

# Image Warping, Compositing & Morphing

Adam Finkelstein  
Princeton University  
COS 426, Spring 2003



## Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warp
- Combining
  - Composite
  - Morph



## Image Processing

- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warp
- Combining
  - Composite
  - Morph



## Image Warping

- Move pixels of image
  - Mapping
  - Resampling



Source image

Warp



Destination image



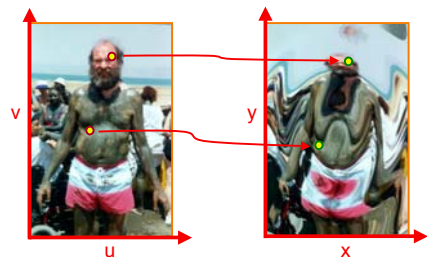
## Overview

- Mapping
  - Forward
  - Reverse
- Resampling
  - Point sampling
  - Triangle filter
  - Gaussian filter



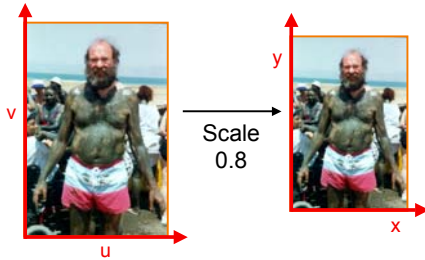
## Mapping

- Define transformation
  - Describe the destination  $(x,y)$  for every location  $(u,v)$  in the source (or vice-versa, if invertible)



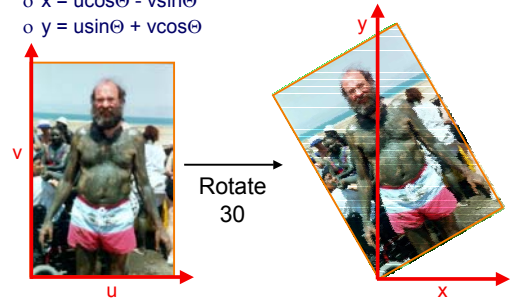
## Example Mappings

- Scale by *factor*:
  - $x = factor * u$
  - $y = factor * v$



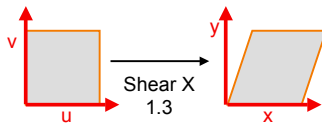
## Example Mappings

- Rotate by  $\Theta$  degrees:
  - $x = u \cos \Theta - v \sin \Theta$
  - $y = u \sin \Theta + v \cos \Theta$

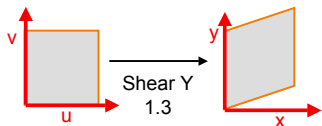


## Example Mappings

- Shear in X by *factor*:
  - $x = u + factor * v$
  - $y = v$



- Shear in Y by *factor*:
  - $x = u$
  - $y = v + factor * u$



## Other Mappings

- Any function of  $u$  and  $v$ :
  - $x = f_x(u, v)$
  - $y = f_y(u, v)$



Fish-eye



"Swirl"



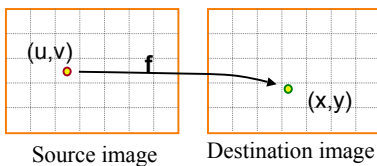
"Rain"

## Image Warping Implementation I

- Forward mapping:

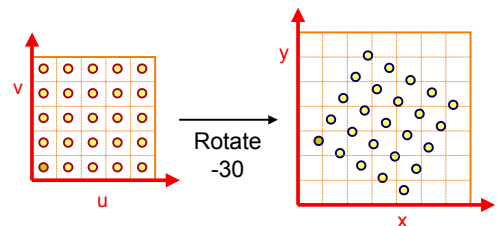
```

for (int u = 0; u < umax; u++) {
    for (int v = 0; v < vmax; v++) {
        float x = f_x(u, v);
        float y = f_y(u, v);
        dst(x, y) = src(u, v);
    }
}
    
```



