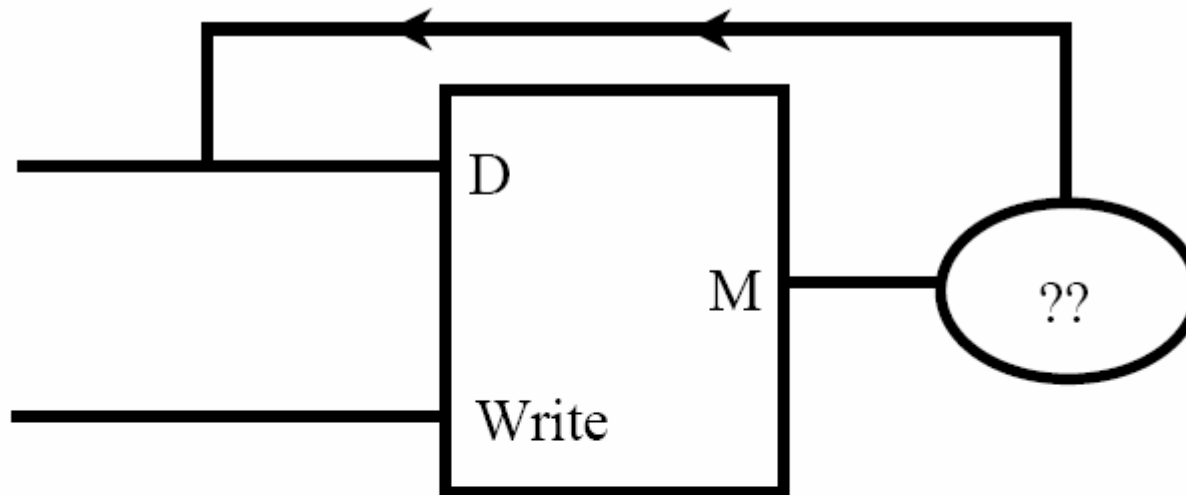
- Nothing happens unless $Write = 1$

The D Flip-Flop



- Nothing happens unless $Write = 1$
- If $Write = 1$, then M becomes set to D
- Once $Write = 0$ again, M just keeps its value. (It ignores D.)

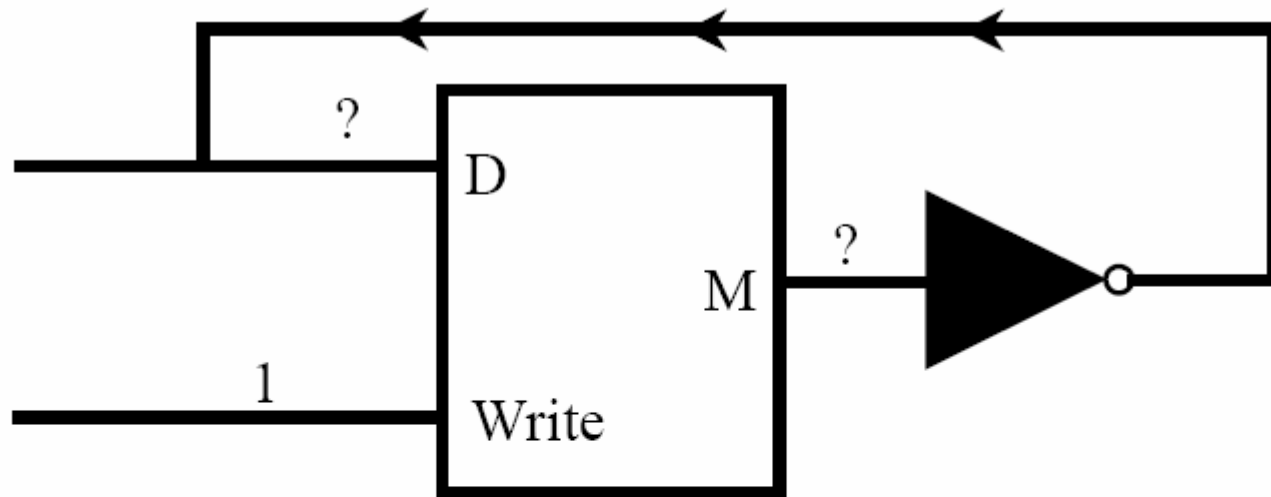
A Subtle Problem



“Race condition.”

- When $Write = 1$, then $M = D$.
- If we have some feedback between M and D , then circuit could go haywire.

