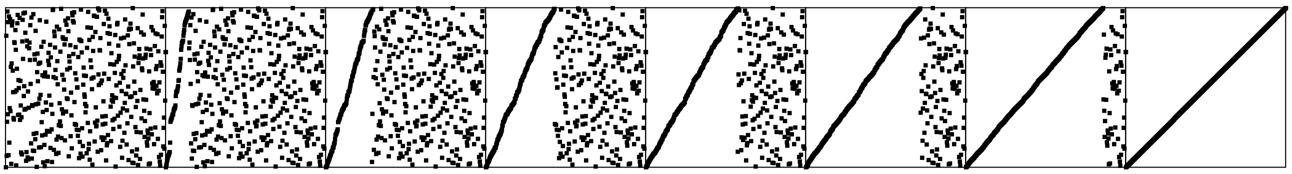
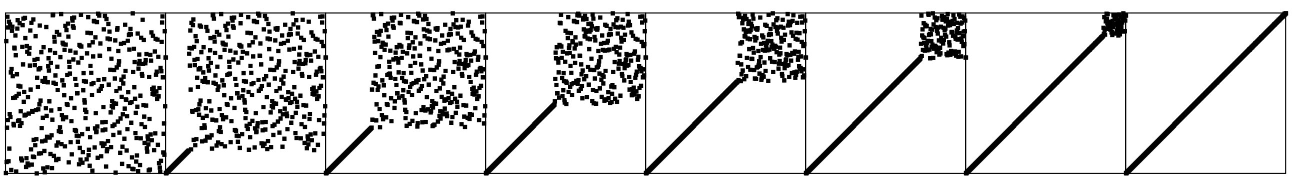
COS 226 Lecture 2: Elementary sorting algs

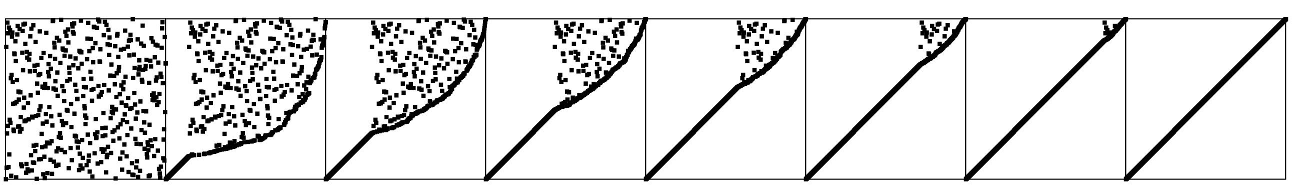
Insertion sort



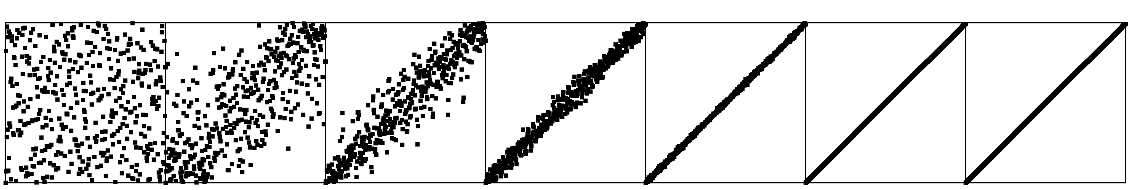
Selection sort



Bubble sort



Shellsort



Why study elementary algorithms?

Easy to code

Fastest for small files

Context for developing ground rules

Fastest in some special situations

May not be so elementary

Ground rules

FILES of RECORDS containing KEYS

File fits in memory

Use abstract comparison, exchange

```
typedef int itemType
#define less(A, B)  (A < B)
#define exch(A, B)
        { itemType t = A; A = B; B = t; }
```

Macros or subroutines?

