Readings and Discussion points for February 16 2006

Conway's Game of Life

You know at the back of your mind that computers are used for large-scale simulations, say a simulation of the US economy to predict next year's growth rate, or a simulation of the earth's atmosphere to predict the weather.

Today we will think about simulations in the context of a simple example: Princeton math professor John H. Conway's *Game of Life*. Conway invented "Life" in the 1960s as a "solitaire game" that would be played out by hand on a board with a square grid and round marker pieces. As computers became cheaper and plentiful in the ensuing decades, it became possible to simulate "Life" very fast. The game became an obsession for high school kids and serious scientists alike. It was used as a metaphor for (and a tool to study) complex behavior arising from simple local rules.

Read the handout, an excerpt from *The cellular automaton offers a model of the world and a world unto itself* by Brian Hayes, Scientific American March 1984 (pages 1 and 2 give a good introduction, page 3 is somewhat technical, but page 4 might be useful.)

Imagine an extremely large square grid ($N \times N$) on each of whose squares there may be exactly one or zero organisms (hereafter, "critters"). Each square has 8 neighbors, namely, the squares above and below it, to its left and right, plus four at the corners. (Note that the squares at the corners and edges of the grid have fewer neighbors.) This population of critters evolves, generation by generation, as follows:

1. *Survival:* A critter survives to the next generation if it has exactly 2 or 3 neighbors.
2. *Death:* A critter dies (i.e., does not live on to the next generation and its square becomes empty) from loneliness if it has 1 or fewer neighbors and from overcrowding if it has 4 or more.
3. *Birth:* In an empty square with exactly 3 living neighbors ("parents"?) a critter is born in the next generation.

Take a small grid and a starting pattern of critters, and using a pencil and eraser, simulate its evolution over a few generations.

Think about how you would write pseudocode to simulate the Game of Life on an $N \times N$ grid for T steps, where T is a number stored in a variable. Assume that the grid is represented by an $N \times N$ array $L[1,\ldots,N][1,\ldots,N]$, where $L[i][j]$ corresponds to the square at the intersection of the ith row and jth column. Use the convention that $L[i][j] = 1$ if this cell currently contains a critter, and $L[i][j] = 0$ if it is empty. (Do not worry about how L was initialized.) Use any other variables, arrays etc. as needed.

When first writing your pseudocode ignore the fact that the squares at the corners and edges do not have 8 neighbors. Then think about how to make your algorithm correct for those squares as well.

Finally, visit `http://www.math.com/students/wonders/life/life.html` to see some simulations of actual populations.