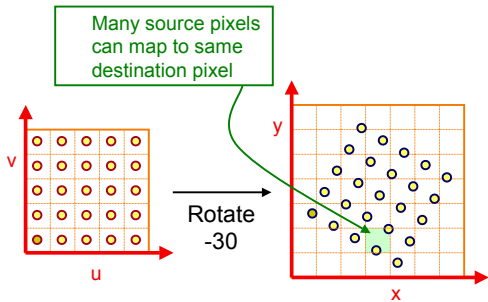
## Forward Mapping

- Iterate over source image



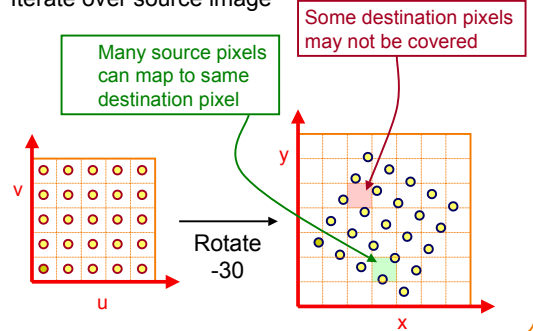
## Forward Mapping - NOT

- Iterate over source image



## Forward Mapping - NOT

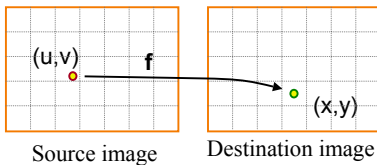
- Iterate over source image



## Image Warping Implementation II

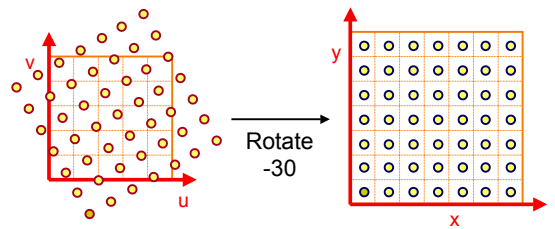
- Reverse mapping:

```
for (int x = 0; x < xmax; x++) {
  for (int y = 0; y < ymax; y++) {
    float u = f_x^{-1}(x,y);
    float v = f_y^{-1}(x,y);
    dst(x,y) = src(u,v);
  }
}
```



## Reverse Mapping

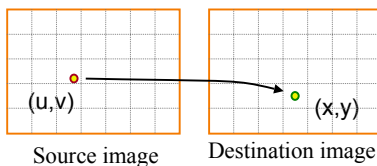
- Iterate over destination image
  - Must resample source
  - May oversample, but much simpler!



## Resampling

- Evaluate source image at arbitrary  $(u,v)$

$(u,v)$  does not usually have integer coordinates



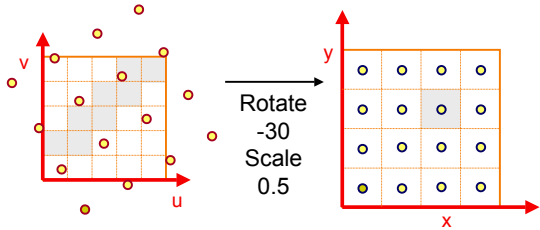
## Overview

- Mapping
  - Forward
  - Reverse
- » Resampling
  - Point sampling
  - Triangle filter
  - Gaussian filter

## Point Sampling

- Take value at closest pixel:
  - $\text{int } iu = \text{trunc}(u+0.5);$
  - $\text{int } iv = \text{trunc}(v+0.5);$
  - $\text{dst}(x,y) = \text{src}(iu,iv);$

This method is simple, but it causes aliasing



## Triangle Filtering

- Convolve with triangle filter

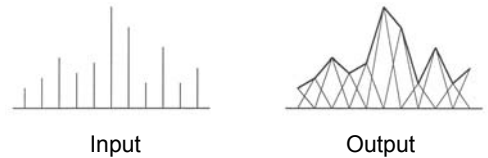
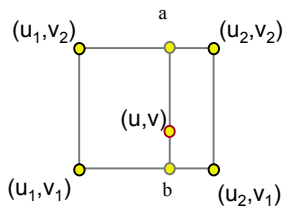


