# COS 226 Lecture 11: String searching

TEXT: N characters

PATTERN: M characters

(C occurrences of pattern in text)

Existence

Any occurrence of pattern in text?

Enumerate

How many occurrences of pattern in text?

Search

Find an occurence of pattern in text

Search all

Find all occurences of pattern in text

Three parameters N, M, C

start with N >> M >> C

Ex: N = 100,000, M = 100, C = 5

Other factors

multiple patterns (preprocessing)

binary vs. ascii text

avoid backup in text

## String searching examples

Text string: find gjkmxorzeoa in

**kvjlixapejrbxeenpphkhthbkwyrwamnugzhppfx**
**iyjyanhapfwbghxmshwrlyujfjhrsovkvveylnbx**
**nawavgizyvmfohigeabgksfnbkmffxjffqbualey**
**tqrphyrbjqdjqavctgxjifqgfgydhoiwhrvwqbxg**
**rixydzbpajnhopvlamhhfavoctdfytvvggikngkw**
**zixgjtlxkozjlefilbrboignbzsudssvqymnapbp**
**qvlubdoyxkkwhcoudvtkmikansgsutdjythzlapa**
**wlvliygjkmxorzeoafeoffbfxuhkzukeftnrfmoc**
**ylculksedgrdivayjpgkrtedehwhrvvbbltdkctq**

Binary: find 10100111101011011 in

**10001001010110100010011010110101101101100**
**01111111101010011101010000001011110100110**
**10111000000110001100001001100001111011010**
**11000000110111101011111011110001111000010**
**10111110011100110011110110010000001110100**
**11111000010110011100110100101101010010010**
**00011111111111011100111101100101010011110**
**11110011101100100100100110001000100111000**
**00100000111000101011101010011010010001100**

Long text and pattern

* fixed pattern, random text: return "not found"
* fixed text, random pattern: return "not found"

## Brute-force string searching

Check for pattern at every text position

```
int brutesearch(char *p, char *a)
  {
    int i, j, cnt = 0;
    for (i = 0; i < strlen(a); i++)
      for (j = 0; j < strlen(p); j++)
        {
          if (a[i+j] != p[j]) break;
          if (j == strlen(p)-1) return i;
        }
    return strlen(a)+1;
  }
```

DON'T USE THIS PROGRAM! (Why?)

In C, strlen takes time proportional string length

- running time at least N^2
- even for simpler programs (count the a's)

Performance matters in ADT design!

Exercise: implement string ADT with fast strlen

- need space to store length
- need to update length when changing string
- ...

## Brute-force algorithm (continued)

Easy alternative (if bug is discovered!)

```
int brutesearch(char *p, char *a)
   { int i, j;
      int M = strlen(p), N = strlen(a), cnt = 0;
      for (i = 0; i < N; i++)
         for (j = 0; j < M; j++)
            {
               if (a[i+j] != p[j]) break;
               if (j == M-1) return i;
            }
      return N+1;
   }
```

Different implementation (same algorithm)

char match: increment both i and j

char mismatch: set j to 0, reset i

```
int brutesearch(char *p, char *a)
   {
      int i, j, M = strlen(p), N = strlen(a);
      for (i = 0, j = 0; j < M && i < N; i++, j++)
         while (a[i] != p[j]) { i -= j-1; j = 0; }
      if (j == M) return i-M; else return i;
   }
```

## Analysis of brute-force algorithm

Running time depends on text and pattern

Worst case: search for 000001

```
.    00000000000000000000000000000001
.    00000*
.     00000*
.      00000*
.       00000*
.        00000*
```

M*N bit compares

Average case (fixed pattern, random text)

```
.    10011010010100010100111000111
.    *
.     00*
.      0*
.       *
.        *
.         0*
.          *
.           001
```

long pattern: 2*N compares

short pattern:

* precise cost depends on pattern
* (first 001 appears before first 000, on average)

Ref: Flajolet and Sedgewick

## Rabin-Karp algorithm

Idea: Use hashing

* compute hash function for each text position
* no table! (just compare with pattern hash)

