COS 226 Lecture 13: File compression

Compression: reduce the size of a file
- to save TIME when transmitting the file
- to save SPACE when storing it away

Widely used
- despite technological advances
- (Why?)

Basic concepts ancient (1952)
Best technology recently developed

Simple methods
Huffman codes
Information theory
Arithmetic codes
Adaptive (LZ) codes

Simple methods

Step 1: avoid doing something dumb
Step 2: try to do something clever

Character count encoding
    Ex: ASCII binary file
    Ex: ACGT molecular biology sequences
    Ex: five-bit code for text

Run length encoding
    Ex: blanks in text files
    Ex: black-and-white graphics
How to represent counts?
    escape characters
    separate alphabet (may waste bits)
    different language entirely

General representation problem
    annoying practical problems
    usually work with either
    * bitstream/bytestream
    * separate language entirely

Special hacks
    Ex: encode /usr/dict/words

Run-length encoding

Remove blanks in files

ex: tabular data without tabs

ex: (ancient) card images

Record lengths of 0s and 1s in binary files

ex: black-and-white graphics

```
.00000000000001111111111111000000000 13 14  9
.00000000000111111111111111110000000 11 18  7
.00000001111111111111111111111110000  8 24  4
.00000011111111111111111111111111000  7 26  3
.00000111111111111111111111111111110  5 30  1
.00001111111000000000000000001111111  4  7 18 7
.00001111100000000000000000000011111  4  5 22 5
.00001110000000000000000000000000111  4  3 26 3
.00001110000000000000000000000000111  4  3 26 3
.00001110000000000000000000000000111  4  3 26 3
.00001110000000000000000000000000111  4  3 26 3
.00001111000000000000000000000001110  5  4 23 3 1
.00000011100000000000000000000111000  7  3 20 3 3
.01111111111111111111111111111111111  1 35
.01111111111111111111111111111111111  1 35
.01111111111111111111111111111111111  1 35
.01111111111111111111111111111111111  1 35
.01111111111111111111111111111111111  1 35
.01100000000000000000000000000000011  1  2 31 2
```

Compression factor increases with resolution

No help for random files!

Huffman code

Variable length code, defined by trie

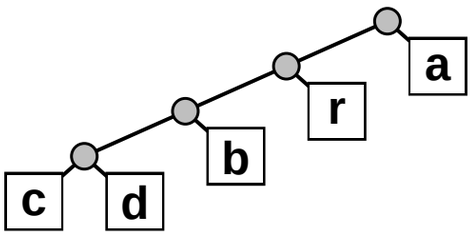
Ex: abracadabra

88 bits in byte code

55 bits in 5-bit code

33 bits in 3-bit code (only 5 different letters)

Huffman code:

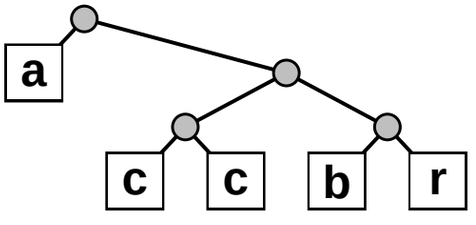


a	**1**	**5**	**5**
b	**001**	**2**	**6**
c	**0000**	**1**	**4**
d	**0001**	**1**	**4**
r	**01**	**2**	**4**
			23 bits

1*001*01*0*0000*0*0001*0*001*01*0

10010110000100011001011

not necessarily unique



a	**0**	**5**	**5**
b	**110**	**2**	**6**
c	**100**	**1**	**3**
d	**101**	**1**	**3**
r	**111**	**2**	**6**
			23 bits

01101110100010101101110

Depends on encoding frequently used chars

with fewer bits than infrequently used chars

Huffman code implementation

ENCODE:

- count frequency of occurrence of characters
- build trie by combining two smallest frequencies
- trie defines code
- pass through source, output code

DECODE:

