Class Meeting #8
8 Puzzle

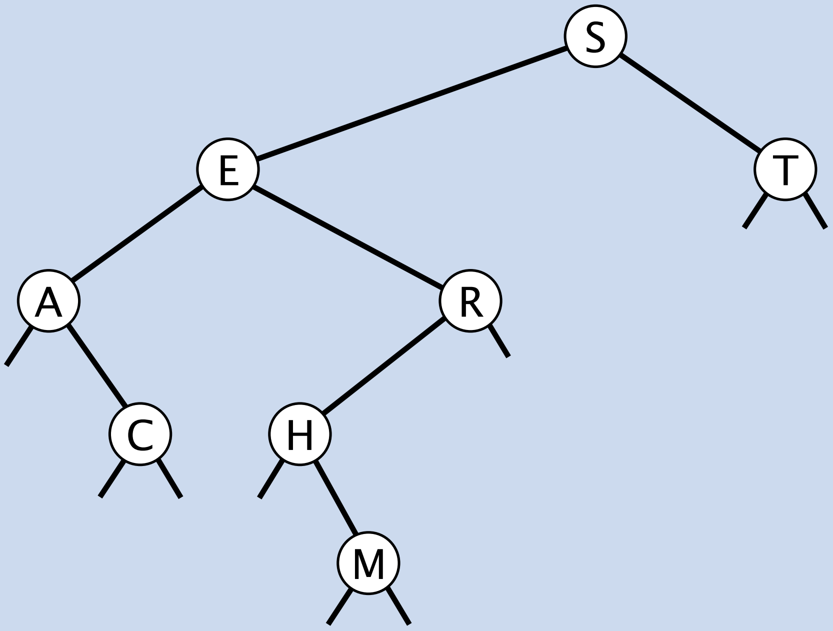
COS 226

Based in part ok slides by Jérémie Lumbroso and Kevin Wayne

LEVEL-ORDER TRAVERSAL

Level-order traversal of a binary tree.

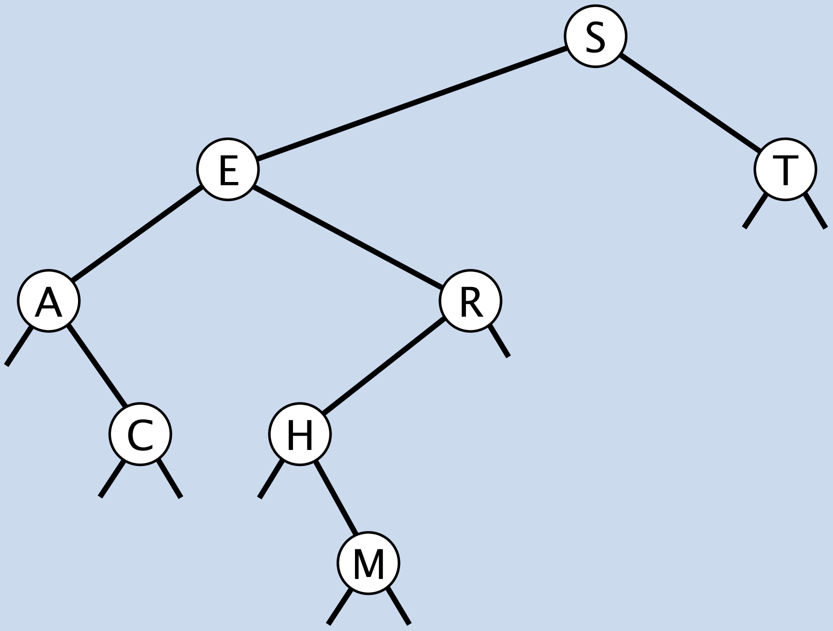
- Process root.
- Process children of root, from left to right.
- Process grandchildren of root, from left to right.
- …



level-order traversal: **S E T A R C H M**

LEVEL-ORDER TRAVERSAL

Q1. Given binary tree, how to compute level-order traversal?



