COS 226 Lecture 12: Pattern matching

Generalize string-searching problem to include
incompletely specified patterns

TEXT: N characters
PATTERN: M-character REGULAR EXPRESSION
(C occurrences of pattern in text)

Ex:
Is some substring of
cdbcaaaaabcddbbc
in the language described by the RE
(a*b + ac)d ?

As with string matching,
extend to count matches, return all matches

egrep

Practical application of core principles
of the theory of computation
Based on simulating
NONDETERMINISTIC finite-state automata
Essential concept in computer science
build intermediate abstractions
pick the right ones!

"man grep" excerpts

Name

grep - search file for regular expression

Syntax

grep [option...] expression [file...]

Description

Commands of the grep family search the input files (standard input default) for lines matching a pattern. Normally, each line found is copied to the standard output.

Take care when using the characters $ * [^ | ( ) and in the expression because they are also meaningful to the Shell. It is safest to enclose the entire expression argument in single quotes ' '.

Basic elements of expressions (grep)

```
c       any non-special char matches itself
r*      zero or more occurences of r
```

"Extended" regular expressions (egrep)

```
(r)
r1 | r2
```

Regular expression

Theoretician:

    language accepted by FSA

Programmer:

    compact description of multiple patterns

Concatenate

abcda	**abcda**

OR

a+b	**a b**
c(a+b)d	**cad cbd**
(ac+b)d	**acd bd**
(a+c)(b+d)	**ab ad cb cd**

Closure

a*	**. a aa aaa aaaa aaaaa**
ca*b	**cb cab caab caaab caaaab**
(a+b)*	**. a b aa ab ba bb aaa aab aba ...**
c(a+b)*d	**cd cad cbd caad cabd cbad cbbd ...**

String searching FSAs

Recall:

    KMP algorithm: FSA for one pattern
    AC algorithm: FSA for multiple patterns
    purpose of FSA is to avoid backing up in text

RE searching is much harder

    check each text position for RE match
    N(N-1)/2 possible matches

Ex: Is some prefix of

```
.     cdbcaaaaabcddbbc
.      dbcaaaaabcddbbc
.       bcaaaaabcddbbc
.        caaaaabcddbbc
.          ...
```

in the language described by the RE

(a*b + ac)d ?

THM: There exists an FSA for every RE

Ex: FSA for acb (string match)

	0	1	2	3
a	1	1	1	3
b	0	0	3	3
c	0	2	0	3

C program to simulate FSAs

Ex: FSA for c(a+b)d

.		0	1	2	3	4
.	a	0	3	0	0	4
.	b	0	2	0	0	4
.	c	1	1	1	1	4
.	d	0	0	3	3	4

```
struct state { char c; int next[256]; }
state fsa[1000];

int t = start;
while ((c = getchar()) != EOF)
  t = fsa[t].next[c];
if (t == final) printf("Accepted ");
else printf("Rejected ");
```

Different type of FSA can avoid large next table
- character in state
- one state on match
- another state on mismatch

Possible approach to implementing grep:
- build FSA from RE
- use C program to simulate FSA

Problems:
- FSA can be exponentially large
- how to build FSA from RE?

Nondeterministic FSAs

Appropriate intermediate abstraction for egrep
- small in size
- can build directly from RE

FSA that can guess the right answer
More than one successor state

Simplest version
- character in state
  - next state on match
- NULL states
  - two possible successor states

Ex: NDFSA for ca*b

.		0	1	2	3	4
.	**ch**	**c**		**a**	**b**	
.	**next1**	**1**	**2**	**1**	**4**	**4**
.	**next2**	**1**	**3**	**1**	**4**	**4**

Ex: NDFSA for c(a+b)d

.		0	1	2	3	4	5
.	**ch**	**c**		**a**	**b**	**d**	
.	**next1**	**1**	**2**	**4**	**4**	**5**	**5**
.	**next2**	**1**	**3**	**4**	**4**	**5**	**5**

NDFSAs and FSAs

Nondeterminism does not extend
descriptive power of FSAs

THM: Given an nondeterministic FSA,
there is a deterministic FSA that recognizes
the same language.

Possible approach to implement grep:
- build NDFSA from RE
- transform to FSA
- use C program to simulate FSA

Standard construction for NDFSA to FSA
- make one FSA state for
  - each possible subset of NDFSA states

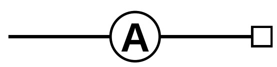
Problem:
- FSA may be exponential in size

Easier approach:
- use C program to simulate NDFSA

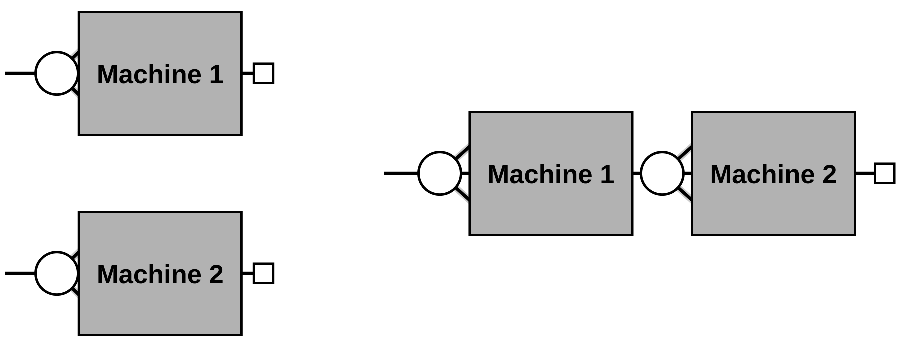
Building a NDFSA from a regular expression

