Homework 1

Out: *February 13* Due: *February 25*

You are encouraged to collaborate with your classmates, but be sure to list your collaborators with your answer. Remember: False proofs cause brain damage. Do not ever attempt to write down a proof that you are unsure about. It is better to write down your thoughts, and point out where you are stuck. Please be concise in your answers. If you suspect a problem is open, just say so and give reasons for your suspicion.

§1 Show that $\Sigma_2^p = \mathbf{NP}^{\mathrm{SAT}}$.

§2 Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$. (Hint: Use padding, mentioned in the notes for Lecture 1.)

§3 Show that there exists an oracle A such that $\mathbf{NP}^A \neq \mathbf{coNP}^A$.

§4 Given a language $L \subseteq \{0,1\}^*$, let $L_n = L \cap (\cup_{i=0}^{n}(\{0,1\}^i))$. We say that L is *self-reducible* if there exists a polynomial time oracle Turing machine M such that for every $x \in \{0,1\}^n$,

$$x \in L \quad \Leftrightarrow \quad M^{L_{n-1}}(x) \text{ accepts.}$$

(a) Given an example of a self-reducible language.

(b) Prove that if L is self-reducible, then L is in **PSPACE**.

§5 Suppose we define the *logspace hierarchy* in analogy with the polynomial hierarchy using logspace machines that can use alternation. Does this hierarchy collapse by Immerman's theorem ($\mathbf{NL} = \mathbf{coNL}$)?