- use bits to guide trie search
- output char when reached, restart at top

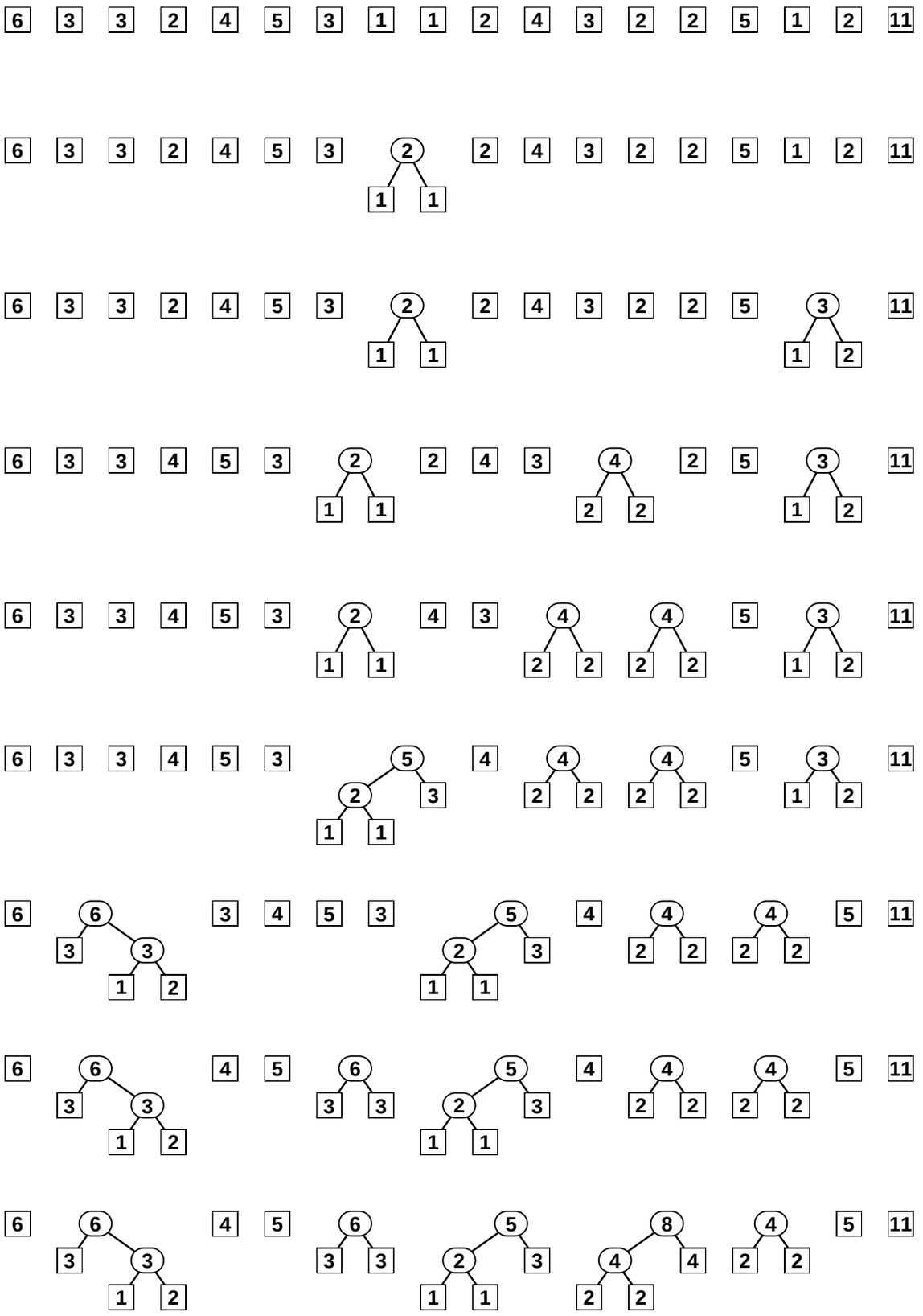
Have to transmit trie!