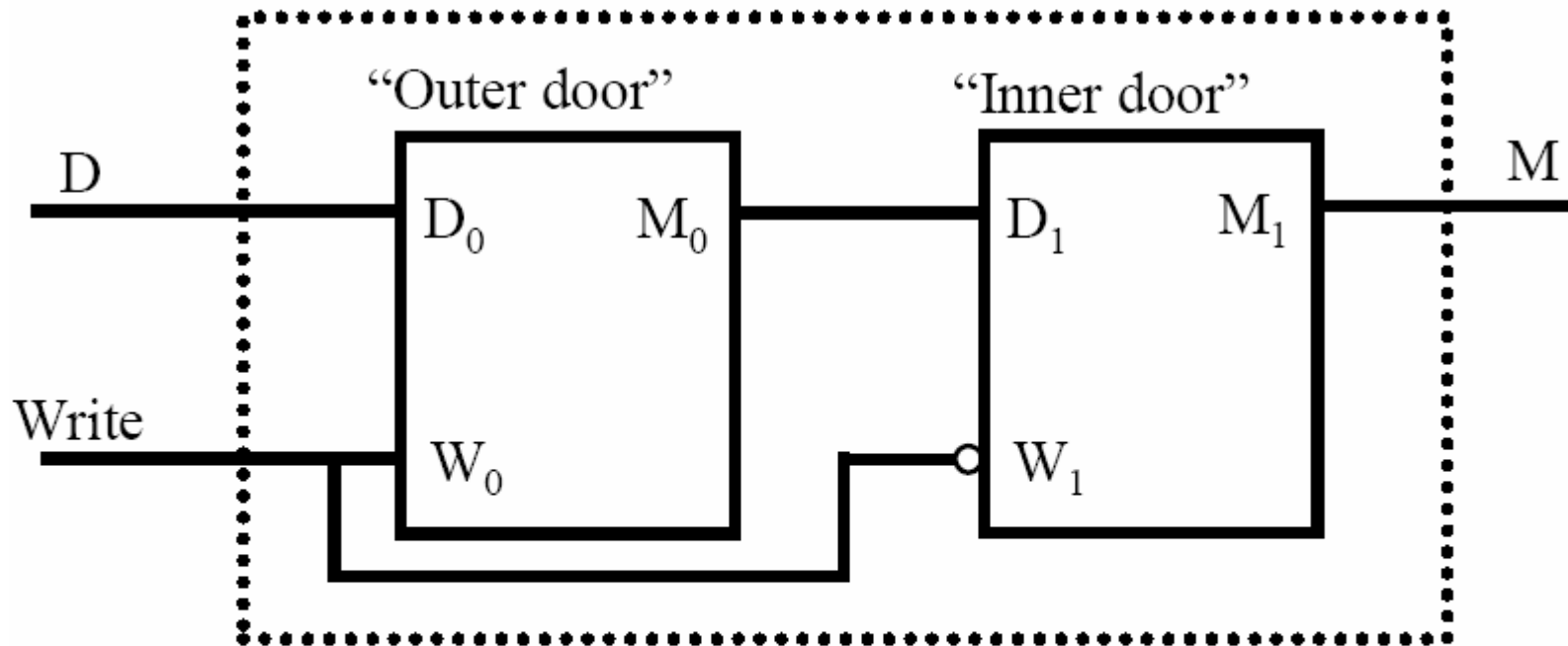
Example



- For example, suppose a NOT gate connects M and D.
- When Write = 1, M and D keep changing. We have no control.

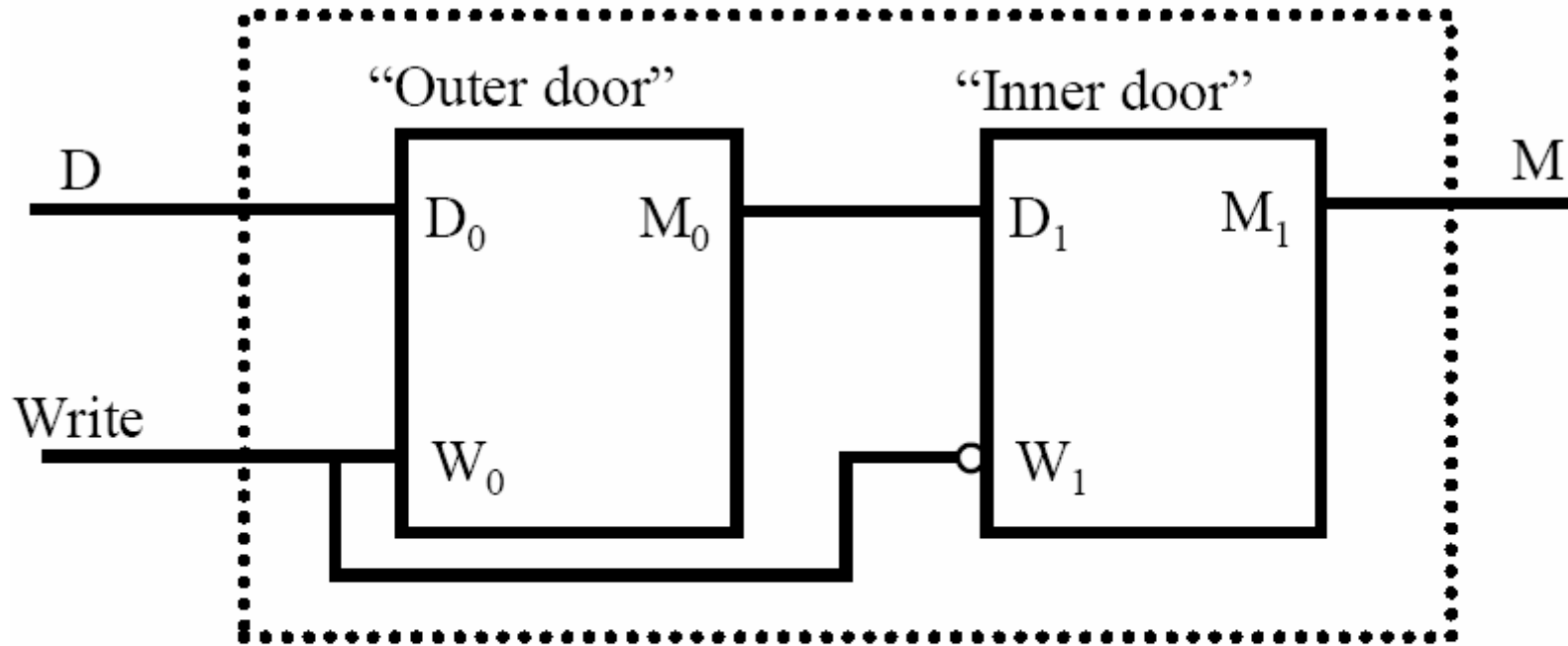
Desired: M should invert only once when we make Write = 1

