Outline

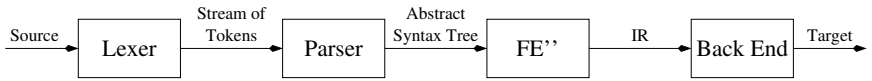
- ML-Yacc
- Recursive Descent Parser Generation

Syntax Analysis

Front End:

- Lexical Analysis - Break source into *tokens*.
- Syntax Analysis - Parse phrase structure.
- Semantic Analysis - Calculate meaning.

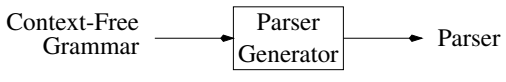
Our Compiler:

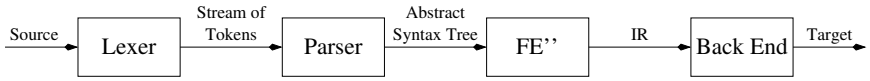


Parser Functions:

- Verify that token stream is valid.
- If it is not valid, report syntax error and recover.
- Build Abstract Syntax Tree (AST).

ML-YACC ("Yet Another Compiler-Compiler")





- Input to **ml-yacc** is a context-free grammar specification.
- Output from **ml-yacc** is a shift-reduce parser in ML.

Context-Free Grammar Specification

- CFG specification consists of 3 parts:

  User Declarations
  %%
  ML-YACC Definitions
  %%
  Rules

- **User Declarlarions:** define various values that are available to *rules* section.
- **ML-YACC Definitions:** declare terminal and non-terminal symbols; declare precedences for terminals that help resolve shift-reduce conflicts.
- **Rules:** specify productions of grammar and *semantic actions* associated with productions.

ML-YACC Declarations

- Need to specify type associated with positions of tokens in input file

```
%pos int
```

- Need to specify terminal and non-terminal symbols (no symbols can be in both lists)

```
%term IF | THEN | ELSE |...
%nonterm prog | stmt | expr |...
```

- Optionally specify end-of-parse symbol - terminals which may follow start symbol

```
%eop EOF
```

- Optionally specify start symbol - otherwise, LHS non-terminal of first rule is taken as start symbol

```
%start prog
```

Attribute Grammar

- ML-YACC employs *attribute grammar* scheme
  - Each terminal or non-terminal symbol may have associated attribute/value.
  - When parser reduces using production $A \rightarrow \alpha$, semantic action associated with production is exectued in order to compute value for A based on the values of symbols in α.
  - Parser returns value associated with start symbol. (If no attribute, `()` is returned.)
- Can specify *types* of atttributes associtated with symbols.

```
%term     ID of string | NUM of int | IF | THEN | ...
%nonterm  prgm | stmt | expr of int | ...
```

Rules

$symbol_0$: $symbol_1$ $symbol_2$ ... $symbol_n$ (*semantic_action*)

- Semantic action typically builds piece of AST corresponding to derived string
- Can access attribute/value of RHS symbol X using X<n>, where n specifies a particular occurrence of X on RHS.

```
%term    PLUS | MINUS | NUM of int | ...
%nonterm exp of int | ...

exp: exp PLUS exp     (exp1 + exp2)
|    exp MINUS exp    (exp1 - exp2)
|    NUM              (NUM)
```

- Type of value computed by semantic action must match type of value associated with LHS non-terminal.

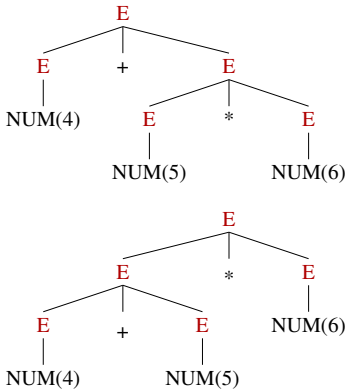
Example

```
%%

%term    ID | NUM | PLUS | MINUS | MULT | DIV | EOF
%nonterm expr
%pos int
%start expr
%eop EOF
%verbose

%%

expr : ID               ()
     | NUM              ()
     | expr PLUS expr   ()
     | expr MINUS expr  ()
     | expr MULT expr   ()
     | expr DIV expr    ()
```

ML-YACC and Ambiguous Grammars

- A grammar is ambiguous if it can derive a string of tokens with two or more different parse trees.
- Consider: $4 + 5 * 6$, `NUM(4) PLUS NUM(5) MULT NUM(6)`

$$
\begin{aligned}
expr &\rightarrow ID \\
expr &\rightarrow NUM \\
expr &\rightarrow expr + expr \\
expr &\rightarrow expr * expr
\end{aligned}
$$



- We perfer to bind "*" tighter than "+".