- suffices to transmit code
- relatively insignificant for big messages

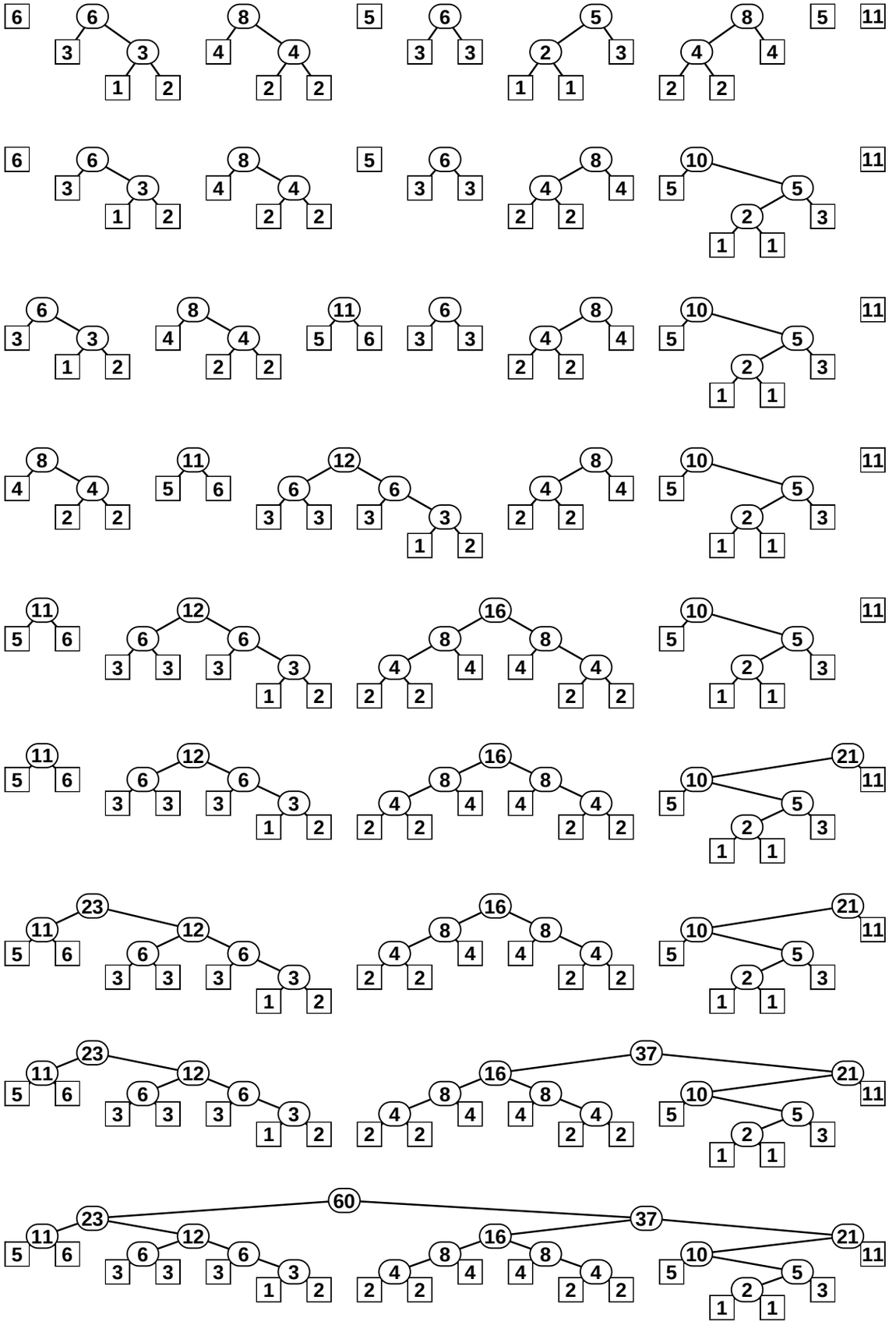
Issues:

- trie representation
- data structure to hold nodes being processed

Huffman code construction example



Huffman code construction example (cont.)



Huffman code implementation (continued)

Trie representation
- parent links
- right-left bit

Data structure for nodes being processed
- "indirect" priority queue

To build code:
- put all chars with nonzero counts in PQ
- while PQ has two or more nodes
  - take two smallest off PQ
  - make new node with sum of counts
  - put on PQ

Another algorithm to build code:
- sort initial frequencies
- merge with generated frequencies
- no PQ needed

THM (Huffman, 1952): Huffman code is optimal
(no variable-length code uses fewer bits)

Problems:
- have to transmit code
- expensive computationally
- two-pass
- not optimal (!)