The “Airlock” Flip-Flop



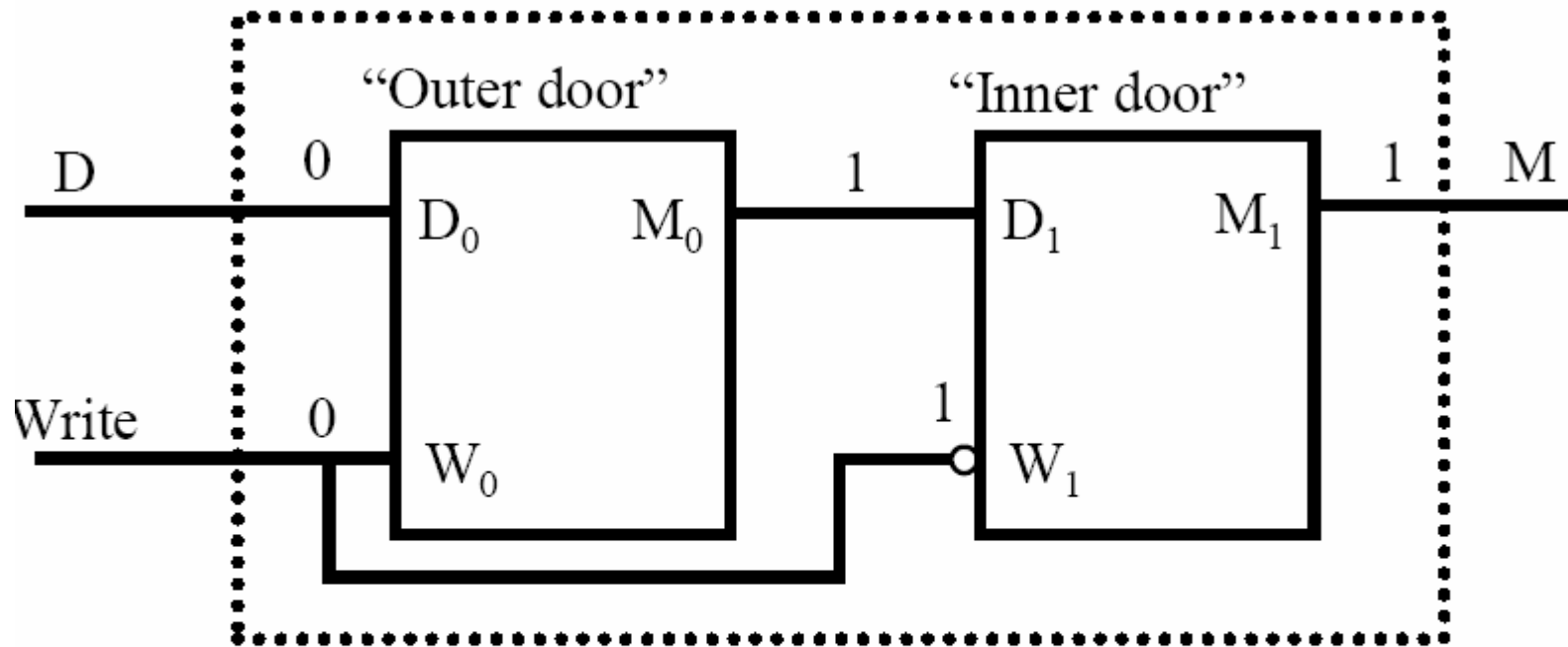
- Two-Stage System to prevent D ever passing through directly to M (W_0, W_1 connected by NOT, so never 1 at the same time)

