Consistency
March 30th+31st, 2022
Consistency Models

- Strict Serializability
- Linearizability
- Sequential
- Causal+
- Eventual

- Stronger
- Weaker
Consistency Models

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Strict Serializability

- **Transactions**: Operations can span multiple objects (e.g., keys in KV store)
- **Total order**: There exists some legal total ordering of transactions.
  - Legal: In the total ordering, a read operation sees the latest write operation.
- **Preserves real-time ordering**: Any transaction $A$ that completes before transaction $B$ begins, occurs before $B$ in the total order.
- **Properties**:
  - Writes in a completed transaction appear to all future reads
  - Once a read sees transaction and completes, all future reads must see new transaction

**Pros**: Easily reason about correctness of transactions

**Cons**: High read and write latencies
# Strict Serializability Example

<table>
<thead>
<tr>
<th>Strictly Serializable?</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1:</td>
<td>{W(x)b, W(y)b}</td>
</tr>
<tr>
<td>P2: {W(x)a}</td>
<td></td>
</tr>
<tr>
<td>P3:</td>
<td>{R(x)a}   {R(x)b}</td>
</tr>
<tr>
<td>P4:</td>
<td>{R(x)b}   {R(y)b}</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Strictly Serializable?</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1:</td>
<td>{W(x)b, W(y)b}</td>
</tr>
<tr>
<td>P2: {W(x)a}</td>
<td></td>
</tr>
<tr>
<td>P3:</td>
<td>{R(y)b}   {R(x)a}</td>
</tr>
<tr>
<td>P4:</td>
<td>{R(x)b}   {R(y)b}</td>
</tr>
</tbody>
</table>
Consistency Models

- Strict Serializability
- Linearizability
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- Eventual

Strength:
- Stronger
- Weaker
Linearizability

- **Total order**: There exists some legal total order of operations
- **Preserves real-time ordering**: Any operation $A$ that completes before operation $B$ begins, occurs before $B$ in the total order.
- **Difference from **strict serializability**?**
  - Single-object operations! No transactions!
- **Properties**
  - A completed write appears to all future reads
  - Once a read sees a new value, all future reads must return the new value (until new write)

**Pros**: Easy to reason about correctness

**Cons**: High read and write latencies
# Linearizability Example

<table>
<thead>
<tr>
<th>Linearizable?</th>
<th>No</th>
<th>Linearizable?</th>
<th>Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1:</td>
<td>W(x)a</td>
<td>P1:</td>
<td>W(x)a</td>
</tr>
<tr>
<td>P2:</td>
<td>W(x)b</td>
<td>P2:</td>
<td>W(x)b</td>
</tr>
<tr>
<td>P3:</td>
<td>R(x)b</td>
<td>P3:</td>
<td>R(x)a</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x)b</td>
<td>P4:</td>
<td>R(x)a</td>
</tr>
</tbody>
</table>
Consistency Models

- **Strict Serializability**
- **Linearizability**
- **Sequential**
- **Causal+**
- **Eventual**

**Stronger** → **Sequential** → **Weaker**
Sequential Consistency

- **Total order**: There exists some legal total order of operations.
- **Preserves process ordering**: Total order respects order of each process’s operations.
- **Difference from linearizability?**
  - Order of ops across processes not determined by real-time

**Pros**: Can allow more orderings than linearizability $\rightarrow$ better performance

**Cons**: Many possible sequential executions $\rightarrow$ increased application complexity
### Sequential Consistency Example

<table>
<thead>
<tr>
<th>P1:</th>
<th>W(x)a</th>
<th>P1:</th>
<th>W(x)a</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2:</td>
<td>W(x)b</td>
<td>P2:</td>
<td>W(x)b</td>
</tr>
<tr>
<td>P3:</td>
<td>R(x)b, R(x)a</td>
<td>P3:</td>
<td>R(x)b, R(x)a</td>
</tr>
<tr>
<td>P4:</td>
<td>R(x)b, R(x)a</td>
<td>P4:</td>
<td>R(x)a, R(x)b</td>
</tr>
</tbody>
</table>

#### Sequentially Consistent? Yes

#### Sequentially Consistent? No
Consistency Models

Strict Serializability  Linearizability  Causal+  Sequential  Eventual

Stronger  Weaker
Causal+ Consistency

- **Partial order**: Order causally related ops the same way across all processes
- **+**: Replicas eventually converge
- **Difference from **sequential consistency**?**
  - Only causally related ops need to be ordered: no total order
  - Concurrent ops may be ordered differently across different processes

**Pros**: Preserves causality while improving efficiency

**Cons**: Need to reason about concurrency
<table>
<thead>
<tr>
<th>Ops</th>
<th>Concurrent</th>
</tr>
</thead>
<tbody>
<tr>
<td>a,b</td>
<td>No</td>
</tr>
<tr>
<td>a,e</td>
<td>Yes</td>
</tr>
<tr>
<td>a,g</td>
<td>No</td>
</tr>
<tr>
<td>c,e</td>
<td>Yes</td>
</tr>
<tr>
<td>c,d</td>
<td>No</td>
</tr>
<tr>
<td>d,g</td>
<td>No</td>
</tr>
<tr>
<td>d,f</td>
<td>No</td>
</tr>
<tr>
<td>e,g</td>
<td>No</td>
</tr>
<tr>
<td>a,d</td>
<td>No</td>
</tr>
</tbody>
</table>
Causal+ Consistency Example