a simple string to be encoded using a minimal number of bits

c f p d g l r u a b o t m s e n i *
1 1 1 2 2 2 2 2 3 3 3 3 4 4 5 5 6 11

1 1 1 2 2 2 2 2 3 3 3 3 4 4 5 5 6 11
1 2 2 2 2 2 3 3 3 3 4 4 5 5 6 11 * 2
2 2 2 2 3 3 3 3 4 4 5 5 6 11 * 2 3
2 2 3 3 3 3 4 4 5 5 6 11 * 2 3 4
3 3 3 3 4 4 5 5 6 11 * 2 3 4 4

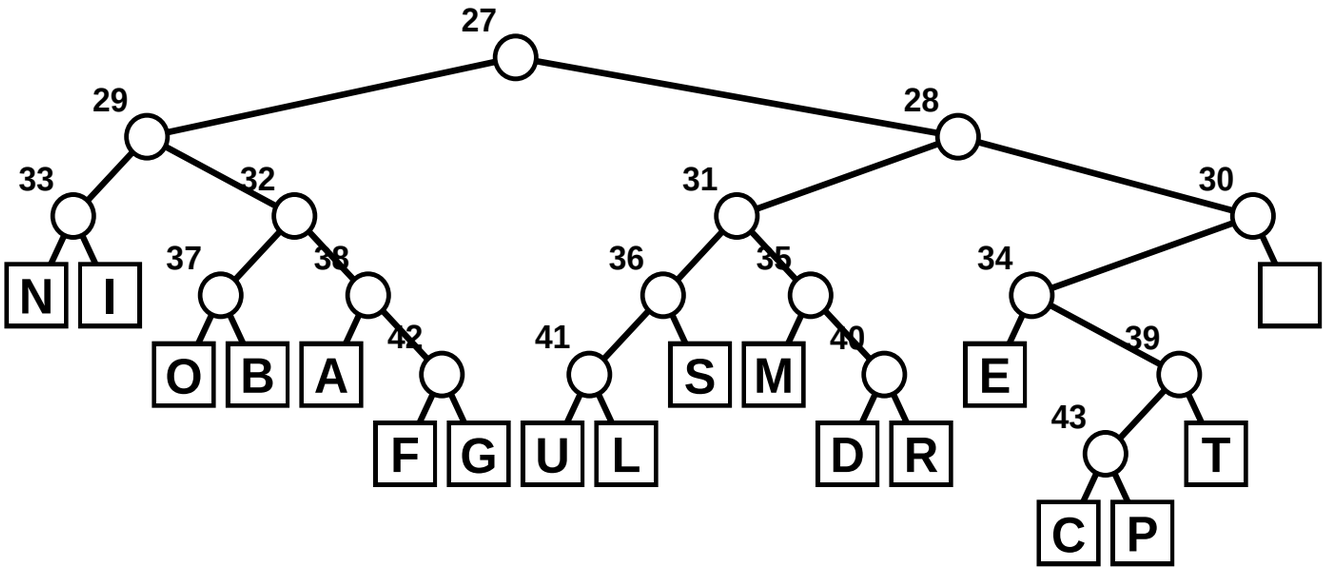
3 3 3 4 4 5 5 6 11 * 3 4 4 5
3 4 4 5 5 6 11 * 3 4 4 5 6
4 4 5 5 6 11 * 4 4 5 6 6
5 5 6 11 * 4 4 5 6 6 8
5 5 6 11 * 5 6 6 8 8
6 11 * 5 6 6 8 8 10
11 * 6 6 8 8 10 11
11 * 8 8 10 11 12
11 * 10 11 12 16
*** 11 12 16 21**

16 21 23
37 23
60

Huffman trie representation

Links point up the tree

- one index per node to parent
- one bit per node (left or right child)



.	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
.	27	27	28	28	29	29	30	31	31	32	32	34	35	36	38	39
.	*		*		*			*			*	*	*		*	

To get code for given char

- * move up the trie
- * collect left-right bits
- * reverse string when top reached

Entropy

Basic concept of information theory

formal model of source

formal concept of "information content"

First order model: chars generated randomly

with fixed probability (independently)

THM: Huffman is optimal in first order model

Entropy concept confirms intuition

* random data cannot be compressed
* data compressed by one optimal compressor

won't be compress further by another

* no guarantee of specific performance on all data

BUT

* "average" case results
* depends on mathematical model
* real data may not fit any model

Example:

short program generates large data file

compression alg has to discover the program!