The “Airlock” Flip-Flop



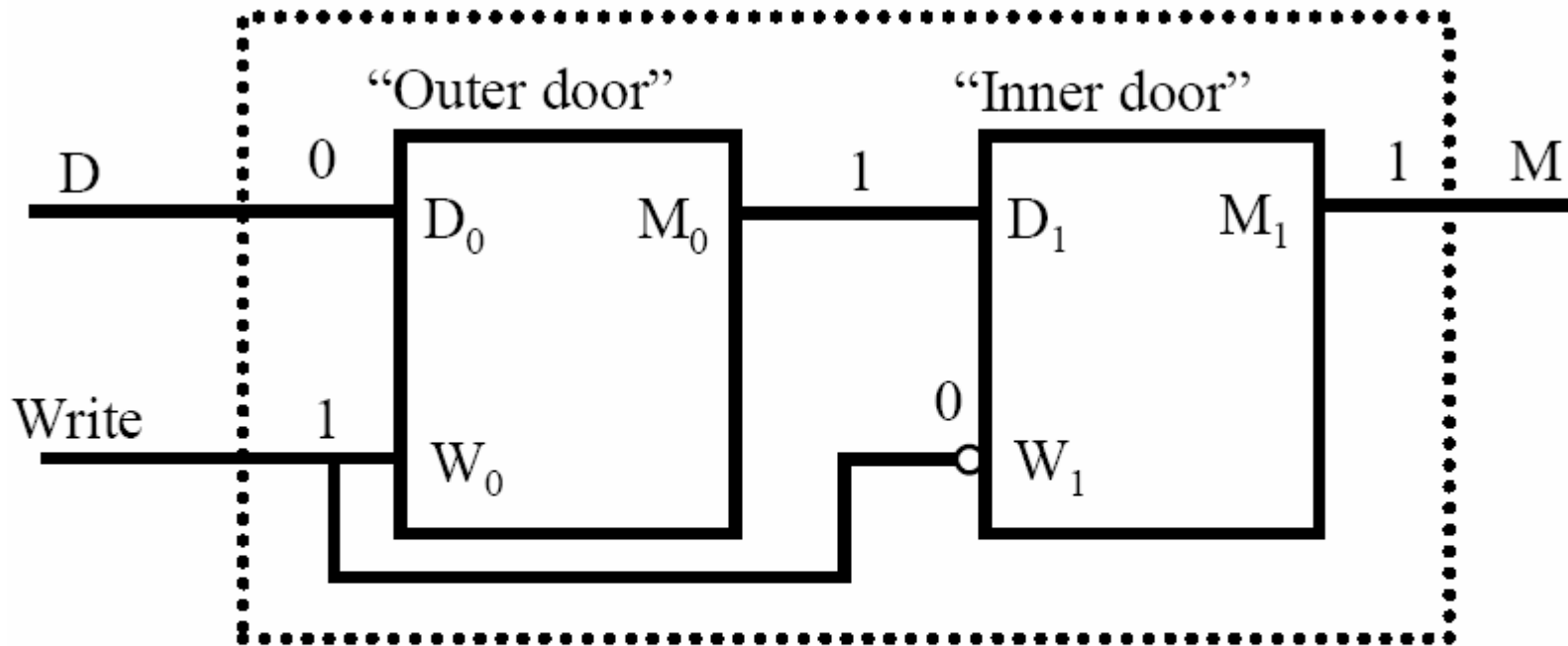
- We start with $Write = 0$.
- Let's say D is always NOT M ; i.e. connected by NOT gate. Start with $D = 0, M = 1$.

