Assignment 4: Solitons in Automata

COS 496 Due: Tues., April 23

Basic Part: Parity-Rule Filter Automata

Mariusz described the 1834 discovery of solitons by Scott-Russell in an English canal. Since then they have popped up in many places, including optical fibers. There is evidence that the Great Spot on Jupiter is a soliton. In 1985 they were discovered lurking in the zero-one world of computer scientists, in *filter automata* [1].

A one-dimensional, binary, filter automaton is defined for an infinite array of bits, a_i^t, which is the binary value of cell i at time t. Its time evolution determines the value of cell i at time $t+1$ by the next-state rule

$$a_i^{t+1} = F(a_{i-r}^{t+1}, a_{i-r+1}^{t+1}, \ldots, a_{i-1}^{t+1}, a_i^t, \ldots, a_{i+r}^t)$$

Notice that the new value at position i depends on newly computed values on the left, and old values on the right. Update from the leftmost 1, and assume we always start with a finite number of 1s. This is analogous to the operation of a feedback digital filter. A filter automaton is just a simple variation of the usual cellular automaton.

I'll distribute copies of [1] in class. It contains a recipe for a class of one-dimensional filter automata, the *parity rule filter automata*, which support solitons. Implement them and run them for a few values of neighborhood radii, verifying the general results reported in this paper. For example: (a) a random initial condition should break up into solitons; (b) some collisions between solitons should preserve the shape of the colliding particles, and some should change their shape; (c) the faster soliton should be shifted forward by a collision, and the slower one backward.

Extra Credit

A. Try to extend the parity rule filter automata in a natural way to k-valued site values. Use a color display.

B. Try to extend the parity rule filter automata in a natural way to two-dimensions.

C. Play with variations of the update rule of the parity rule filter automata, to see if you can find similar behave in other automata. Start with the suggestion in Eq. 5 of [1].

References

[1] J. K. Park, K. Steiglitz, and W. P. Thurston, "Soliton-like Behavior in Automata," *Physica*, **19D** (1986) 423–432.

[2] N. Islam, J.P. Singh, and K. Steiglitz, "Soliton Phase Shifts in a Dissipative Lattice," *Journal of Applied Physics*, **62**(2) (1987) 689–693.

[3] K. Steiglitz, I. Kamal, and A. Watson, "Embedding Computation in One-dimensional Automata by Phase Coding Solitons," *IEEE Transactions on Computers*, **37**(2) (1988) 138–145.

[4] C. H. Goldberg, "Parity Filter Automata," *Complex Systems*, **2** (1988) 91–141.

[5] A. S. Fokas, E. Papadopoulou, and Y. Saridakis, "Particles in Soliton Cellular Automata," *Complex Systems*, **3** (1989) 615–633.

[6] A. S. Fokas, E. Papadopoulou, and Y. Saridakis, "Coherent Structures in Cellular Automata," *Physics Letters*, **147A**(7) (1990) 369–379.

[7] M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equations, and Inverse Scattering* (Cambridge University Press, Cambridge, UK, 1991) (see pp. 152-162).

[8] M. Bruschi, P. M. Santini, and O. Ragnisco, "Integrable Cellular Automata," *Physics Letters A* **169** (3) (1992) 151-160.