Statistical modeling

find short program (or model parameters)

ex: speech recognition

A difficult file to compress

One million random characters

ylculksedgrdivayjpgkredehwhrvvbbltdkctqtoctnnhylmsgvwac
dytmoajjsorphchnxlygnezneofgidqfyqfmvldxreuseufgryyvuzs
ezslumidtljqydtbprazipawngrubtqyjfiowyaktrwuopoohtkuaen
ndcobuekavqepoxmidcmdrahhuzdjmjxkeoyimflsoyxknqkqsgwgjo
uungwqzzhllznusllaepdphwojuiorbpsuxxebnszlktzbapzpscprl
pspqslilkpktzjarozmfvaqjmnlqpesbvdgpkkxrxeaphrlfabegwpr
qfnehrquwdyiuurcrpylxzzkaystirtemsgaplcpcfnwmgfokcfyigs
...

A lower bound for compression: 130 chars!

```
#include "math.h"

main ()
   { int i, j; char x;
      for (i = 0; i < 12500; i++)
         {
            for (j = 0; j < 80; j++)
               { x = 'a' + 26*drand48();
                 printf("%c", x);
               }
         }
   }
```

Ex:

Postscript file??

Arithmetic Coding

Represent source with a real number!

Ex: abracadabra

a	**0.00000**	**.45454**	**5/11 chars are a's**
b	**.45454**	**.63636**	**2/11 chars are b's**
c	**.63636**	**.72727**	
d	**.72727**	**.81818**	
r	**.81818**	**1.00000**	

ENCODE: rescale interval for each letter

Ex: code for "abracadabra" is .2787887

a	**.00000000**	**.45454547**	**(0, 4/11) in (0, 1)**
b	**.20661159**	**.28925622**	**(5/11, 7/11) in (0, 4/11)**
r	**.27422991**	**.28925622**	
a	**.27422991**	**.28106004**	
c	**.27857634**	**.27919728**	
a	**.27857634**	**.27885857**	
d	**.27878159**	**.27880725**	
a	**.27878159**	**.27879325**	
b	**.27878690**	**.27878901**	
r	**.27878863**	**.27878901**	
a	**.27878863**	**.27878881**	

Can rescale and output code digit when

leading digits of intervals match

(have to do so to handle long messages)

Arithmetic Coding (decode)

Given code (real number between 0 and 1)

- it falls in one of the intervals (l, r)
  - associated with the characters
- output that character
- rescale to (0,1)

  code = (code-l)/(r-l);
- repeat to get next char

Ex:

.27878872 a since .27... is in (0, 5/11)
.61333513 b (.27...- 0)/(5/11); in (5/11, 7/11)
.87334329 r (.61...- 5/11)/(2/11); in (9/11, 1)
.30338812 a
.66745383 c
.34199208 a
.75238258 d
.27620819 a
.60765803 b
.84211922 r
.13165572 a

Need to adjust algorithm to match rescaling used on decode (can be tricky to do)

Bottom line:

- slightly better than Huffman
- like having fractional bits

Pure Adaptive Coding (LZ)

To encode the next character(s)

* if no match of length >1 in previous string, output the next char
* otherwise, output
  - pointer to longest previous match
  - length of longest previous match
  - char causing the mismatch

```
.  0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
.  t h i s   i s   t h e   t h i r d   t i m e
.
.  t h i s * 2 3 t h e 4 3 i r d 6 2 i m e
.
.
.  0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
.  0 1 1 0 0 1 1 0 0 0 0 1 1 0 1 1 1 1 0 1
.
.  0 1 1 0 4 4 0 6 5 1 4 2 1 5 3
```

Problems:

- big dictionary (N^2 prev substrings)
- online dictionary update not easy

Dictionary-based Adaptive Coding (LZW)