Figure 2.4 Wolberg

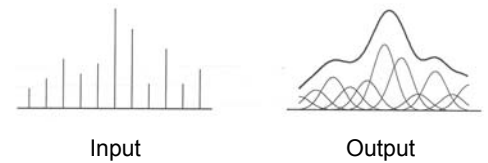
## Triangle Filtering

- Bilinearly interpolate four closest pixels
  - $a = \text{linear interpolation of } \text{src}(u_1, v_2) \text{ and } \text{src}(u_2, v_2)$
  - $b = \text{linear interpolation of } \text{src}(u_1, v_1) \text{ and } \text{src}(u_2, v_1)$
  - $\text{dst}(x,y) = \text{linear interpolation of "a" and "b"}$



## Gaussian Filtering

- Convolve with Gaussian filter

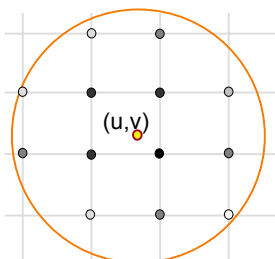


Width of Gaussian kernel affects blurriness

Figure 2.4 Wolberg

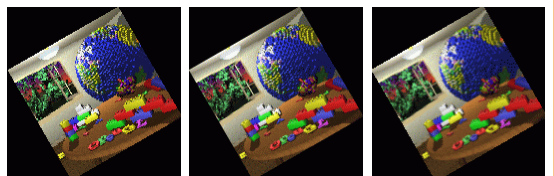
## Gaussian Filtering

- Compute weighted sum of pixel neighborhood:
  - Weights are normalized values of Gaussian function



## Filtering Methods Comparison

- Trade-offs
  - Aliasing versus blurring
  - Computation speed



Point

Bilinear

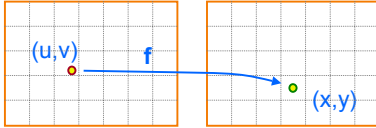
Gaussian

## Image Warping Implementation



- Reverse mapping:

```
for (int x = 0; x < xmax; x++) {
    for (int y = 0; y < ymax; y++) {
        float u =  $f_x^{-1}(x,y)$ ;
        float v =  $f_y^{-1}(x,y)$ ;
        dst(x,y) = resample_src(u,v,w);
    }
}
```



Source image

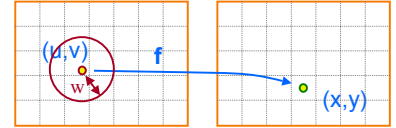
Destination image

## Image Warping Implementation



- Reverse mapping:

```
for (int x = 0; x < xmax; x++) {
    for (int y = 0; y < ymax; y++) {
        float u =  $f_x^{-1}(x,y)$ ;
        float v =  $f_y^{-1}(x,y)$ ;
        dst(x,y) = resample_src(u,v,w);
    }
}
```



Source image

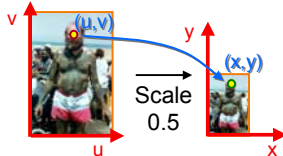
Destination image

## Example: Scale



- Scale (src, dst, sx, sy):

```
float w = max(1.0/sx, 1.0/sy);
for (int x = 0; x < xmax; x++) {
    for (int y = 0; y < ymax; y++) {
        float u = x / sx;
        float v = y / sy;
        dst(x,y) = resample_src(u,v,w);
    }
}
```

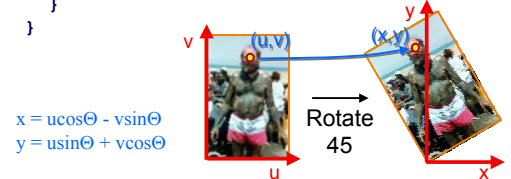


## Example: Rotate



- Rotate (src, dst, theta):

```
for (int x = 0; x < xmax; x++) {
    for (int y = 0; y < ymax; y++) {
        float u = x*cos(-theta) - y*sin(-theta);
        float v = x*sin(-theta) + y*cos(-theta);
        dst(x,y) = resample_src(u,v,w);
    }
}
```



$$x = u \cos \theta - v \sin \theta$$

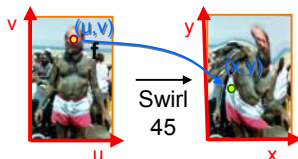
$$y = u \sin \theta + v \cos \theta$$

## Example: Fun



- Swirl (src, dst, theta):

```
for (int x = 0; x < xmax; x++) {
    for (int y = 0; y < ymax; y++) {
        float u = rot(dist(x,xcenter)*theta);
        float v = rot(dist(y,ycenter)*theta);
        dst(x,y) = resample_src(u,v,w);
    }
}
```