ML-YACC and Ambiguous Grammars

- Similarly Consider: $4 + 5 + 6$, `NUM(4) PLUS NUM(5) PLUS NUM(6)`
- We perfer to bind left "+" first.
- ML-YACC will report *shift-reduce* conflicts when parsing strings.
  - $4 + 5 * 6$, `NUM(4) PLUS NUM(5) MULT NUM(6)`
    - At some point, E + E will be on top of stack, "*" will be the current token-type in stream.
    - Parser can reduce by rule E → E + E, or shift. Prefer *shift*.
  - $4 + 5 + 6$, `NUM(4) PLUS NUM(5) PLUS NUM(6)`
    - At some point, E + E will be on top of stack, "+" will be the current token-type in stream.
    - Parser can reduce by rule E → E + E, or shift. Prefer *reduce*.

Directives

Three Solutions:

1. Let YACC complain, but demonstrate that its choice (to shift) was correct.
2. Rewrite grammar to eliminate ambiguity.
3. Keep grammar, but add *precedence directives* which enable conflicts to be resolved. Use `%left, %right, %nonassoc`
   - For this grammar:
     ```
     %left PLUS MINUS
     %left MULT DIV
     ```
   - PLUS, MINUS are left associative, bind equally tightly
   - MULT, DIV are left associative, bind equally tightly
   - MULT, DIV bind tighter than PLUS, MINUS

Directives

- Given directives, ML-YACC assigns precedence to each *terminal* and *rule*
  - Precedence of terminal based on order in which associativity specified
  - Precedence of rule is the precedence of right-most terminal. For example, precedence(E → E + E) = precedence(PLUS).
- Given shift-reduce conflict, ML-YACC performs the following:
  1. Find precedence of rule to be reduced, terminal to be shifted.
  2. prec(terminal) > prec(rule) ⇒ shift.
  3. prec(rule) > prec(terminal) ⇒ reduce.
  4. prec(terminal) = prec(rule), then:
     - assoc(terminal) = left ⇒ reduce.
     - assoc(terminal) = right ⇒ shift.
     - assoc(terminal) = nonassoc ⇒ report as error.

Precedence Examples

1

```
input: 4 + 5 * 6
               |
stack: 4 + 5
action: prec(*) > prec(+) -> shift
```

2

```
input: 4 * 5 + 6
               |
stack: 4 * 5
action: prec(*) > prec(+) -> reduce
```

3

```
input: 4 + 5 + 6
               |
stack: 4 + 5
action: assoc(+) = left -> reduce
```

Default Behavior

What if directives not specified?

- shift-reduce: report error, *shift* by default.
- reduce-reduce: report error, reduce by rule that occurs first.

What to do:

- shift-reduce: acceptable in well defined cases (dangling else).
- reduce-reduce: unnacceptable. Rewrite grammar.

Direct Rule Precedence Specification

Can assign *specific* precedence to rule, rather than precedence of last terminal.

- Use the %prec directive.
- Commonly used for the *unary minus* problem.

```
%left PLUS MINUS
%left MULT DIV
```

- Consider $-4 * 6$, `MINUS NUM(4) MULT NUM(6)`
- We perfer to bind left unary minus ("-") tighter. Here, precedence of MINUS is lower than MULT, so we get $-(4 * 6)$, not $(-4) * 6$.
- Solution:

```
%left PLUS MINUS
%left MULT DIV
%left UMINUS

exp : MINUS expr %prec UMINUS ()
    | expr PLUS expr ()...
```

Syntax vs. Semantics

Consider language with two classes of expressions

- *Arithmetic* expressions (ae)

```
ae : ae PLUS ae ()
   | ID         ()
```

- *Boolean* expressions (be)

```
be : be AND be ()
   | be OR be  ()
   | be EQ be  ()
   | ID        ()
```

- Consider: `a := b`, ID(a) ASSIGN ID(b):
  - Reduce-reduce conflict - parser can't choose between be → ID or ae → ID.
  - For now ae and be should be aliased - let semantic analysis (next phase) determine that `a & b + c` is a type error.
  - Type checking cannot be done easily in context free grammars.