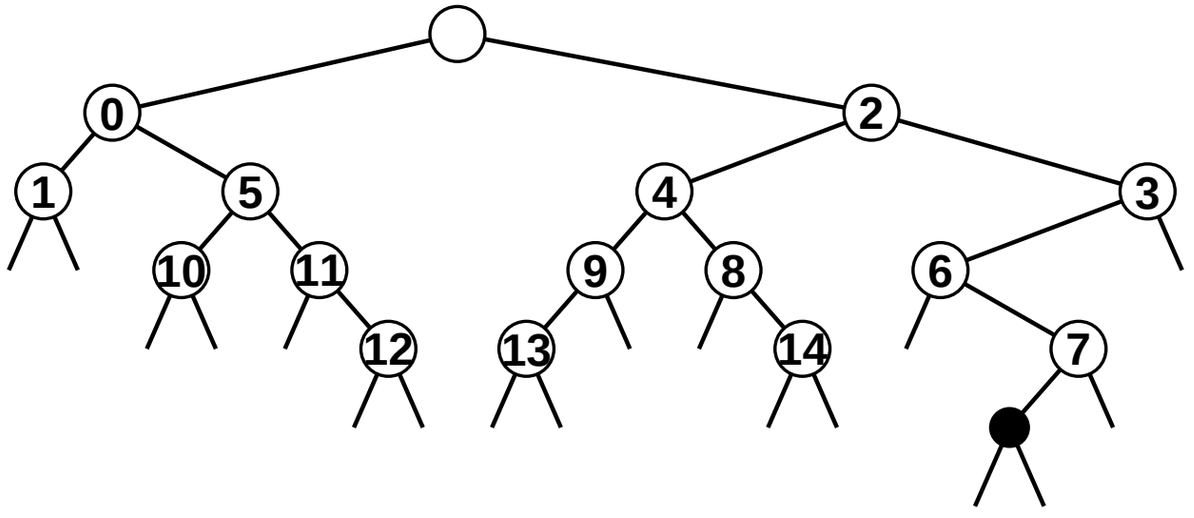
```
.  0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
.  t h i s   i s   t h e   t h i r d   t i m e
.
.   0 t       t
.   1 h       h
.   2 i       i
.   3 s       s
.   4 *       *
.   5 is    2 i
.   6 *t    4 t
.   7 he    1 e
.   8 *th   6 h
.   9 ir    2 r
.  10 d       d
.  11 *ti   6 i
.  12 m       m
.  13 e       e
.
.  t h i s * 2 i 4 t 1 e 6 h 2 r d 6 i m e
```

Binary example of LZW

```
.  0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9
.  0 1 1 0 0 1 1 0 0 0 0 1 1 0 1 1 1 1 0 1
.
.  0   0 1 0
.  1   1 1 1
.  2   2 2 10     1 0
.  3   4 2 01     0 1
```

LZW implementation

Use trie for dictionary (standard representation)



.000111100111011011011000100110111100010111101 0

ENCODE loop

- lookup string suffix in trie
- output dictionary index at bottom
- add new node to trie

DECODE options

- build trie (faster, takes space)
- build dictionary (slower)

Empirical results

Methods are available as UNIX tools

- pack: Huffman
- compress: LZ
- gzip: LZW

Experiment: Compress the text of "Moby Dick"

	original	**1220K**				
	5-bit	**778K**	**63%**			
	Huffman	**692K**	**56%**			
	LZ	**523K**	**42%**			
	LZW	**493K**	**40%**			
		gzip the results			pack the results	
	Huffman	**692K**	**624K**	**10%**	**692K**	**687K** **1%**
	LZ	**523K**	**520K**		**523K**	**"no savings"**
	LZW	**493K**	**493K**		**493K**	**"no savings"**

Experiment: Compress a 1MB file of random chars

original	**1012K**		
5-bit	**638K**	**63%**	
Huffman	**610K**	**60%**	
LZ	**695K**	**68%**	
LZW	**646K**	**63%**	

Summary

Don’t ignore the obvious

Huffman:

- represent fixed-length bitstrings
- with variable-length codes

Lempel-Ziv:

- represent variable-length bitstrings
- with fixed-length codes

Not covered: codes like JPEG

- "lossy" codes for graphics
- (don’t really need all the bits)

Other questions

- compute on compressed data?
- archiving
  - compression code must be truly portable
  - Ex: replace processor
  - Ex: send files to different processor
- massive compression needed on some data