- Macros: low cost, simple
- Subroutines: more general

Selection sort

A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	A	O	R	T	I	N	G	E	X	S	M	P	L	E
A	A	E	R	T	I	N	G	O	X	S	M	P	L	E
A	A	E	E	T	I	N	G	O	X	S	M	P	L	R
A	A	E	E	G	I	N	T	O	X	S	M	P	L	R
A	A	E	E	G	I	N	T	O	X	S	M	P	L	R
A	A	E	E	G	I	L	T	O	X	S	M	P	N	R
A	A	E	E	G	I	L	M	O	X	S	T	P	N	R
A	A	E	E	G	I	L	M	N	X	S	T	P	O	R
A	A	E	E	G	I	L	M	N	O	S	T	P	X	R
A	A	E	E	G	I	L	M	N	O	P	T	S	X	R
A	A	E	E	G	I	L	M	N	O	P	R	S	X	T
A	A	E	E	G	I	L	M	N	O	P	R	S	X	T
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X

```
void selection(itemType a[], int l, int r)
  { int i, j;
    for (i = l; i < r; i++)
      { int min = i;
        for (j = i+1; j <= r; j++)
            if (less(a[j], a[min])) min = j;
        exch(a[i], a[min]);
      }
  }
```

Insertion sort

A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	O	S	R	T	I	N	G	E	X	A	M	P	L	E
A	O	R	S	T	I	N	G	E	X	A	M	P	L	E
A	O	R	S	T	I	N	G	E	X	A	M	P	L	E
A	I	O	R	S	T	N	G	E	X	A	M	P	L	E
A	I	N	O	R	S	T	G	E	X	A	M	P	L	E
A	G	I	N	O	R	S	T	E	X	A	M	P	L	E
A	E	G	I	N	O	R	S	T	X	A	M	P	L	E
A	E	G	I	N	O	R	S	T	X	A	M	P	L	E
A	A	E	G	I	N	O	R	S	T	X	M	P	L	E
A	A	E	G	I	M	N	O	R	S	T	X	P	L	E
A	A	E	G	I	M	N	O	P	R	S	T	X	L	E
A	A	E	G	I	L	M	N	O	P	R	S	T	X	E
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X

```
void insertion(itemType a[], int l, int r)
  { int i, j;
    for (i = l+1; i <= r; i++)
      { itemType v = a[i];
        j = i;
        while (j > l && less(v, a[j-1]))
          { a[j] = a[j-1]; j--; }
        a[j] = v;
      }
  }
```

Bubble sort

A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	A	S	O	R	T	I	N	G	E	X	E	M	P	L
A	A	E	S	O	R	T	I	N	G	E	X	L	M	P
A	A	E	E	S	O	R	T	I	N	G	L	X	M	P
A	A	E	E	G	S	O	R	T	I	N	L	M	X	P
A	A	E	E	G	I	S	O	R	T	L	N	M	P	X
A	A	E	E	G	I	L	S	O	R	T	M	N	P	X
A	A	E	E	G	I	L	M	S	O	R	T	N	P	X
A	A	E	E	G	I	L	M	N	S	O	R	T	P	X
A	A	E	E	G	I	L	M	N	O	S	P	R	T	X
A	A	E	E	G	I	L	M	N	O	P	S	R	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
A	A	E	E	G	I	L	M	N	O	P	R	S	T	X

```
void bubble(itemType a[], int l, int r)
  { int i, j;
     for (i = l; i < r; i++)
        for (j = r; j > i; j--)
              compexch(a[j], a[j-1]);
  }
```

Improvements:

* add a test to exit if no exchanges
* go back and forth

Properties of elementary sorts

All quadratic running time

Selection sort

    comparisons: N-1 + N-2 + ... + 2 + 1 = N^2/2
    exchanges: N

Insertion sort

    comparisons: (N-1 + N-2 + ... + 2 + 1)/2 = N^2/4
    exchanges: N^2/4

Bubble sort

    comparisons: N-1 + N-2 + ... + 2 + 1 = N^2/2
    exchanges: about N^2/2

Large records, small keys

    selection sort linear in amount of data
    N records M words (1-word keys)
        comparison cost N^2/2
        exchange cost NM
    if N proportional to M
        costs and amount of data proportional to M^2

Files nearly in order

    bubble and insertion sort can be linear
    (quicksort can be quadratic)

Pointer sort

Sort large records by swapping references to the records, not the records themselves

1	9	Fox	1
2	4	Quilici	1
3	8	Chen	2
4	3	Furia	3
5	1	Kanaga	3
6	5	Andrews	3
7	10	Rohde	3
8	6	Battle	4
9	2	Aaron	4
10	7	Gazsi	4

Trivial to implement: change abstract comparison

Array indices

```
typedef int itemType
#define less(A, B) (key(A) < key(B))
#define exch(A, B)
   { itemType t = A; A = B; B = t; }
```

True pointers

```
typedef keyType* itemType
#define less(A, B) (*A < *B)
#define exch(A, B)
   { itemType t = A; A = B; B = t; }
```