Ex: "table" size 97, M = 5

Search for 15926 = 18 (mod 97)

```
.      31415926535897932384626433
.      31415 = 84 (mod 97)
.       14159 = 94 (mod 97)
.        41592 = 76 (mod 97)
.         15926 = 18 (mod 97)
.          59265 = 95 (mod 97)
```

Problem: hash function depends on M characters

Solution: compute hash for i+1 from hash for i

(improves running time from N*M to N+M)

**31415 = 84 (mod 97)**

**14159 = (31415 - 30000)*10 + 9**

**= (84 - 3*9)*10 + 9 (mod 97)**

**= 579 = 94 (mod 97)**

**41592 = (94 - 1*9)*10 + 2 = 76 (mod 97)**

**15926 = (76 - 4*9)*10 + 6 = 18 (mod 97)**

**59265 = (18 - 1*9)*10 + 5 = 95 (mod 97)**

Problem: need full compare on collision

**92653 = (95 - 5*9)*10 + 3 = 18 (mod 97)**

Solution: use giant (virtual) table!

## RK algorithm implementation

Take table size as big as possible
(not so big that overflow happens)

```
#define q 3355439
#define d 256
int rksearch(char *p, char *a)
  {
    int i, j, dM = 1, h1 = 0, h2 = 0;
    int M = strlen(p), N = strlen(a);
    for (j = 1; j < M; j++) dM = (d*dM) % q;
    for (j = 0; j < M; j++)
      {
        h1 = (h1*d + p[j]) % q;
        h2 = (h2*d + a[j]) % q;
      }
    for (i = M; i < N; i++)
      {
        if (h1 == h2) return i-M;
        h2 = (h2 + d*q - a[i-M]*dM) % q;
        h2 = (h2*d + a[i]) % q;
      }
    return N;
  }
```

Randomized algorithm:
take random (big) table size

## Knuth–Morris–Pratt algorithm

Text characters that match are in the pattern!
Can precompute what to do on mismatch

Ex: searching for 000001

- mismatch 00000* implies
  - * text had 000000
  - * if next text char is 1, full match found
  - * if next text char is 0, back in same state
- mismatch 000* implies
  - * text had 0001
  - * restart search on next text char

Finite-state automaton for 000001

| . | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| . | 0 | 1 | 2 | 3 | 4 | 5 | 5 |
| . | 1 | 0 | 0 | 0 | 0 | 0 | 6 |

