2-D Shape Analysis

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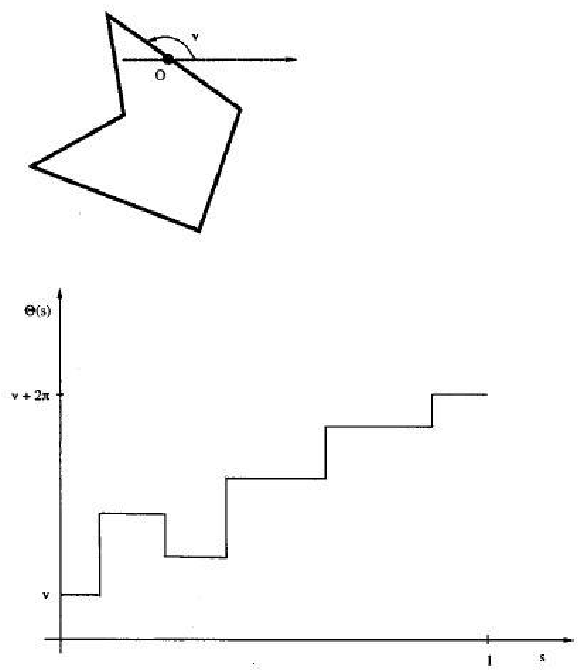
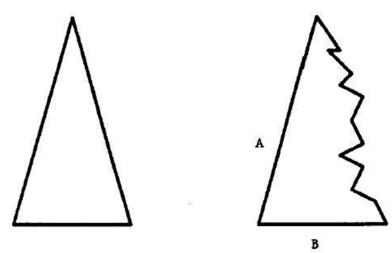
Comparing polygonal shapes using turning function

- A standard method is represent polygon by a list of vertices
- An alternative way is to define the turning function $\theta_A(s)$
- The turning function measures the angle v of tangent as a function of arc length s
- The turning function increases with left hand turns and decrease with right hand turns



Turning function (continued)

- For convex polygon A, $\theta_A(1) = \theta_A(0)+2\pi$
- It is piecewise constant for polygons
- It is invariant under translation and scaling
- It may be unstable under non-uniform noise



A polygon distance function

- Two polygons A and B with turning function $\theta_A(s)$ and $\theta_B(s)$, define L_P Distance between $\theta_A(s)$ and $\theta_B(s)$

$$\delta_p(A,B) = \| \theta_A - \theta_B \|_P = \left(\int_0^1 | \theta_A(s) - \theta_B(s) |^p \, ds\right)^{\frac{1}{p}}$$

- The distance function is sensitive to both the rotation of polygons and choice of reference point

Distance function (continued)

- It makes more sense to consider the minimum distance over all the choices

$$d_p(A,B) = \left(\min_{t\in[0,1]} \int_0^1 | \theta_A(s+t) - \theta_B(s) + \theta |^p \, ds\right)^{\frac{1}{p}}$$

$$= \left(\min_{t\in[0,1]} D_p^{A,B}(t,\theta)\right)^{\frac{1}{p}}$$

$$D_p^{A,B}(t,\theta) = \int_0^1 | \theta_A(s+t) - \theta_B(s) + \theta |^p \, ds$$

Distance function (continued)

- To minimize , $h(t,\theta) = D_2^{A,B}(t,\theta)$ the best value of θ is given by

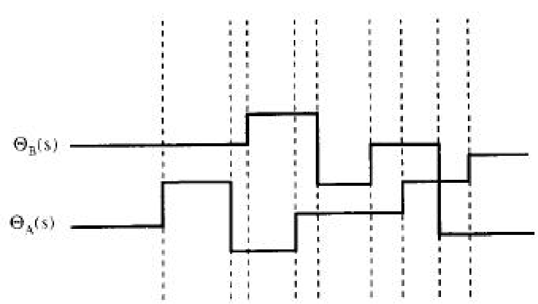
$$\theta(t) = \int_0^1 (g(s) - f(s+t))ds = \alpha - 2\pi t$$

- where $\alpha = \int_0^1 g(s)ds - \int_0^1 f(s)ds$

and $f(s) = \theta_A(s)$, $g(s) = \theta_B(s)$

Critical events

- Critical event is a value of t where breakpoints of f collides with breakpoints of g
- There are mn critical events for m breakpoints in f and n breakpoints in g



Complexity of the algorithm

- For one-variable minimization problem

$$d_2(A,B) = \{\min_{t\in[0,1]}[\int_0^1 [f(s+t) - g(s)]^2 ds - [\theta(t)]^2]\}^{\frac{1}{2}}$$