Stable sorting for two-key records

Sort on the first key, then on the second

Aaron	**4**	**Fox**	**1**
Andrews	**3**	**Quilici**	**1**
Battle	**4**	**Chen**	**2**
Chen	**2**	**Furia**	**3**
Fox	**1**	**Kanaga**	**3**
Furia	**3**	**Andrews**	**3**
Gazsi	**4**	**Rohde**	**3**
Kanaga	**3**	**Battle**	**4**
Quilici	**1**	**Aaron**	**4**
Rohde	**3**	**Gazsi**	**4**

Stable sort: stays sorted on first key where equal on second

Fox	**1**
Quilici	**1**
Chen	**2**
Andrews	**3**
Furia	**3**
Kanaga	**3**
Rohde	**3**
Aaron	**4**
Battle	**4**
Gazsi	**4**

Which of the elementary methods are stable?

4-sorting

Divide into 4 subfiles

every 4th element starting at the 1st

every 4th element starting at the 2nd

every 4th element starting at the 3rd

every 4th element starting at the 4th

A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	S	O	R	E	I	N	G	T	X	A	M	P	L	E
A	S	O	R	E	I	N	G	P	X	A	M	T	L	E

A	I	O	R	E	S	N	G	P	X	A	M	T	L	E
A	I	O	R	E	S	N	G	P	X	A	M	T	L	E
A	I	O	R	E	L	N	G	P	S	A	M	T	X	E

A	I	N	R	E	L	O	G	P	S	A	M	T	X	E
A	I	A	R	E	L	N	G	P	S	O	M	T	X	E
A	I	A	R	E	L	E	G	P	S	N	M	T	X	O

A	I	A	G	E	L	E	R	P	S	N	M	T	X	O
A	I	A	G	E	L	E	M	P	S	N	R	T	X	O

A I A G E L E M P S N R T X O

Interleaved 4-sorting

Use insertion sort with an "increment" of 4

A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
A	I	O	R	T	S	N	G	E	X	A	M	P	L	E
A	I	N	R	T	S	O	G	E	X	A	M	P	L	E
A	I	N	G	T	S	O	R	E	X	A	M	P	L	E
A	I	N	G	E	S	O	R	T	X	A	M	P	L	E
A	I	N	G	E	S	O	R	T	X	A	M	P	L	E
A	I	A	G	E	S	N	R	T	X	O	M	P	L	E
A	I	A	G	E	S	N	M	T	X	O	R	P	L	E
A	I	A	G	E	S	N	M	P	X	O	R	T	L	E
A	I	A	G	E	L	N	M	P	S	O	R	T	X	E
A	I	A	G	E	L	E	M	P	S	N	R	T	X	O

```
h = 4;
for (i = l+h; i <= r; i++)
   { itemType v = a[i];
      j = i;
      while (j >= l+h && less(v, a[j-h]))
         { a[j] = a[j-h]; j -= h; }
      a[j] = v;
   }
```

Shellsort

Use a decreasing sequence of increments

	A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
13	A	S	O	R	T	I	N	G	E	X	A	M	P	L	E
	A	E	O	R	T	I	N	G	E	X	A	M	P	L	S
	A	E	O	R	T	I	N	G	E	X	A	M	P	L	S
4	A	E	O	R	T	I	N	G	E	X	A	M	P	L	S
	A	E	N	R	T	I	O	G	E	X	A	M	P	L	S
	A	E	N	G	T	I	O	R	E	X	A	M	P	L	S
	A	E	N	G	E	I	O	R	T	X	A	M	P	L	S
	A	E	N	G	E	I	O	R	T	X	A	M	P	L	S
	A	E	A	G	E	I	N	R	T	X	O	M	P	L	S
	A	E	A	G	E	I	N	M	T	X	O	R	P	L	S
	A	E	A	G	E	I	N	M	P	X	O	R	T	L	S
	A	E	A	G	E	I	N	M	P	L	O	R	T	X	S
	A	E	A	G	E	I	N	M	P	L	O	R	T	X	S
	A	E	A	G	E	I	N	M	P	L	O	R	T	X	S
1	A	A	E	G	E	I	N	M	P	L	O	R	T	X	S
	A	A	E	G	E	I	N	M	P	L	O	R	T	X	S
	A	A	E	E	G	I	N	M	P	L	O	R	T	X	S
	A	A	E	E	G	I	N	M	P	L	O	R	T	X	S
	A	A	E	E	G	I	N	M	P	L	O	R	T	X	S
	A	A	E	E	G	I	M	N	P	L	O	R	T	X	S
	A	A	E	E	G	I	M	N	P	L	O	R	T	X	S
	A	A	E	E	G	I	L	M	N	P	O	R	T	X	S
	A	A	E	E	G	I	L	M	N	O	P	R	T	X	S
	A	A	E	E	G	I	L	M	N	O	P	R	T	X	S
	A	A	E	E	G	I	L	M	N	O	P	R	T	X	S
	A	A	E	E	G	I	L	M	N	O	P	R	T	X	S
	A	A	E	E	G	I	L	M	N	O	P	R	S	T	X
	A	A	E	E	G	I	L	M	N	O	P	R	S	T	X

