Assignment 8

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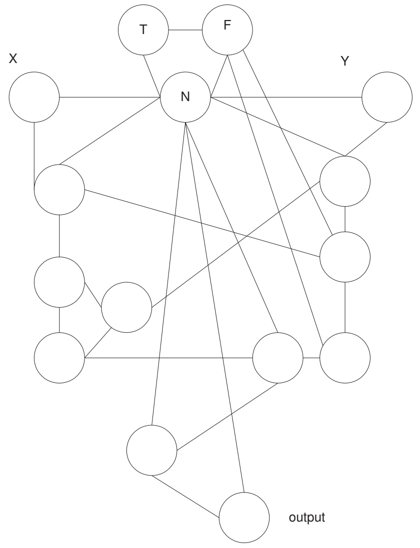
The solution set this time will be a little bit different since the problems were challenging. In addition to offering solutions, I will take you through the thinking process I used to arrive at them.

1. Coloring Circuits and Factoring

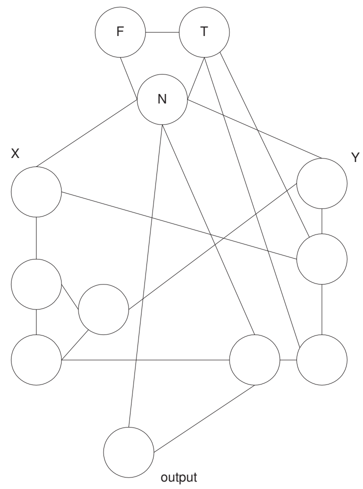
   (a) The first thing I notice is the hint along with the problem set. I also remember how we often build things of smaller parts. Looking into past lectures I notice that an AND gate is an OR gate with both of its inputs inverted and its ouput inverted. I can put this together from three NOT gate circle collections and one OR circle collection.

   Solution using the NOT collections with the OR.



   While this is a perfectly valid answer intuition tells me that an AND gate probably should not take more circles than an OR gate to make. After thinking things over a bit more I think that an OR gate pretty much says "if either input is true then the output is true" and an AND gate says "if either input is false then the output is false". Therefore if I just switch which nodes the true and false circles are connected to on the OR collection I can get an AND collection.

   Solution switching TRUE and FALSE around.



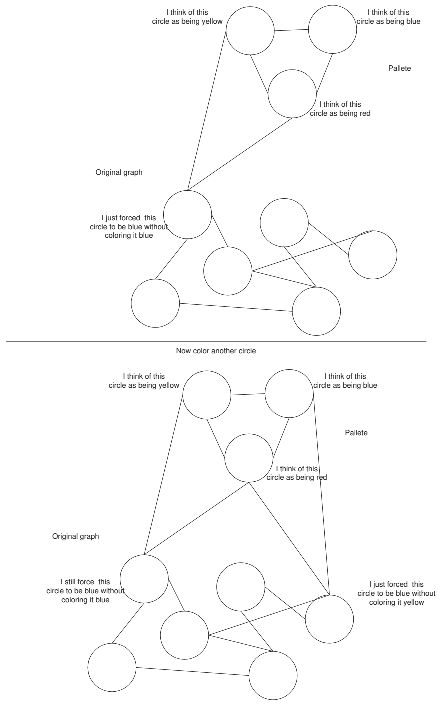
With either approach I want to check my answer. This means that I need to check all four possibilites of the truth table (X=false Y=false), (X=true, Y=false), (X=false, Y= true), (X=true, Y=true). For each possibility I need to check that a valid coloring exists. Take the case X=true, Y=false. First I take my AND collection and color the input X true and the input Y false, then I color the output false since I know true AND false = false. I then check that a valid coloring exists. I also want to make sure that there are possibilities for a wrong answer. To do this I color the inputs as before and then color the output as true. Then I check that *no* valid coloring exists since the existance of one would mean by AND gate did not work. I repeat the same *no* test for making the output the nothing color. Either one of the answers gets full credit. In the figures I just use T for true color, F for false color, and N for null color. For coloring problems I can just pretend these are colors. The figures follow.

(b) First I think about what the problem is actually asking of me. I am given a n+1 bit number that I know is the product of two prime numbers. Next I think about what I can do with the circuit. I remember that every circuit can be made from AND, OR, and NOT gates. I also know how to make each gate into a circle collection. Therefore I can make the whole circuit into one big circle collection. This is one step. Normally I take two inputs and multiply them to get an output, but this is sort of the opposite. I want to see what two numbers could have given me the output. So in this case I take my multiplier collection and color the output of the multiplier to be the binary represenation of the n+1 bit number. This is the second step. Next I run the coloring program to color the collection. The program will color the whole collection including the inputs. I can now read the inputs and record them as my answer for my final step. They will multiply to the product since the collection was a multiplier circuit. A few notes on the process. We only try this on n+1 bit numbers of the form N=pq where p and q are primes. Since p and q exist for N there should always be a coloring. What if we get N = N X 1 as a coloring? That cannot happen since the largest input is a n bit number and the product is a n+1 bit number. If the inputs were X and Y would I get X=p, Y=q or X=q, Y=p? Either one could happen. They are both valid colorings.