## Image Processing



- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warp
- Combining
  - Composite
  - Morph

## Overview: combining images



- Image compositing
  - Blue-screen mattes
  - Alpha channel
  - Porter-Duff compositing algebra
- Image morphing
  - Specifying correspondences
  - Warping
  - Blending

## Even CG folks can win an Oscar



Smith Duff Catmull Porter

## Image Compositing



- Separate an image into “elements”
  - Render independently
  - Composite together
- Applications
  - Cel animation
  - Chroma-keying
  - Blue-screen matting



Dobkin meets the King

## Blue-Screen Matting



- Composite foreground and background images
  - Create background image
  - Create foreground image with blue background
  - Insert non-blue foreground pixels into background

Problem: no partial coverage!



## Alpha Channel



- Encodes pixel coverage information
  - $\alpha = 0$ : no coverage (or transparent)
  - $\alpha = 1$ : full coverage (or opaque)
  - $0 < \alpha < 1$ : partial coverage (or semi-transparent)
- Example:  $\alpha = 0.3$



Partial Coverage

or

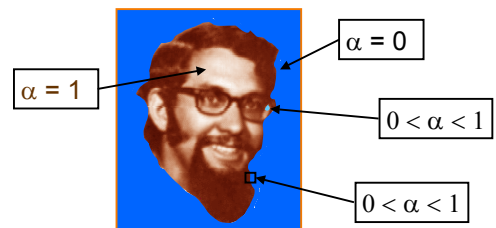


Semi-Transparent

## Compositing with Alpha



Controls the linear interpolation of foreground and background pixels when elements are composited.



## Pixels with Alpha



- Alpha channel convention:
  - $(r, g, b, \alpha)$  represents a pixel that is  $\alpha$  covered by the color  $C = (r/\alpha, g/\alpha, b/\alpha)$ 
    - » Color components are premultiplied by  $\alpha$
    - » Can display  $(r,g,b)$  values directly
    - » Closure in composition algebra
- What is the meaning of the following?
  - $(0, 1, 0, 1) = ?$
  - $(0, 1/2, 0, 1)$  Full green, full coverage
  - $(0, 1/2, 0, 1/2)$  Half green, full coverage
  - $(0, 1/2, 0, 0) = ?$  Full green, half coverage
  - No coverage

## Semi-Transparent Objects



- Suppose we put A over B over background G



- How much of B is blocked by A?
 
$$\alpha_A$$

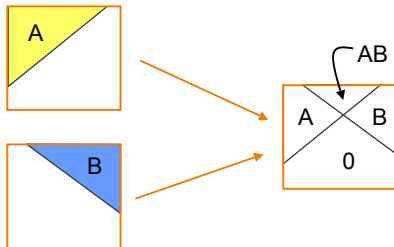
- How much of B shows through A?
 
$$(1-\alpha_A)$$

- How much of G shows through both A and B?
 
$$(1-\alpha_A)(1-\alpha_B)$$

## Opaque Objects



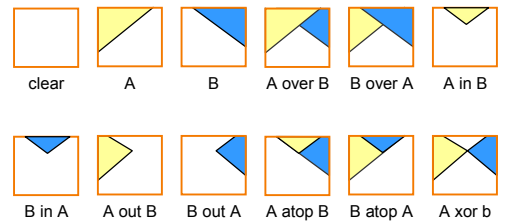
- How do we combine 2 partially covered pixels?
  - 3 possible colors (0, A, B)
  - 4 regions (0, A, B, AB)



## Composition Algebra



- 12 reasonable combinations



Porter & Duff '84

## Example: $C = A$ Over $B$



- For colors that are not premultiplied:

$$C = \alpha_A A + (1-\alpha_A) \alpha_B B$$

$$\alpha = \alpha_A + (1-\alpha_A) \alpha_B$$

- For colors that are premultiplied:

$$C' = A' + (1-\alpha_A) B'$$

$$\alpha = \alpha_A + (1-\alpha_A) \alpha_B$$



A over B

Assumption:  
coverages of A and B  
are uncorrelated  
for each pixel

## Image Composition Example



Jurassic Park

## Overview

- Image compositing
  - Blue-screen mattes
  - Alpha channel
  - Porter-Duff compositing algebra
- Image morphing
  - Specifying correspondences
  - Warping
  - Blending