Each pass makes the next easier

Shellsort implementation

```
void shellsort(itemType a[], int l, int r)
  { int i, j;
    int incs[16] = { 1391376, 463792, 198768,
                       86961, 33936, 13776, 4592,
                       1968, 861, 336, 112, 48,
                       21, 7, 3, 1 };
    for ( k = 0; k < 16; k++)
      { int h = incs[k];
        for (i = l+h; i <= r; i++)
          { itemType v = a[i];
            j = i;
            while (j >= h && less(v, a[j-h]))
              { a[j] = a[j-h]; j -= h; }
            a[j] = v;
          }
      }
  }
```

Need a sort routine, fast? Use Shellsort!

* not much code
* best method for small and medium files
* still OK even for giant files

How do we know what increments to use?

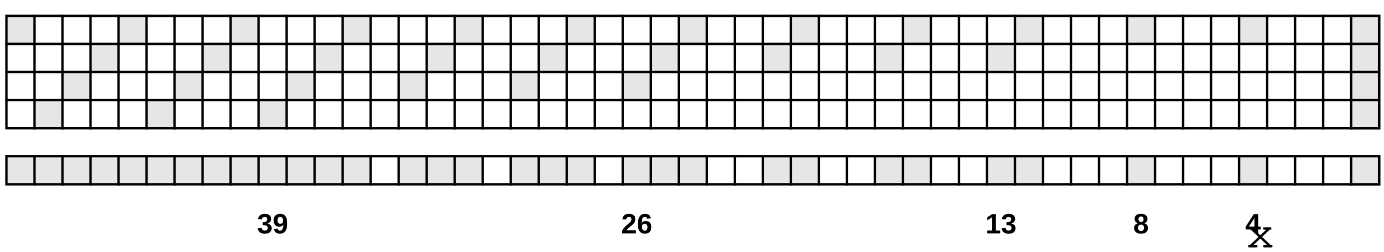
* plenty of proven winners to use
* easiest: 1, 4, 13, 40, 121, 363, 1090, ...

Relatively prime increment sequences

When we h-sort a file that is k-sorted,
    it stays k-sorted
(Know an easy proof? SEND MAIL)

Only 18N comparisons are needed to 1-sort a file
    that is 4-sorted and 13-sorted

Elements to the left of x that could be greater:



In general, if h and k are relatively prime:

Only (h-1)(k-1)N comparisons to 1-sort a file
    that is h-sorted and k-sorted
Only (h-1)(k-1)N/g comparisons to g-sort a file
    that is h-sorted and k-sorted

Big increments, small files: h(N/h)^2 = N^2/h
Small increments, use theorem: h^2N/h   = Nh

Tradeoff best bounds: N^(3/2) total
Similar methods (harder proofs) give 4/3, 5/4, 6/5 ...

More increment sequences

On the other hand, common divisors are good:

Only N comparisons to 1-sort a file that is
2-sorted and 3-sorted
Only N comparisons to 2-sort a file that is
4-sorted and 6-sorted
Only N comparisons to 3-sort a file that is
6-sorted and 9-sorted

```
.                  1
.                2   3
.              4   6   9
.            8   12  18  27
.          16  24  36  54  81
.        32  48  72  108 162 243
.      64  96  144 216 324 486 729
```

Total time: N (log N)(log N)

Too many increments for real sizes
- start with bigger numbers than 2 and 3
- mix up with primes

Have a better idea for an increment sequence?
SEND MAIL if it beats 1 2 7 21 48 112 336 ...