Problem Set No. 1

1. Describe a $\Theta(n \lg n)$-time algorithm that, given a set S of n real numbers and another real number x, determines whether or not there exist two elements in S whose sum is exactly x.

2. Prove that for $i \geq 0$, the $(i+2)$nd Fibonacci number satisfies $F_{i+2} \geq \phi^i$.

3. Show that $\sum_{k=1}^{n} 1/k^2$ is bounded above by a constant.

4. Solve the recurrence $T(n) = 2T(\sqrt{n}) + 1$ by making a chnage of variables. Do not worry about whether values are integral.

5. **Finding the missing integer**

   An array $A[1 \ldots n]$ contains all the integers from 0 to n except one. It would be easy to determine the missing integer in $O(n)$ time by using an auxiliary array $B[0 \ldots n]$ to record which numbers appear in A. In this problem, however, we cannot access an entire integer in A with a single operation. The elements of A are represented in binary, and the only operation we can use to access them is "fetch the j^{th} bit of $A[i]$," which takes constant time.

   Show that if we use only this operation, we can still determine the missing integer in $O(n)$ time.