Recursive Descent/Predictive/LL(1) Parser Generation

Grammar:

A → S EOF
S → *id* := E
S → *print* (L)
E → *id*
E → *num*
L → E
L → L, E

```
fun A() =              (S(); eat(EOF))
and S() = case !tok of
            ID      => (eat(ID); eat(ASSIGN); E())
            PRINT   => (eat(PRINT); eat(LPAREN);
                        L(); eat(RPAREN))
and E() = case !tok of
            ID      => (eat(ID))
            NUM     => (eat(NUM))
and L() = case !tok of
            ID      => (?????)
            NUM     => (?????)
```

Problem

- Based on current function and next token-type in input stream, parser must predict which production to use.
- If !tok = ID, parser cannot determine which production to use:
  L → E (E could be ID)
  L → L, E (L could be ID)
- Predictive parsing only works for grammars where first terminal symbol of each subexpression provides enough information to choose which production to use.

Algorithms for Constructing Predictive Parsers

To construct a predictive parser, what we want to know is which rule to apply when

- We have a particular non-terminal (say X) that needs to be expanded, and
- we see a particular terminal symbol (say a).

In general, we will apply the rule $X \rightarrow \gamma$ when

- a is the *first* symbol that can be generated by the string γ, or
- γ reduces to the empty string (is *nullable*) and a is the first symbol in a string that can *follow* X.

Algorithms for Constructing Predictive Parsers

Figuring out these First and Follow sets isn't quite as easy as it sounds at first. Consider this grammar:

1. $Y \rightarrow$
2. $Y \rightarrow bb$
3. $X \rightarrow c$
4. $X \rightarrow YZ$
5. $Z \rightarrow d$

Non-terminal to expand	Terminal Seen	Rule to Apply
X	c	?
X	b	?
X	d	?

Algorithms for Constructing Predictive Parsers

To construct a predictive parser:

- Let γ be a string of terminal and non-terminal symbols
- Need to compute 3 values:
  1. For each γ corresponding to RHS of production, must determine if γ can derive empty string (ϵ) $\rightarrow$ **nullable**.
  2. For each γ corresponding to RHS of production, must determine set of all terminal symbols that can begin any string derived from $\gamma \rightarrow$ **first(γ)**.
  3. For each non-terminal X in grammar, must determine set of all terminal symbols that can immediately follow X in a derivation $\rightarrow$ **follow(X)**.

Iterative Analysis Techniques

Iterative analysis techniques often come in handy when constructing compilers.

- Interative analysis applies when:
  - We want to compute the set of all objects that have some property P.
  - The property P is defined *inductively*. In other words, there are some base cases and some inductive cases that define the objects in the property:
    - Base cases: o_1, o_2, o_3 obviously have property P.
    - Inductive cases (inductive rules): if some objects (o_4, o_5 for example) have the property, then we can find other objects that also have the property.
  - We are guaranteed that the total number of objects in the set is *finite*. This guarantees that our interative analysis will eventually terminate.

Iterative Analysis Techniques

Iterative analysis algorithms have the form:

- Initialize the set S that we are looking for with all of the base cases.
- Apply the inductive rules over and over again (this is the iteration), adding all of the new objects until you reach a *fixed point*.

A fixed point is a set that does not change (does not gain new elements) when you apply any of the inductive rules.

- We will be able to construct Nullable, First, and Follow sets using iteration.
- Many program optimizations (dataflow analyses) often use iteration as well.
- Although the worst-case complexity of iterative techniques can be very bad, it often turns out that they are very efficient in practice and terminate in only a couple of iterations.

Computing Nullable Sets

Computation of Nullable:

- Base Case: If a non-terminal S can derive ϵ directly, it is nullable.
- Inductive Case: A non-empty string γ is nullable if every symbol S $\in \gamma$ is nullable.

The Nullable set can be computed first before the First and Follow sets.

Computing First Sets

- Base Case: If T is a terminal symbol, then first(T) = $\{T\}$.
- Inductive Case: If X is a non-terminal and $X \rightarrow Y_1 Y_2 Y_3 ... Y_n$, then

  first(Y_1) $\in$ first(X)
  first(Y_2) $\in$ first(X), if Y_1 is nullable
  first(Y_3) $\in$ first(X), if Y_1, Y_2 is nullable
  ⋮
  first(Y_n) $\in$ first(X), if Y_1, Y_2, ... Y_{n-1} is nullable

- Let $\gamma = S_1 S_2 ... S_k$. Then,

$$\text{first}(\gamma) = \begin{cases} \text{first}(S_1) \\ \text{first}(S_2), \text{if } S_1 \text{ is nullable} \\ \text{first}(S_3), \text{if } S_1, S_2 \text{ is nullable} \\ \vdots \\ \text{first}(S_k), \text{if } S_1, S_2, ..., S_{k-1} \text{ is nullable} \end{cases}$$

(Notice computing First depends upon computing Nullable.)