- The basic algorithm compute the minimum value in O(mn(m+n)) time
- A refined version of algorithm runs in O(mnlog(mn)) time

Comparing polygons with graph matching

- Divide contour K by straight-lines which are parallel to X-axis
- The divided graph is called the segmentation E(K) of contour K

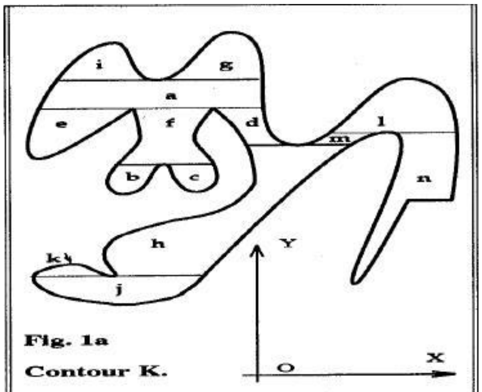


Fig. 1a
Contour K.

G-graph

- The areas restricted by lines and contours are called lumps
- Lumps with more than one point in common are called adjacent
- Among adjacent lumps, parent lump is higher in Y direction
- E(K) induces an associated graph G(K), which is called G-graph

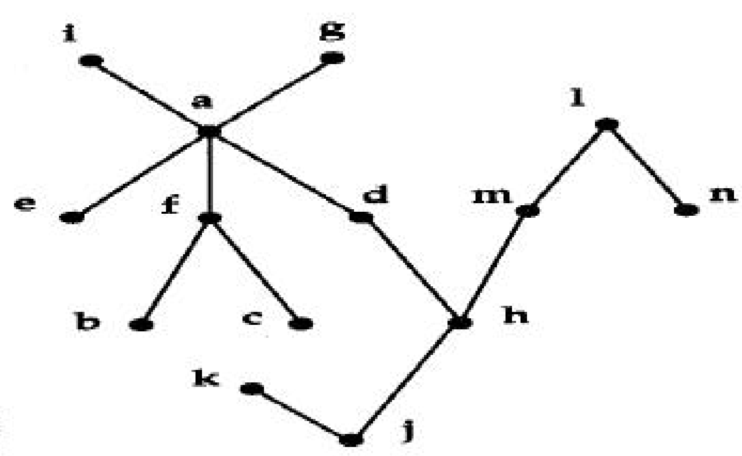


Fig. 1b

Graph G(K).

G-graph (continued)

- Siblings are in the relation ‘left of’ or ‘right of’
- A weight function w(x) is defined on vertices
- There are no pair of vertices x, y such that x is the unique parent of y and y is the unique parent of x
- 2 G-graph G1 and G2 are isomorphic are denoted as G1≅G2

Simplified contours

- Choose an arbitrary leaf in K’ and cut it off
- Repeat the cut-off

and get a family of simplified contours

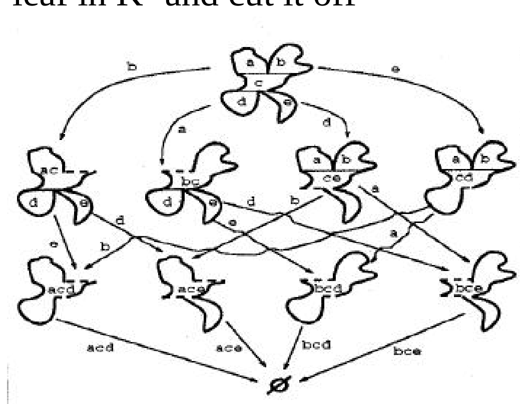


Fig. 2a. **Segmentation lattice M(E(K)).**

Reduction to simplified contour similarity

- Define the depth DEP(K') of a node K' as the sum of weights of the leaves which have been cut off
- If G(K1) ≅ G(K2), Define the simple distance
  $D(K1,K2) = \sum |w(v) - w(\lambda(v))| \quad v \in V(G(K))$
- The function $DSIM_Y(K1,K2)$ measures the similarity between K1 and K2 in Y direction
  $DSIM_Y(K1,K2) = \min F(K1',K2')$
  $F(K1',K2') =$
  $c(DEP(K1')+DEP(K2'))+D(K1',K2')$