## Image Morphing

- Animate transition between two images

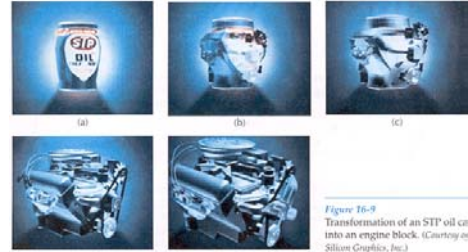


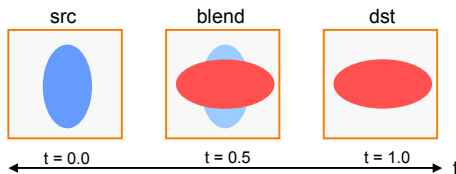
Figure 16-9  
Transformation of an STP oil can  
into an engine block. (Courtesy of  
Silicon Graphics, Inc.)

H&B Figure 16.9

## Cross-Dissolving

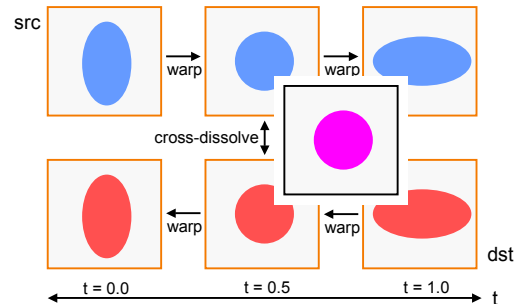
- Blend images with “over” operator
  - alpha of bottom image is 1.0
  - alpha of top image varies from 0.0 to 1.0

$$\text{blend}(i,j) = (1-t) \text{src}(i,j) + t \text{dst}(i,j) \quad (0 \leq t \leq 1)$$



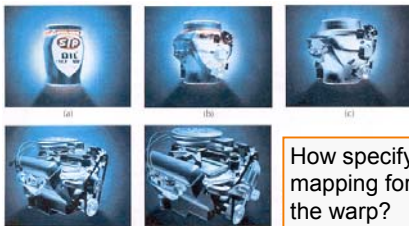
## Image Morphing

- Combines warping and cross-dissolving



## Image Morphing

- The warping step is the hard one
  - Aim to align features in images



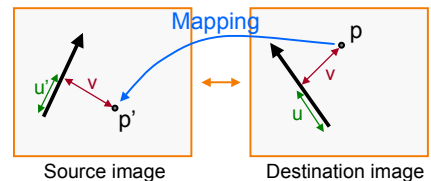
How specify  
mapping for  
the warp?

H&B Figure 16.9



## Feature-Based Warping

- Beier & Neeley use pairs of lines to specify warp
  - Given  $p$  in dst image, where is  $p'$  in source image?



$u$  is a fraction  
 $v$  is a length (in pixels)

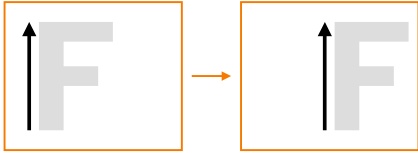
Beier & Neeley  
SIGGRAPH 92





### Warping with One Line Pair

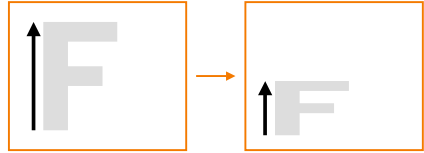
- What happens to the “F”?



Translation!

### Warping with One Line Pair

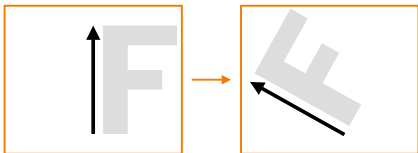
- What happens to the “F”?



Scale!

### Warping with One Line Pair

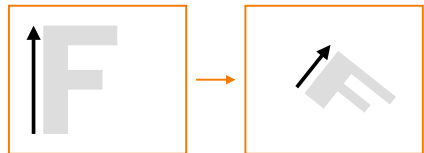
- What happens to the “F”?