Computing Follow Sets

Let X, Y be non-terminals; γ, γ_1, and γ_2 be strings of terminals and non-terminals

- Base Case: Follow(X) is the empty set, for all non-terminals X.
- Inductive Case: if grammar includes production: $X \rightarrow \gamma Y$
  $\rightarrow$ follow(X) $\in$ follow (Y).
- Inductive Case: if grammar includes production: $X \rightarrow \gamma_1 Y \gamma_2$
  $\rightarrow$ first (γ_2) $\in$ follow (Y)
  $\rightarrow$ follow (X) $\in$ follow (Y), if γ_2 is nullable.

Building Predictive Parser

$Z \rightarrow XYZ$ $\quad$ $Y \rightarrow$ c $\quad$ $X \rightarrow$ a
$Z \rightarrow$ d $\quad$ $Y \rightarrow \epsilon$ $\quad$ $X \rightarrow$ b Y e

Initial:

	nullable	first	follow
Z	no		
Y	no		
X	no		

First step: Find nullable symbols.

Building Predictive Parser

$Z \rightarrow XYZ$ $\quad$ $Y \rightarrow$ c $\quad$ $X \rightarrow$ a
$Z \rightarrow$ d $\quad$ $Y \rightarrow \epsilon$ $\quad$ $X \rightarrow$ b Y e

Results of finding nullable symbols:

	nullable	first	follow
Z	no		
Y	yes		
X	no		

Next: do first iteration of first and follow sets.

Building Predictive Parser

$Z \rightarrow XYZ$ $\quad$ $Y \rightarrow$ c $\quad$ $X \rightarrow$ a
$Z \rightarrow$ d $\quad$ $Y \rightarrow \epsilon$ $\quad$ $X \rightarrow$ b Y e

Results of first iteration of first and follow:

	nullable	first	follow
Z	no	d	
Y	yes	c	e
X	no	a,b	

Do second iteration of first and follow (since there were changes).

Building Predictive Parser

$Z \rightarrow XYZ$ $\quad$ $Y \rightarrow$ c $\quad$ $X \rightarrow$ a
$Z \rightarrow$ d $\quad$ $Y \rightarrow \epsilon$ $\quad$ $X \rightarrow$ b Y e

Results of second iteration:

Iteration 3:

	nullable	first	follow
Z	no	d,a,b	
Y	yes	c	e,d,a,b
X	no	a,b	c,d,a,b

There were changes. Try another iteration....

Ok, don't bother, there are no more changes. We have found the fixed point.

Predictive Parsing Table

	nullable	first	follow
Z	no	d,a,b	
Y	yes	c	e,d,a,b
X	no	a,b	c,d,a,b

Build *predictive parsing table* from nullable, first, and follow sets.

	a	b	c	d	e
Z	$Z \rightarrow XYZ$	$Z \rightarrow XYZ$		$Z \rightarrow d$	
Y	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$	$Y \rightarrow c$	$Y \rightarrow \epsilon$	$Y \rightarrow \epsilon$
X	$X \rightarrow a$	$X \rightarrow bYe$			

- Enter $S \rightarrow \gamma$ in row S, column T: for each $T \in$ first(γ).
- If γ is nullable, enter $S \rightarrow \gamma$ in row S, column T: for each $T \in$ follow(S).
- Entry in row S, column T tells parser which clause to execute if current function is $S()$ and next token-type is T
- Blank entries are syntax errors.

Predictive Parsing Table

If the predictive parsing table contains *no* duplicate entries, can build predictive parser for grammar.

- Grammar is LL(1) (left-to-right parse, left-most derivation, 1 symbol lookahead).
- Grammar is LL(k) if its LL(k) predictive parsing table has no duplicate entries.
  - Rows correspond to non-terminals, columns correspond to every possible sequence of k terminals.
  - The first(γ) = set of all k-length terminal sequences that can begin any string derived from γ.
  - LL(k) paring tables can be too large.
  - Ambiguous grammars are not LL(k), for any k.