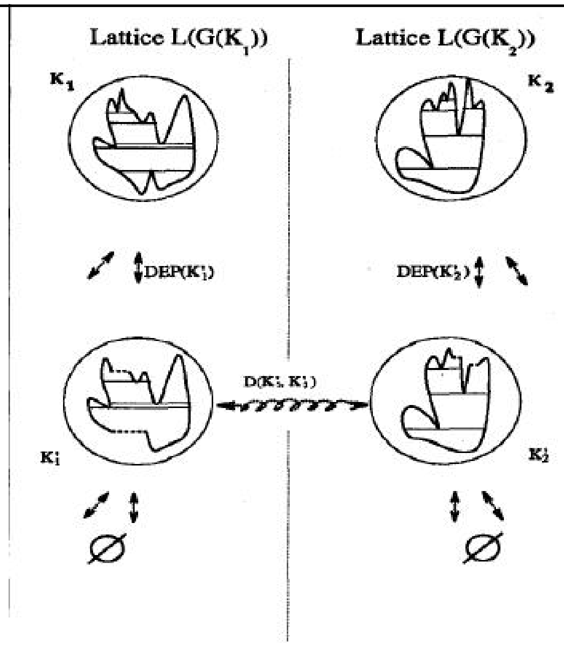


Fig. 3. The distances between contours.

Simplified contour similarity (continued)

- Similarity between contours should be independent of the Y direction

$$DSIM(K_1, K_2) = \int_0^{\pi} DSIM_Y(K_1\angle\alpha, K_2\angle\alpha)d\alpha$$

- Finally, similarity between K1 and K2 is defined as

$DSIM(K1,K2)=\min DSIM(K1,K2\angle\beta)$

$0<=\beta<2\pi$

Elementary morphism

- An elementary morphism of G is defined as
- 1. The deletion of an edge (z, z1)
- 2. If, after deletion, the property of G-graphs is violated, merge the pair of vertices(z1, z2)

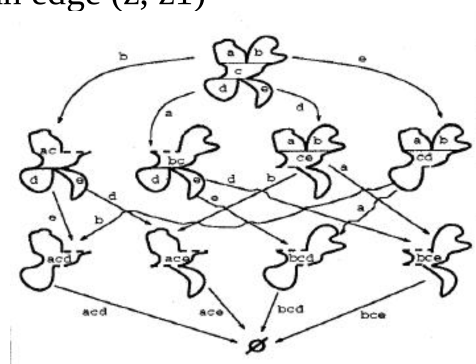


Fig. 2a. **Segmentation lattice M(E(K)).**

Lattice of morphisms

- The lattice of morphisms can be obtained by replacing simplified contours K’ by G(K’)
- For G’, G1’, G2’$\in$ L(G), define DEP(G’) and D(G1’,G2’) analogous to simplified contours
- Reduce this problem to G-graph pair resolution problem

$$DSIM_Y(K1,K2) = DSIM_G(G(K1),G(K2))$$

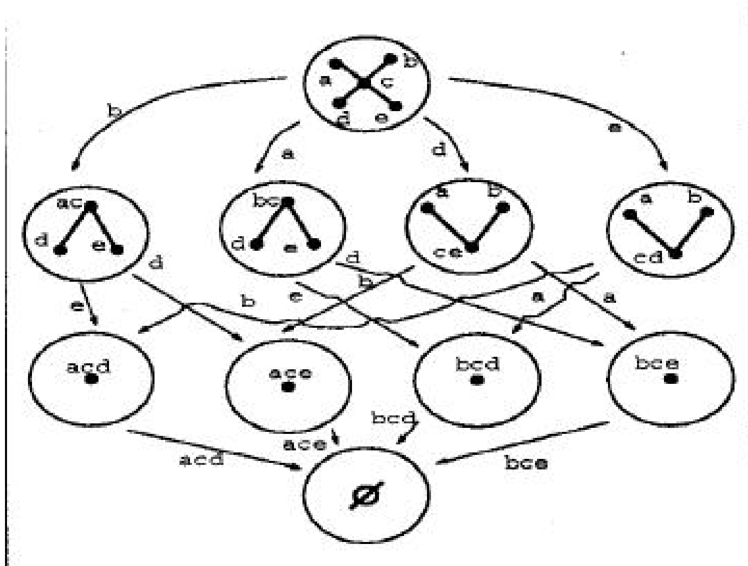


Fig. 2b. Morphism lattice L(G(K)).