Rotation!

### Warping with One Line Pair

- What happens to the “F”?

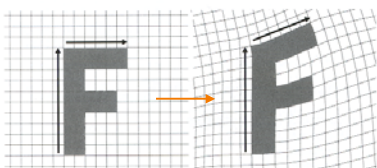


In general, similarity transformations

*What types of transformations can't be specified?*

### Warping with Multiple Line Pairs

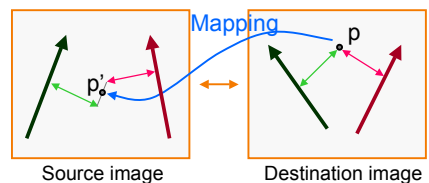
- Use weighted combination of points defined by each pair of corresponding lines



Beier & Neeley, Figure 4

### Warping with Multiple Line Pairs

- Use weighted combination of points defined by each pair of corresponding lines



$p'$  is a weighted average

## Weighting Effect of Each Line Pair

- To weight the contribution of each line pair, Beier & Neeley use:

$$weight[i] = \left( \frac{length[i]^p}{a + dist[i]} \right)^b$$

Where:

- $length[i]$  is the length of  $L[i]$
- $dist[i]$  is the distance from  $X$  to  $L[i]$
- $a, b, p$  are constants that control the warp

## Warping Pseudocode

```

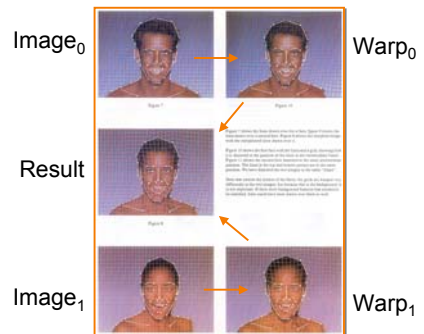
WarpImage(Image, L'[...], L[...])
begin
  foreach destination pixel p do
    psum = (0,0)
    wsum = 0
    foreach line L[i] in destination do
      p'[i] = p transformed by (L[i], L'[i])
      psum = psum + p'[i] * weight[i]
      wsum += weight[i]
    end
    p' = psum / wsum
    Result(p) = Image(p')
  end
end
    
```

## Morphing Pseudocode

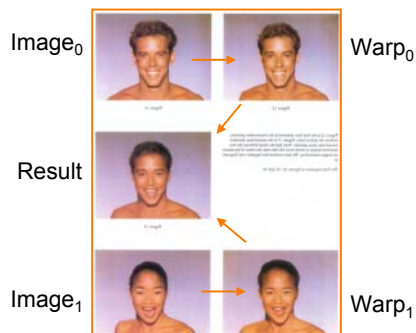
```

GenerateAnimation(Image0, L0[...], Image1, L1[...])
begin
  foreach intermediate frame time t do
    for i = 1 to number of line pairs do
      L[i] = line t-th of the way from L0[i] to L1[i]
    end
    Warp0 = WarpImage(Image0, L0, L)
    Warp1 = WarpImage(Image1, L1, L)
    foreach pixel p in FinalImage do
      Result(p) = (1-t) Warp0 + t Warp1
    end
  end
end
    
```

## Beier & Neeley Example



## Beier & Neeley Example



## CS426 Examples



CS426 Class, Fall198



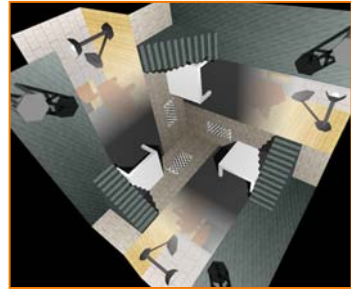
Robert Osada, Fall100

## Image Processing



- Quantization
  - Uniform Quantization
  - Random dither
  - Ordered dither
  - Floyd-Steinberg dither
- Pixel operations
  - Add random noise
  - Add luminance
  - Add contrast
  - Add saturation
- Filtering
  - Blur
  - Detect edges
- Warping
  - Scale
  - Rotate
  - Warp
- Combining
  - Composite
  - Morph

## Next Time: 3D Rendering



Misha Kazhdan,  
CS426, Fall99