Each RE construct
corresponds to the construction
of a piece of the NDFSA

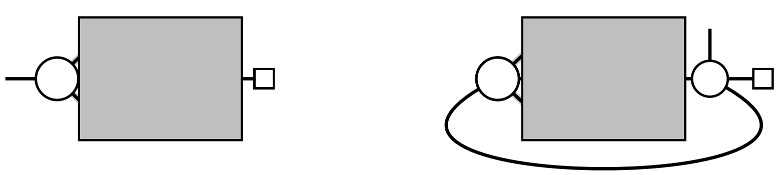
Single character:



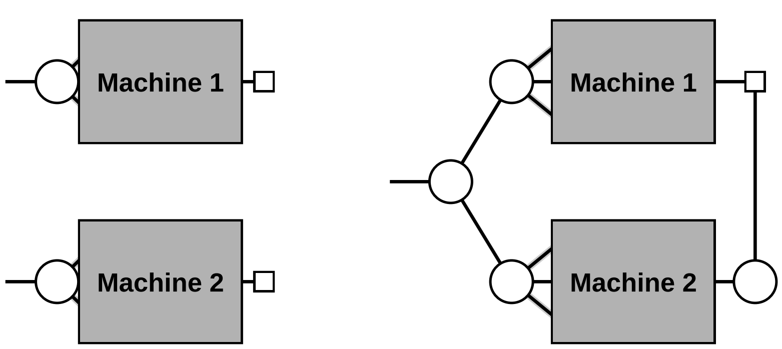
Concatenation:



Closure (*):

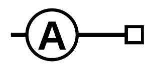


OR:

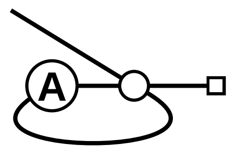


NDFSA construction example

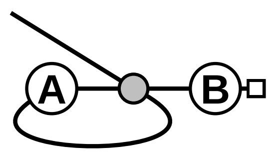
A



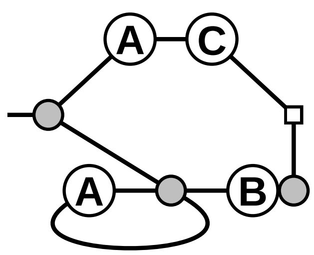
A*



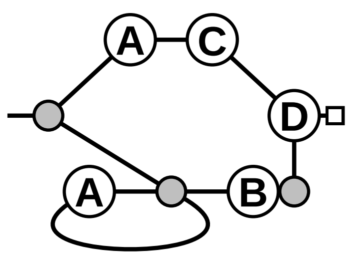
A*B



A*B+AC

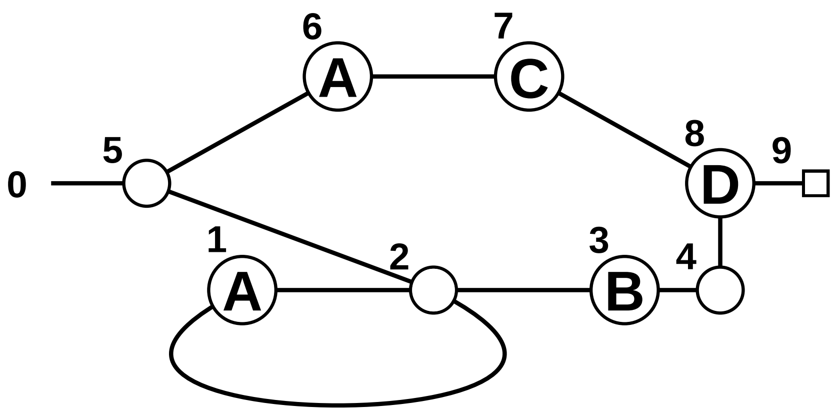


(A*B+AC)D



NDFSA representation

NDFSA for (A*B+AC)D



.		0	1	2	3	4	5	6	7	8	9
.	ch		a		b			a	c	d	
.	next1	5	2	3	4	8	6	7	8	9	0
.	next2	5	2	1	4	8	2	7	8	9	0

Recursive descent RE parser

Goal: program to build NDFSA from RE

Solve easier problem first: is RE legal?