Top-down greedy algorithm

- The difficulty in evaluation of $DSIM_G(G1,G2)$ is the exponential dependence of $|V(L(G))|$ on $|V(G)|$
- The proposed algorithm replaces the lattices by slowest paths in lattices
- The slowest path P(G) in L(G) is defined as

1. The first node G' is the root G
2. Choose leaf $z \in G'$ with minimal weight w(z)
3. Execute elementary morphism G'/z

Comparing polygons by signature

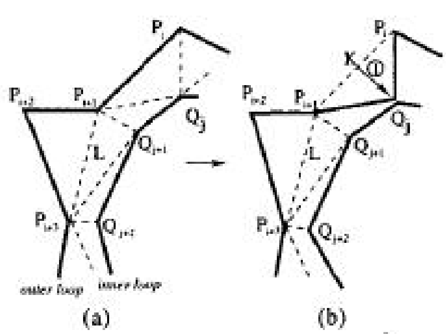
- The method is based on shape deformation
- Enclose the polygon by some predetermined outer polygonal shape
- Shrink the outer polygon into the given shape
- Measure the deformation path length at sample points on perimeter of outer shape
- Compare the path lengths of two polygons at sample points

Shape comparison algorithm

- Normalize the target polygon so that it fits within a bounding box of pre-defined size
- Triangulate zone between the outer and inner polygons
- Eliminate the triangles according to some criteria (snapping)
- Among triangles which can be eliminated, pick the one with largest area

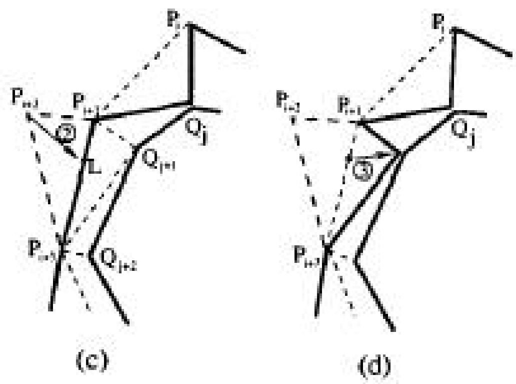
Snapping rules

- Rule 1. One edge P_iP_{i+1} is on the outer loop and opposite vertex Q_j is on Q, snap a point K on P_iP_{i+1} to Q_j



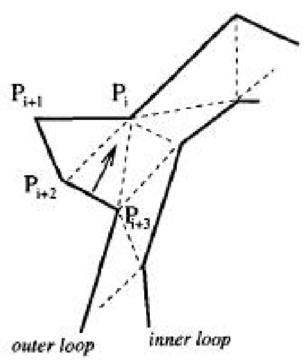
Snapping rules (continued)

- Rule 2. Two edges $P_{i+1}P_{i+2}$, $P_{i+2}P_{i+3}$ are on the outer loop, snap P_{i+2} to a point L on $P_{i+1}P_{i+3}$



Snapping rules (continued)

- Rule 3. Snap operations are to be done such that the resulting outer loop is never intersecting itself

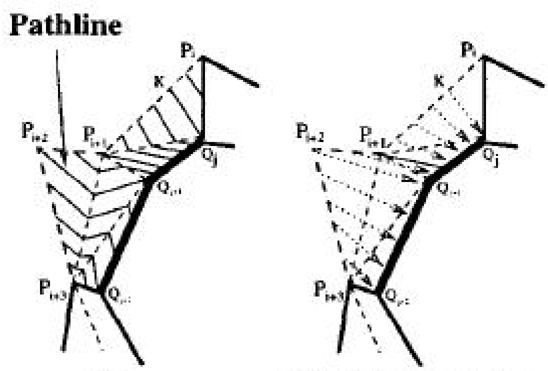


Pathline

- Pathline is the trajectory of a point on outer polygon P to its corresponding position on inner polygon Q
- A pathline begins from or terminates at a vertex of either P or Q is called primary pathline
- Non-primary pathline are derived by interpolating primary pathlines
- All points in zone Z are expressed by $\pi_s(t)$

Postprocessing

- Pathlines can be straightened where possible

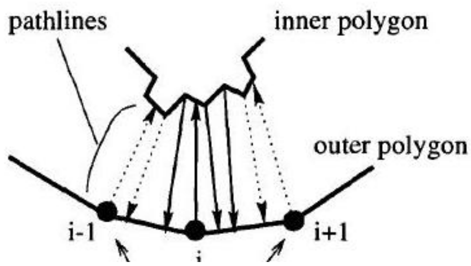


(f) (g) Refined pathlines

Signature file

- A signature file for a polygon is defined as a vector S = (s1,s2,…,sn)

$$s_i = \frac{1}{1+|B|}\left(PathLength(a) + \sum_{b\in B} PathLength(b)\right)$$



Comparison

Two polygons P and Q are compared by computing from their signature files

$$SimlarityIndex_{p,q} = \frac{1}{n}\sum_{i=1}^{n}(S_P(i) - S_Q(i))^2$$

Summary

- Turning function
- Graph matching
- Shape signature by deformation