The “Airlock” Flip-Flop



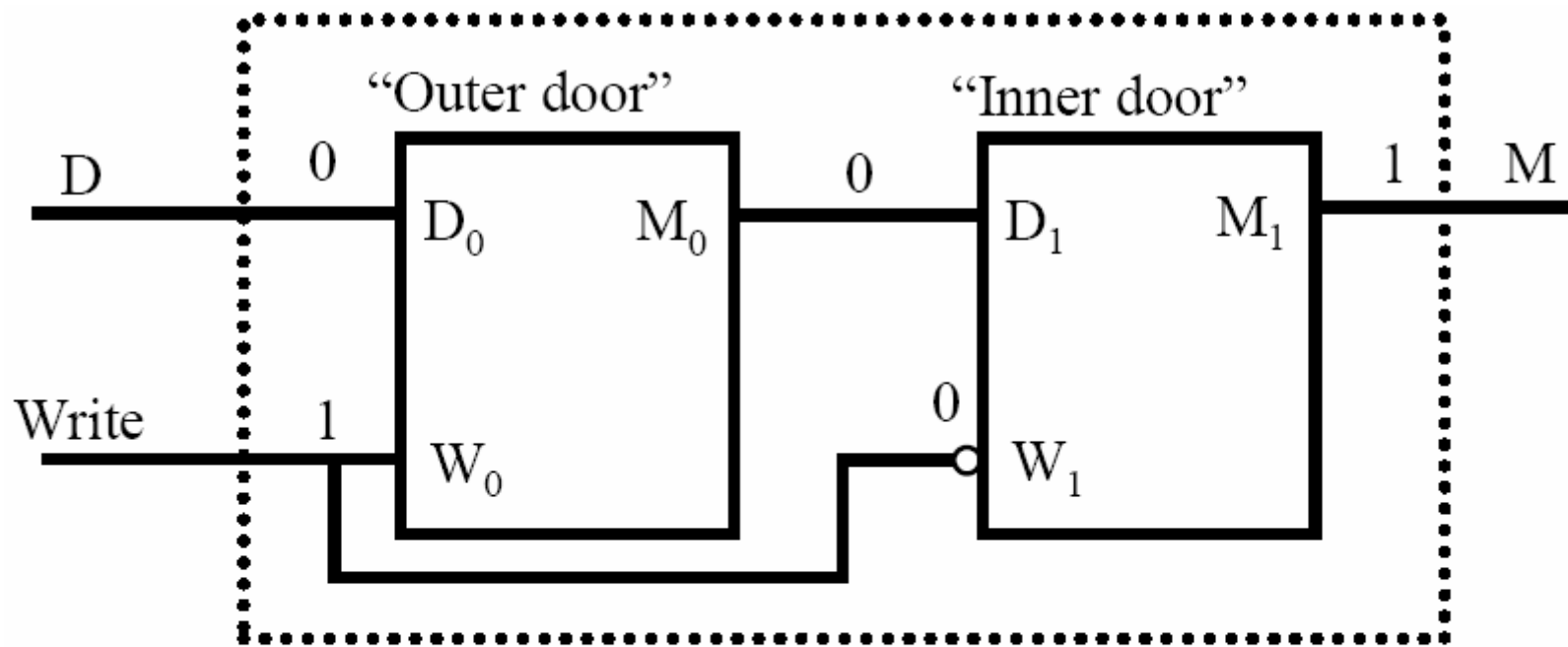
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The “Airlock” Flip-Flop



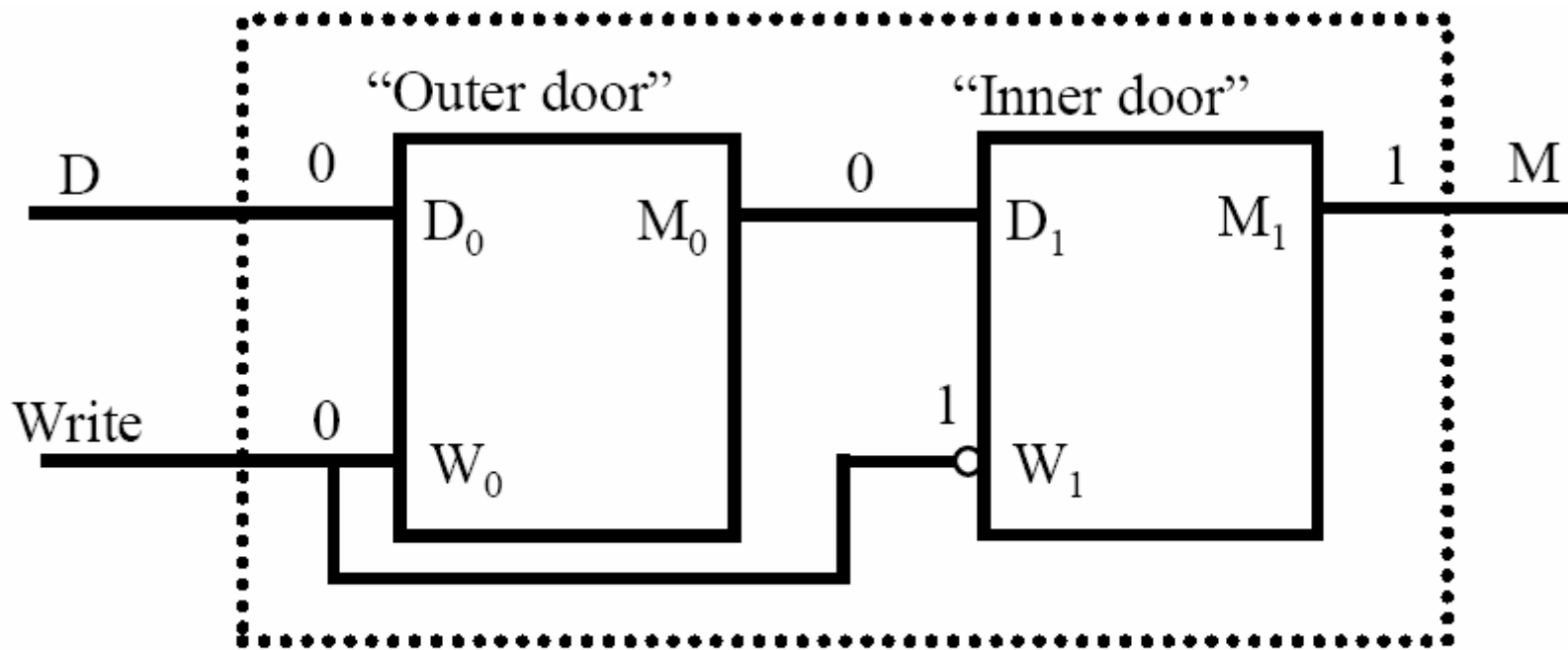
- Want to store D in memory.
- Set $Write$ to 1