Use context free language to describe RE

```
<expr> := <term> | <term> + <expr>
<term> := <fctr> | <fctr><term>
<fctr> := ( <expr> ) | c | ( <expr> )* | c*
```

PARSER: determine structure of legal strings

COMPILER: add semantic information to translate

Top-down recursive descent parser:

recursive programs directly derived from CFL

Expression

```
  <expr> := <term> | <term> + <expr>
expression()
  {
    term();
    if (p[j] == '+')
      { j++;  expression(); }
  }
```

Recursive descent RE parser (continued)

Term

```
  <term> := <fctr> | <fctr><term>
term()
  { factor();
    if ((p[j] == '[') || letter(p[j])) term(); }
  }
```

Factor:

```
  <fctr> := ( <expr> ) | c | ( <expr> )* | c*
factor()
  {
    if (p[j] == '[')
      { j++; expression();
        if (p[j] == ']') j++; else error(); }
    else if (letter(p[j])) j++; else error();
    if (p[j] == '*') j++;
  }
```

Doesn't always work;

```
  <expr> := v | <expr> + <term>
badexpression();
  {
    if (letter(p[j])) j++; else
      {
        badexpression();
        if (p[j] == '+') { j++; term(); }
        else error();
      }
  }
```

leads to infinite recursive loop!

Parsing example

RE: (A*B+AC)D

```
expression()
   term()
      factor()
         [
         expression()
            term()
               factor()  A  *
               term()
                 factor()  B
            +
            expression()
               term()
                 factor()  A
                 term()
                   factor()  C
         ]
      term()
         factor()  D
```

Recursive compiler to build FSA from RE

Augment parser to put out state table for FSA

state: next state to be filled in

setstate: procedure to fill in state values

Recursive routines return index of initial state

```
int expression()
  { int t1 = term(), t2, r = t1;
     if (p[j] == '+')
        {
           j++; state++;
           t2 = state; r = t2; state++;
           setstate(t2, ' ', expression(), t1);
           setstate(t2-1, ' ', state, state);
        }
     return r;
  }
next1[0] = expression();
setstate(0, ' ', next1[0], next1[0]);
setstate(state, ' ', 0, 0);
```

NDFSA for (a*b+ac)

.		0	1	2	3	4	5	6	7	8
.	ch		a		b			a	c	
.	next1	5	2	3	4	8	6	7	8	0
.	next2	5	2	1	4	8	2	7	8	0

Simulating NDFSAs

Maintain a DEQUE (doubly ended queue) with

* possible states for CURRENT char
* a scan marker, *
* possible states for NEXT char

3 6 1 * 4 7 2

Take next state off beginning (stack, queue)

* null: put both states at beginning (stack)
* match: put new state at end (queue)

```
int match(char *a)
{ int j = 0, N = strlen(a), state = next1[0];
   dequeinit(); put(scan);
   while (state)
      {
         if (state == scan) { j++; put(scan); }
         else if (ch[state] == a[j])
            put(next1[state]);
         else if (ch[state] == ' ')
         { push(next1[state]); push(next2[state]); }
         if (dequeempty() || j==N) return 0;
         state = pop();
      }
   return j;
}
```

Don’t bother checking next1==next2. Stay tuned.

NDFSA simulation example

Simulation of NDFSA for pattern (a*b+ac)d
running on text string aabd

```
.         5   a
.     2   6   a
. 1   3   6   a
. 1   3   6   a
.     3   6   a   2
.         6   a   2
.             a   2   7
.                 2   7   a
.             1   3   7   a
.                 3   7   a   2
.                     7   a   2
.                         a   2
.                             2   b
.                         1   3   b
.                             3   b
.                                 b   4
.                                     4   d
.                                     8   d
.                                         d   9
.                                             9   *
.                                             0   *
```

Problem: exponential growth

NDFSA for (a*a)*b

```
.          0   1   2   3   4   5   6
.    ch        a       a       b
.  next1   4   2   3   4   2   6   0
.  next2   4   2   1   4   5   6   0
```

Simulation of machine running on aaaaaaab

```
.         4   a
.     5   2   a
.         2   a
.     1   3   a
.         3   a   2
.             a   2   4
.                 2   4   a
.             1   3   4   a
.                 3   4   a   2
.                     4   a   2   4
.                        ...
                             a   2   4   2   4
                                 2   4   2   4   a
...
.     a   2   4   2   4   2   4   2   4
```

Deque can get exponentially large

Easy fix: no duplicate states on deque

    keep "existence table", indexed by state

    also fixes next1=next2 code hack

Worst-case analysis of grep match

N: length of text

M: length of RE

Number of states in NDFSA:

- single character: 2
- concatenation AB: $|A| + |B| - 1$
- closure A*: $|A| + 1$
- OR A+B: $|A| + |B| + 1$

Total: less than or equal to M+1

Time for match:

- not more than M+1 states for each char
- run alg on each text position
- total not more than

  $(M+1)(N + N-1 + N-2 + ...) = O(MN^2)$
- (linear on real problems)

Quintessential tool, classic algorithm

Context

string searching

    brute force implementation O(MN)
    KMP FSA saves a factor of M

grep implementation

    abstract machine: NDFSA
    pattern (word in CFL) --> NDFSA
    simulate NDFSA

compiler

    abstract machine: computer
    program (word in CFL) --> machine code
    run program (hardware simulation of abstract machine)

grep is simple compiler

    converts CFL program to machine code

parser:

    check if pattern is in CFL

grep:

    check if text is in RE

Same function at different levels of hierarchy

Faster algorithm than grep?

    Yes, using tries
    (ref. Gonnet and Baeza-Yates)