Example

$S' \rightarrow S\$$ $\quad$ $S \rightarrow$ IF E THEN A ELSE A $\quad$ $T \rightarrow$ NUM
$S \rightarrow E$ $\quad$ $E \rightarrow E + T$ $\quad$ $A \rightarrow$ ID = NUM
$S \rightarrow$ IF E THEN A $\quad$ $E \rightarrow T$

Iteration 1:

	nullable	first	follow
S'	no		
S	no	IF	\$
E	no		\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Iteration 2:

	nullable	first	follow
S'	no	IF	
S	no	IF	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Example

$S' \rightarrow S\$$ $\quad$ $S \rightarrow$ IF E THEN A ELSE A $\quad$ $T \rightarrow$ NUM
$S \rightarrow E$ $\quad$ $E \rightarrow E + T$ $\quad$ $A \rightarrow$ ID = NUM
$S \rightarrow$ IF E THEN A $\quad$ $E \rightarrow T$

Iteration 3:

	nullable	first	follow
S'	no	IF	
S	no	IF, NUM	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Iteration 4:

	nullable	first	follow
S'	no	IF, NUM	
S	no	IF, NUM	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

No futher changes

Predictive Parsing Table

	nullable	first	follow
S'	no	IF, NUM	
S	no	IF, NUM	\$
E	no	NUM	\$, THEN, +
T	no	NUM	\$, THEN, +
A	no	ID	\$, ELSE

Build *predictive parsing table* from nullable, first, and follow sets.

	IF	THEN	ELSE	+	NUM	ID	=	\$
S'	$S' \rightarrow S$				$S' \rightarrow S$			
S	$S \rightarrow$ IF E THEN A $S \rightarrow$ IF E THEN A ELSE A				$S \rightarrow E$			
E					$E \rightarrow E + T$ $E \rightarrow T$			
T					$T \rightarrow$ NUM			
A						$A \rightarrow$ ID = NUM		

Table has duplicate entries ⇒ grammar is not LL(1)!

Problems

1. $E \rightarrow E + T$
   $E \rightarrow T$

- first(E+T) = first(T)
- When in function `E()`, if next token is NUM, parser will get stuck.
- Grammar is *left-recursive* - left-recursive grammars cannot be LL(1).
- Solution: rewrite grammar so that it is *right-recursive*.
  $E \rightarrow TE'$
  $E' \rightarrow \epsilon$
  $E' \rightarrow +TE'$
- In general, $\begin{array}{l} X \rightarrow X\gamma \\ X \rightarrow \alpha \end{array}$ derives strings of form $\alpha\gamma^*$ (α doesn't start with X).
  These two productions can be rewritten as follows:
  $X \rightarrow \alpha X'$
  $X' \rightarrow \epsilon$
  $X' \rightarrow \gamma X'$

Problems

2. $S \rightarrow$ IF E THEN A
   $S \rightarrow$ IF E THEN A ELSE A

- Two productions begin with same symbol.
- first(IF E THEN A) = first(IF E THEN A ELSE A)
- Solution: use *left-factoring*
  $S \rightarrow$ IF E THEN $A\ V$
  $V \rightarrow \epsilon$
  $V \rightarrow$ ELSE A

Example

Show that modified grammar is LL(1).

$S' \rightarrow S\$$
$S \rightarrow E$
$S \rightarrow$ IF E THEN $A\ V$
$V \rightarrow \epsilon$
$V \rightarrow$ ELSE A
$E \rightarrow TE'$
$E' \rightarrow \epsilon$
$E' \rightarrow$ + TE
$T \rightarrow$ NUM
$A \rightarrow$ ID = NUM

Example

Show that the grammar is LL(1).

Example

Show that modified grammar is LL(1). Build predictive parsing table.

	nullable	first	follow
S'	no	IF,NUM	
S	no	IF,NUM	$
V	yes	ELSE	$
E	no	NUM	$, THEN
E'	yes	+	$, THEN
T	no	NUM	$, THEN, +
A	no	ID	$, ELSE

	IF	THEN	ELSE	+	NUM	ID	=	$
S'	$S' \rightarrow S$				$S' \rightarrow S$			
S	$S \rightarrow$ IF E THEN $A\ V$				$S \rightarrow E$			
V			$V \rightarrow$ ELSE A					$V \rightarrow \epsilon$
E					$E \rightarrow TE'$			
E'		$E' \rightarrow \epsilon$		$E' \rightarrow$ + TE'				$E' \rightarrow \epsilon$
T					$T \rightarrow$ NUM			
A						$A \rightarrow$ ID = NUM		

Table does not have duplicate entries $\Rightarrow$ modified grammar is LL(1)!