The “Airlock” Flip-Flop



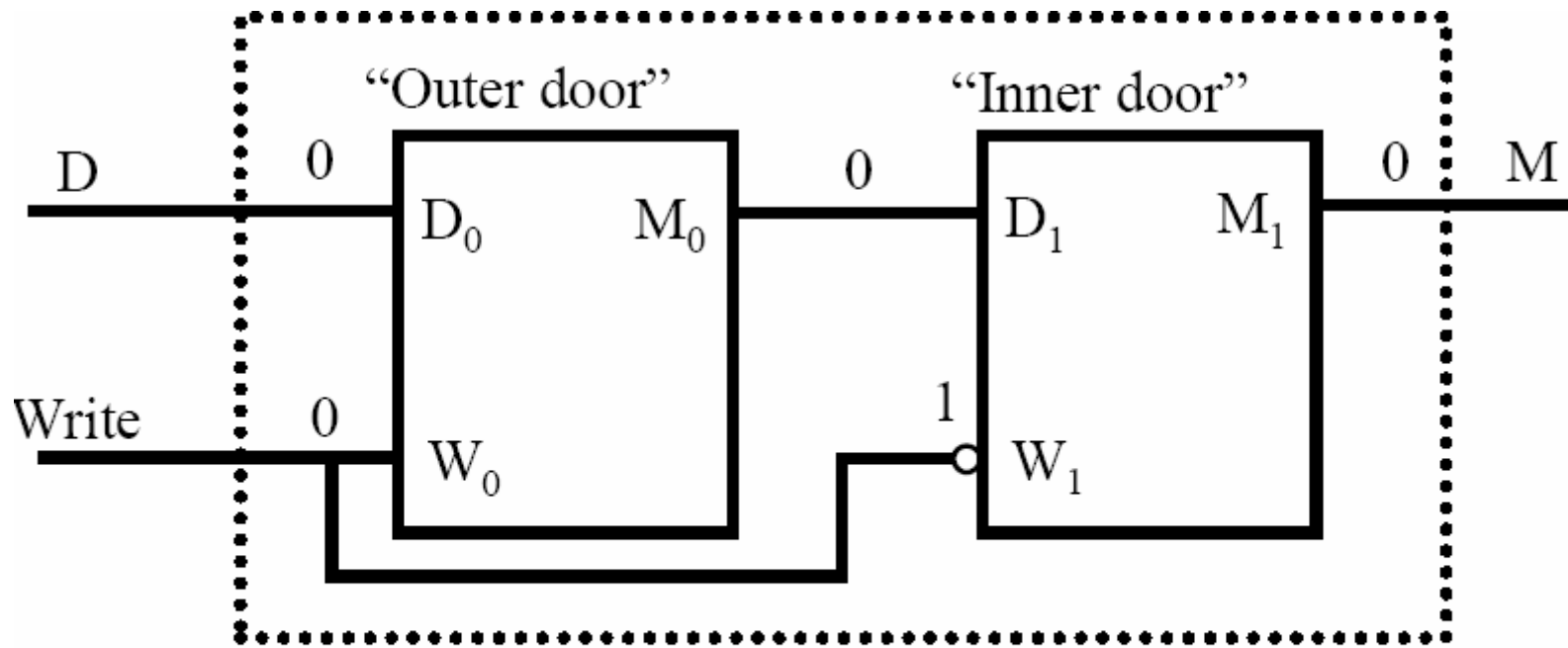
- Want to store D in memory.
- Set $Write$ to 1
- “Outer” flip-flop sets $M_0 = D_0 = 0$
- “Inner” flip-flop ignores D_1 since $W_1 = 0$