(c) Let's say my colors are red, green, and blue. The first thing I do is run the program on the collection with nothing colored. If the program tells me there is no possible coloring I don't waste any more time and say that there is no coloring.
I first pick a circle to focus on. I try to find a color for it. I try pick a color for it and then run the new collection through the program. If the program gives me a no I try another color for that circle. If the program gives me a yes I keep the color for the circle and move on to another one. I continue this process until all the circles have been colored. At each stage there will be a least one valid color for the circle at hand since the previous stage said there still was a coloring.

(d) I want to do pretty much the same task as the previous problem, except I cannot color the circles before I give them to the program. The main trick that I want to use is to somehow be able to do something *like* coloring the circles. I can do this by making a pallete and adding that to the collection that was given. A pallete for three colors is three circles each connected to the other in a triangle. Since the three have edges between them they all have to be different colors. I can now think of each of these circles in the pallete as being a certain color.
Now I need to be able to assign colors like I did before. To give a circle in a collection a color I can force it to be

that color. I do this by connecting the circle of the original collection to the other circles of the pallete except the one I want to force it to be. For example, if I connect a circle to the one designated as blue and the one designated as red, that circe has to be green. Now that I have a tool that allows me to do the same thing as coloring circles I can run the algorithm from the previous problem. *Please note- I never really colored the circles before giving them to the program, I just added to the collection and maintained a mapping outside the program.*
A couple steps of the program.



2. Puzzles

   (a) The barber can never get his hair cut. Suppose he cuts his own hair. Then he doesn't cut his hair becuase he cuts his hair. This is a contradiction, so he cannot cut his own hair. Suppose he did not cut his hair, then he must cut his hair because he cuts everyone's hair who does not cut his own. So we have that he must cut his hair and he must not cut his hair. This is clearly impossible, the problem is actually impossible. Such a barber cannot exist. The question, however, asked "what hairstyle does he have?" not "is this possible" so pretty much any answer received credit.

   (b) This problem was actually in Labyrinth (one of my favorite movies). The question to ask either one is "Is this door (assume you are pointing at a door) the one your brother would say leads to freedom?" The answer will always be a lie since if you are talking to the liar he will lie about what the truthfull one will say and if you talk to the truthful one he will truthfully repeat the lie his brother would tell. Now you may act upon the opposite of their advice.

       Here is another solution. Ask the question "Would your answer be yes if I asked you "Is this door the one that leads to freedom?"?". This question basically turns a liar into a truth-teller. If the door lead to freedom the liar would answer "no" to the question "Is this door the one that leads to freedom", but then he would lie about what he would say and say "yes". If the door did not lead to freedom his answere to the question "Is this door the one that leads to freedom" would be yes, he of course would lie about this and say no. So in the end he by asking him the whole question, you get the true answer to the inside part. The truth teller if asked this question will truthfully say what they would have answered, which was the truth. This technique can be generalized to make a liar into a truth teller.

Extra Credit:

The very first thing I did for this problem is to translate the professor lanaguage of "Can you do it in two tries?" to "It is

possible to do it in two tries, but it is challenging to do so." Of course, don't always rely on this kind of translation.
For this next problem we will use the same technique described above for getting the truth out of a liar. The problem is with dealing with the flimsy sister. We have two sisters we can get a good answer out of and one we cannot, but these sisters know about each other. I will go up to one sister and ask her to pick the other sister that is not flimsy. If it is the truth teller, she will tell me the liar, the liar (when asked the tricky way) will tell me the truth teller, and if I went up to the flimsy sister she would tell me either the truth teller or the liar.
The question I ask is (while pointing at a sister) "Would your answer be yes if I asked you "Is this the flimsy sister"?".
If the answer is "yes" I go to the sister I was pointing at, if the answer is "no" I go to the other sister. Either way I am with a non-flimsy sister. I then finish things off with the question. "Would your answer be yes if I asked you"Is this door the one that leads to freedom?"?". I react to that answer as I did in the second solution to the brother problem.
This problem challenged me also. Thanks to Jeannette Dickerson for her clever solution to the brother problem.