worst-case text

**. 00000000000000000000000000001**

**. 01234555555555555555555555556**

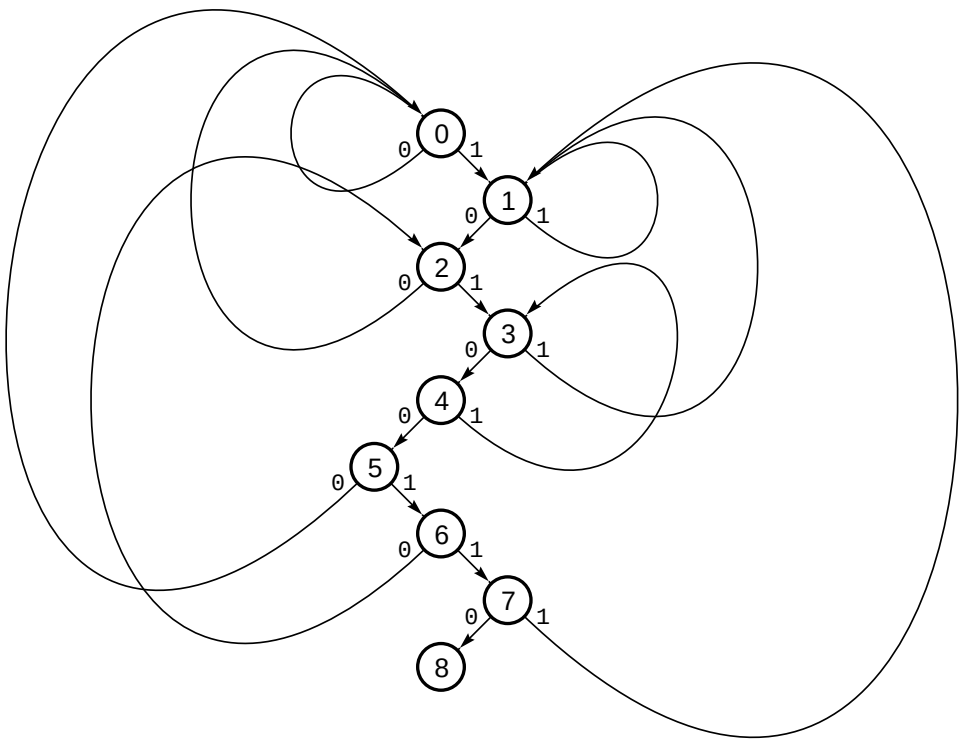
random text

**. 10011101001010001010011100111**

**. 00120001012010123010120001230 0**

# KMP algorithm (continued)

Finite-state automaton for 10100110




| . | | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|---|
| . | 0 | 0 | 2 | 0 | 4 | 5 | 0 | 2 | 8 |
| . | 1 | 1 | 1 | 3 | 0 | 0 | 6 | 7 | 1 |

Search example:

```
.    10011101001010001010011000 0111
.    01201112345623450123456 78
```

KMP algorithm:

- build FSA (or C program) from pattern
- run FSA on text
- N+M running time
- no backup in text

# KMP automaton construction

FSA builds itself (!)

Successors to state i

* match: i+1
* mismatch:
    - FSA state corresponding to
    - * next putative text match
    - * take mismatch into account

Ex: FSA for 10100110

State 6

    1010011: go to state 7

    1010010: go to state for 010010 (state 2)

        remember state for 010011 (X = state 1)

State 7

    10100110: go to state 8

    10100111: go to state for 0100111 (state 1)

        remember state for 0100110 (X = state 2)

main point: these are successors of old X

Algorithm:

    X: mismatch state

    match: FSA[match][i] = i+1

    mismatch:

        FSA[mismatch][i] = FSA[mismatch][X]

        X = FSA[match][X]

# KMP automaton construction example

Pattern: 10100110

```
.         0   1
.    0    0   2
.    1    1   1

.         0   1   2
.    0    0   2   0
.    1    1   1   3

.         0   1   2   3
.    0    0   2   0   4
.    1    1   1   3   0

.         0   1   2   3   4
.    0    0   2   0   4   5
.    1    1   1   3   0   0

.         0   1   2   3   4   5
.    0    0   2   0   4   5   0
.    1    1   1   3   0   0   6

.         0   1   2   3   4   5   6
.    0    0   2   0   4   5   0   2
.    1    1   1   3   0   0   6   7

.         0   1   2   3   4   5   6   7
.    0    0   2   0   4   5   0   2   8
.    1    1   1   3   0   0   6   7   1
```

## KMP implementation

Build FSA from pattern, run FSA on text string

Improvement:
    match state in FSA unnecessary (always i+1)
    use mismatch state only ("next" array)

```
int kmpsearch(char *p, char *a)
{
   int i, j, M = strlen(p), N = strlen(a);
   initnext(p);
   for (i = 0, j = 0; j < M && i < N; i++, j++)
      while ((j >= 0) && (a[i] != p[j])) j = next[j
   if (j == M) return i-M; else return i;
}
```

small hack: next[0] = −1

Ultimate search program for pattern:
    machine language version of

```
int kmpsearch(char *a)
{ int i = 0;
s0: if (a[i] != '1') goto s0; i++;
s1: if (a[i] != '0') goto s1; i++;
s2: if (a[i] != '1') goto s0; i++;
s3: if (a[i] != '0') goto s0; i++;
s4: if (a[i] != '0') goto s0; i++;
s5: if (a[i] != '1') goto s0; i++;
s6: if (a[i] != '1') goto s2; i++;
s7: if (a[i] != '0') goto s1; i++;
     return i-8; }
```

Just another form of FSA[mismatch]

## Right-left pattern scan

Can get sublinear algorithms by moving
right to left in pattern
(left to right in text)

Ex: Find string of 9 consecutive 0s

```
.    1001110100101000101001110001 11
.            0
.           1
.                   1
.                            0
.                           0
.                          1
```

Same idea effective in large alphabet
for every character in alphabet
compute skip when mismatch on that char
(skip at least to next text char!)