The “Airlock” Flip-Flop



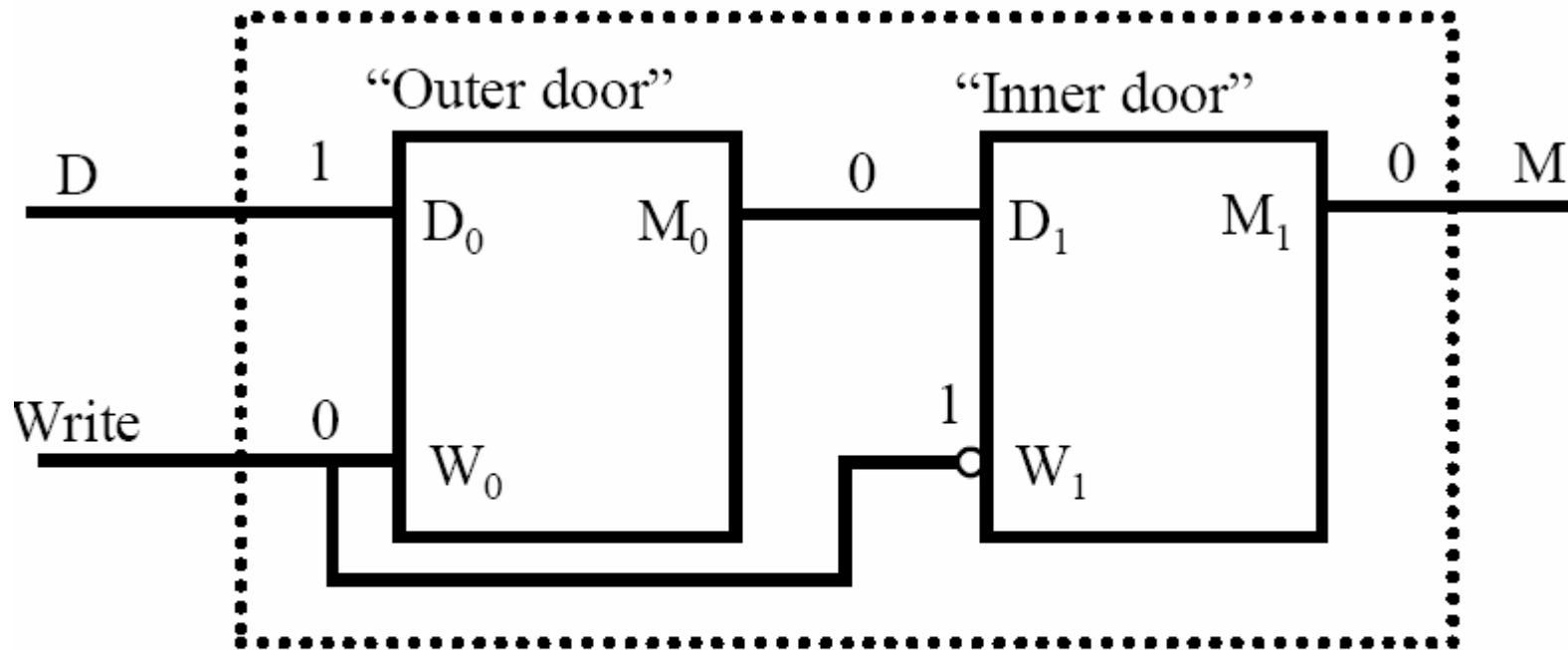
- Now, set Write back to 0

The “Airlock” Flip-Flop



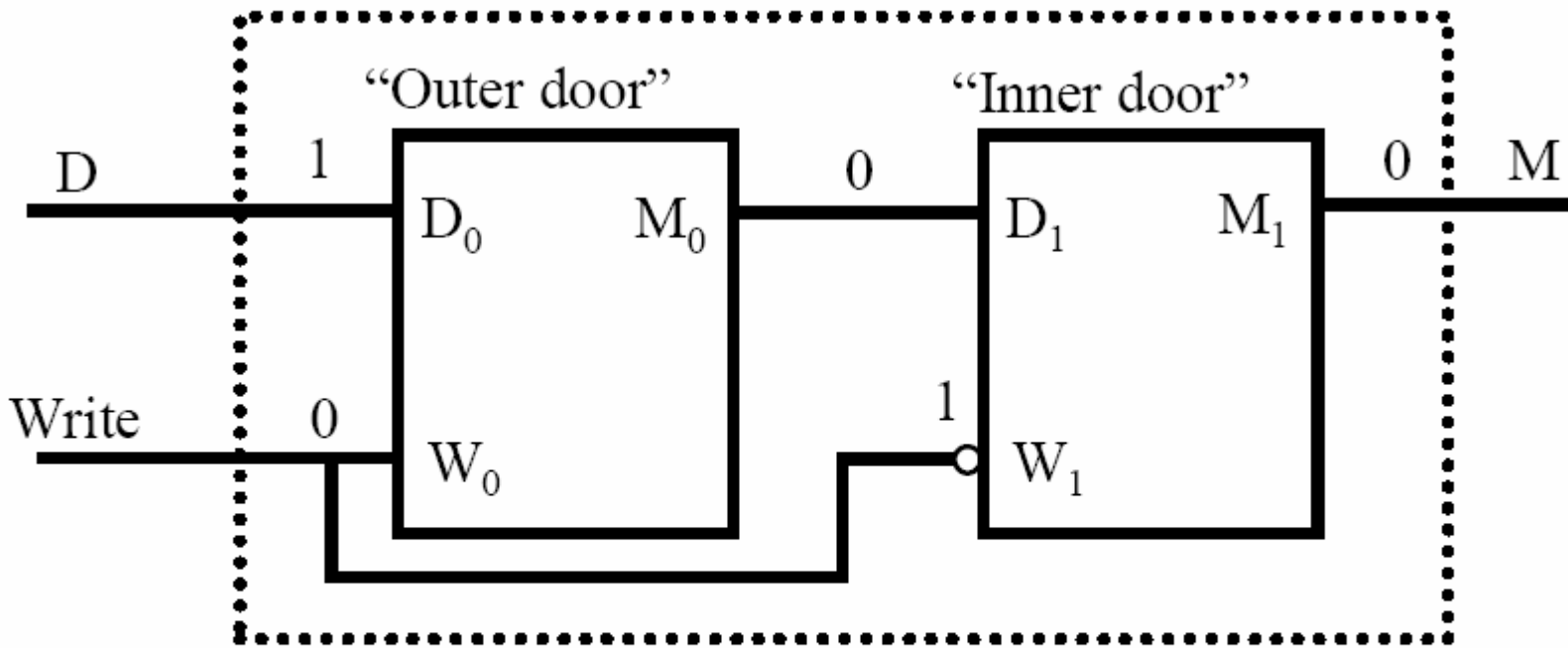
- Now, set $Write$ back to 0
- Now “Inner” flip-flop sets $M = D_1 = 0$