Causally+ Consistent? Yes

P1: W(x)a
P2: W(x)b
P3: R(x)b  R(x)a
P4: R(x)a

Causally+ Consistent? No

P1: W(x)a
P2: R(x)a  W(x)b
P3: R(x)b  R(x)a
P4: R(x)a
Consistency Models

- Strict Serializability
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Stronger to Weaker

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Eventual Consistency

- **Eventual convergence**: If no more writes, all replicas *eventually* agree
- **Difference from *causal consistency*?**
  - Does not preserve causal relationships
  - Is the “+” in causal+
- **Frequently used with application conflict resolution, anti-entropy**

**Pros**: Highly available; think Dynamo

**Cons**: No safety guarantees, need conflict resolution
In a nutshell...

**Strict Serializability**: Total order + real time guarantees over *transactions*

**Linearizability**: Total order + real time guarantees over *operations*

**Sequential consistency**: Total order + process order

**Causal+ consistency**: Causally ordered + replicas eventually converge

**Eventual consistency**: Eventually everyone should agree on state
Exercise 1:

Consistency Model:

- Strictly Serializable: Yes
- Linearizable: Yes
- Sequential: Yes
- Causal+: Yes
- Eventual: Yes

P1:  \{W(x) 1, W(y) 2\} \{R(y) 4\}

P2:  \{W(x) 1, R(y) 4\}

P3:  \{W(x) 0, W(y) 4\}

P4:  \{R(x) 0\} \{R(x) 1\}
Exercise 2:

Consistency Model:

- Linearizable: Yes
- Sequential: Yes
- Causal+: Yes
- Eventual: Yes

P1: W(x) 1  R(y) 4

P2: R(x) 1  R(y) 4

P3: R(x) 1  W(y) 4

P4: R(x) 1  R(y) 4
<table>
<thead>
<tr>
<th>Exercise 3:</th>
<th>Consistency Model:</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1: W(x) 3</td>
<td>Linearizable: No</td>
</tr>
<tr>
<td>W(y) 7</td>
<td>Sequential: Yes</td>
</tr>
<tr>
<td>P2: W(x) 1</td>
<td>Causal+: Yes</td>
</tr>
<tr>
<td>P3: R(x) 1</td>
<td>Eventual: Yes</td>
</tr>
<tr>
<td>R(x) 3</td>
<td></td>
</tr>
<tr>
<td>R(y) 7</td>
<td></td>
</tr>
<tr>
<td>P4: R(x) 1</td>
<td></td>
</tr>
<tr>
<td>R(x) 3</td>
<td></td>
</tr>
<tr>
<td>R(y) 7</td>
<td></td>
</tr>
<tr>
<td>P5: R(x) 1</td>
<td></td>
</tr>
<tr>
<td>R(x) 3</td>
<td></td>
</tr>
<tr>
<td>R(y) 7</td>
<td></td>
</tr>
</tbody>
</table>
Exercise 4:

<table>
<thead>
<tr>
<th></th>
<th>W(x)</th>
<th>W(y)</th>
<th>R(x)</th>
<th>R(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>3</td>
<td></td>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>P2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>P4</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>P5</td>
<td>1</td>
<td>3</td>
<td>7</td>
<td></td>
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**Consistency Model:**
- Linearizable: No
- Sequential: No
- Causal+: Yes
- Eventual: Yes
Exercise 5:

Consistency Model:
- Linearizable: No
- Sequential: No
- Causal+: Yes
- Eventual: Yes

P1: \( W(x) 1 \)
P2: \( W(x) 3 \)
P3: \( W(x) 7 \)
P4: \( R(x) 3 \quad R(x) 7 \quad R(x) 1 \)
P5: \( R(x) 3 \quad R(x) 1 \quad R(x) 7 \)
Exercise 6:

Consistency Model:

- Linearizable: No
- Sequential: No
- Causal+: Yes
- Eventual: Yes

P1: \( W(x) \ 1 \)

P2: \( W(x) \ 3 \)

P3: \( R(x) \ 3 \quad W(x) \ 7 \)

P4: \( R(x) \ 3 \quad R(x) \ 7 \quad R(x) \ 1 \)

P5: \( R(x) \ 3 \quad R(x) \ 1 \quad R(x) \ 7 \)
Exercise 7:

<p>| | | |</p>
<table>
<thead>
<tr>
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<td>P3</td>
<td>R(x) 3</td>
<td>W(x) 7</td>
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<tr>
<td>P5</td>
<td>R(x) 3</td>
<td>R(x) 1</td>
</tr>
</tbody>
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**Consistency Model:**

- **Linearizable**: No
- **Sequential**: No
- **Causal+**: No
- **Eventual**: Yes