Search time proportional to N/M
for practical problems
Time decreases as pattern length increases (!)

Yet another parameter: alphabet size
binary: 2 (group bits for this alg)
text string: 256 (probably less)

## Right-left pattern scan examples

Text character not in pattern

skip forward M chars

```
. now is the time for all good people to come
.       *     *     *     *      e
.                               l
.                              p
.                             o
.                            e
.                           p
```

Text character in pattern

skip forward to putative pattern end

```
. you can fool some of the people some of the time
.       *     *     *      o
.                           e
.                          l
.                         p
.                        o
.                       e
.                      p
```

## Right-left pattern scan implementation

```
initskip(char *p)
   { int j, M = strlen(p);
      for (j = 0; j < 256; j++) skip[j] = M;
      for (j = 0; j < M; j++) skip[p[j]] = M-j-1;
   }

#define max(A, B) (A > B) ? A : B;
int mischarsearch(char *p, char *a)
   { int i, j;
      int M = strlen(p), N = strlen(a);
      initskip(p);
      for (i = M-1, j = M-1; j >= 0; i--, j--)
         while (a[i] != p[j])
            {
               i += max(M-j, skip[a[i]]);
               if (i >= N) return N;
               j = M-1;
            }
      return i+1;
   }
```

Boyer–Moore algorithm

- mismatched character heuristic
- KMP-like computation for skip
- (pick the largest)

# Multiple patterns and text

Related to symbol table ADT

DICTIONARY

build trie from text (preprocess)

pattern lookups in O(lgN) steps

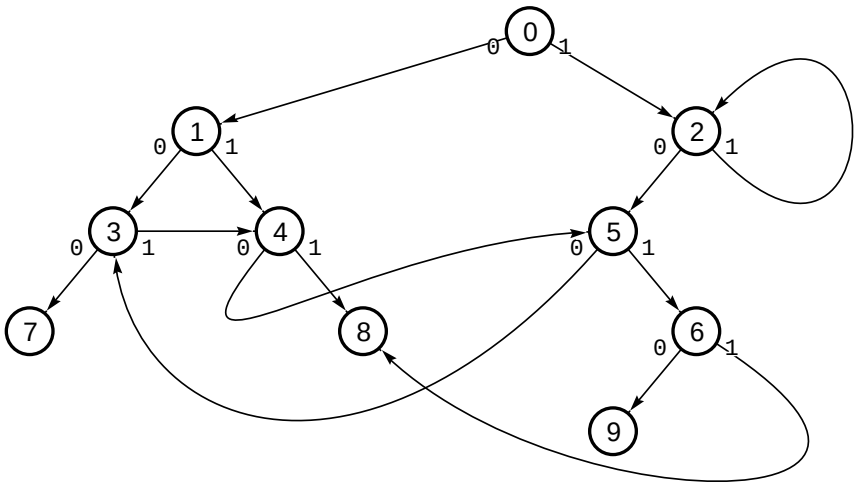
EXCEPTION DICTIONARY

build trie from set of patterns (preprocess)

find patterns in given text in N steps

generalization of KMP

Ex: FSA for 000, 011, and 1010




| . | | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| . | 0 | 1 | 3 | 5 | 7 | 5 | 3 | 9 |
| . | 1 | 2 | 4 | 2 | 4 | 8 | 6 | 8 |

Simultaneous search for all patterns in text

. **1111001001001011101000000**

. **0222253453453456 8**