The “Airlock” Flip-Flop



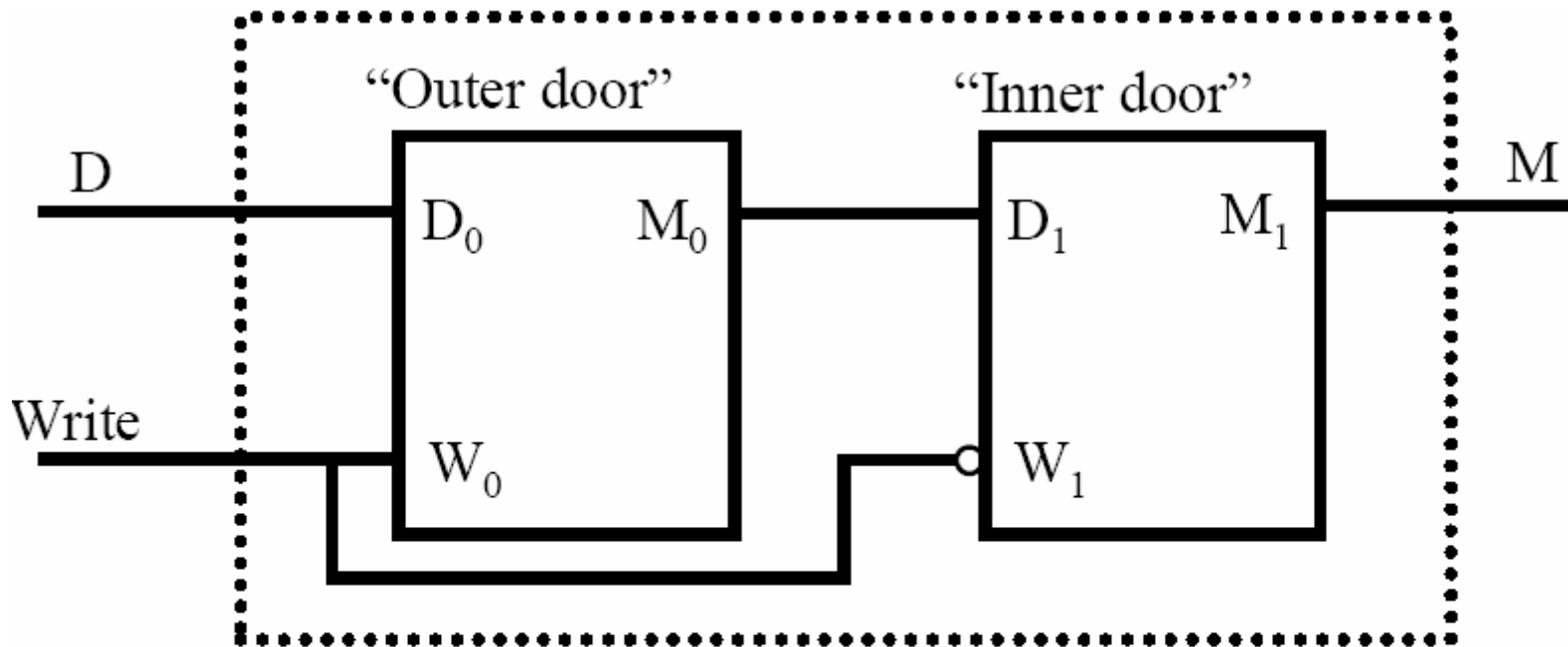
- Because of feedback, D might change to (NOT M), which is 1
- But $Write = 0$, so “Outer” flip-flop ignores D , so M_0 stays 0.

The “Airlock” Flip-Flop



- So memory does not change until we “toggle” Write.
- (“toggle” means change from 0 to 1 or vice versa)

The “Airlock” Flip-Flop



- This is Real Memory!

What controls the “Write” signal?

- Often, the system clock!
- “clock” = device that sends out a fluctuating voltage signal that looks like this



“Computer speed” often refers to the clock frequency (e.g. 2.4GHz)



Next time

Finite State Machines and Clocked Circuits

Memory “Register”: 4 bits