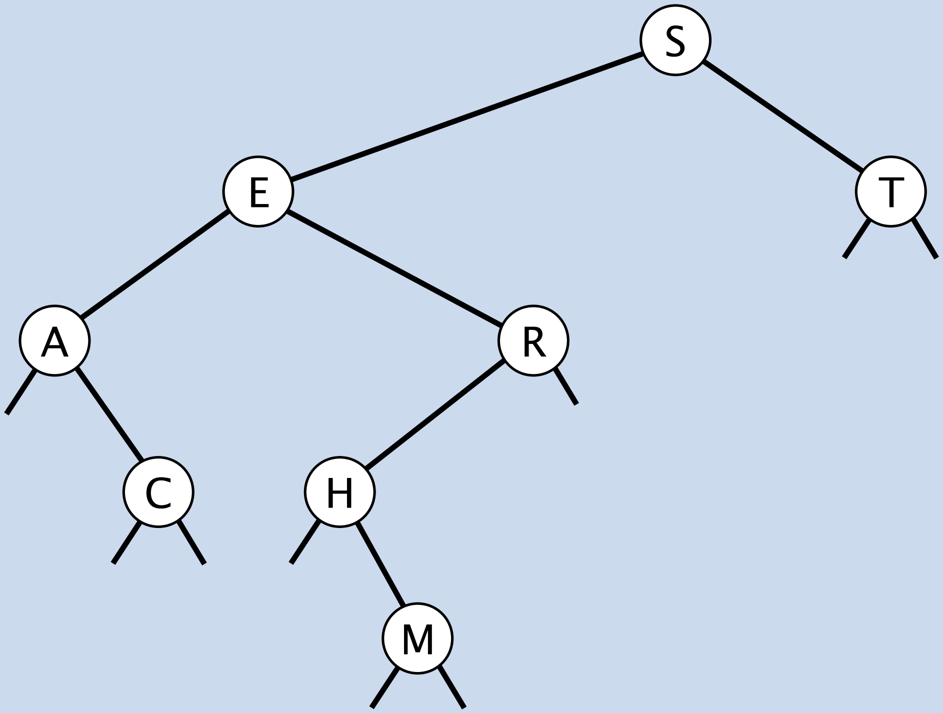

```
queue.enqueue(root);
while (!queue.isEmpty())
{
   Node x = queue.dequeue();
   if (x == null) continue;
   StdOut.println(x.item);
   queue.enqueue(x.left);
   queue.enqueue(x.right);
}
```

level-order traversal: **S E T A R C H M**

LEVEL-ORDER TRAVERSAL

Q2. Given the level-order traversal of a BST, how to (uniquely) reconstruct?

Ex. ~~S~~ ~~E~~ ~~T~~ ~~A~~ ~~R~~ ~~C~~ ~~H~~ ~~M~~

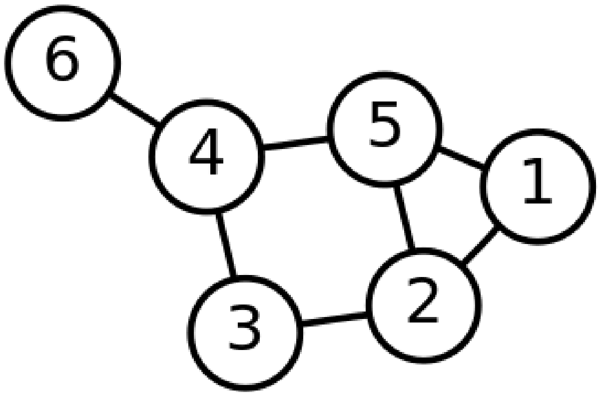


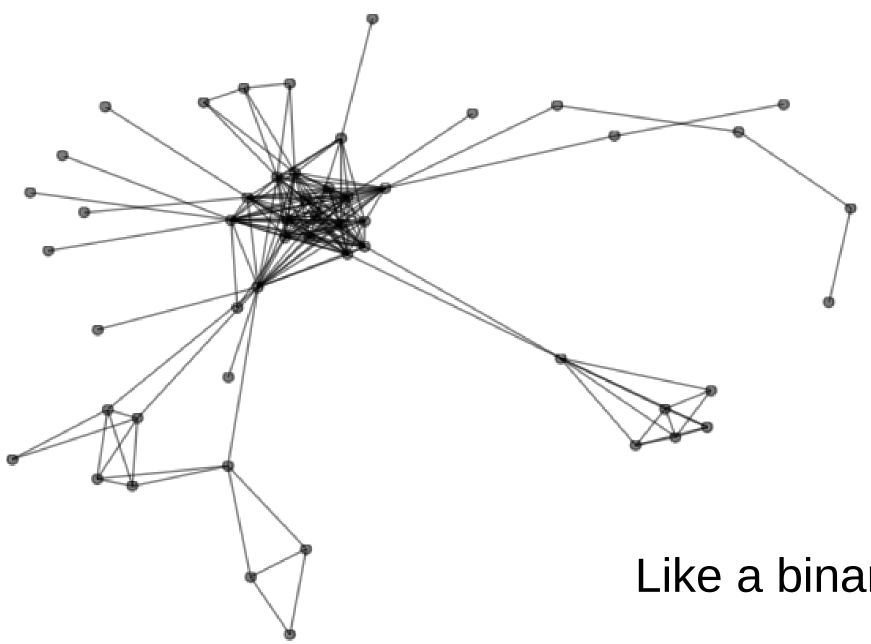
EVEN-DRIVEN SIMULATION DEMO

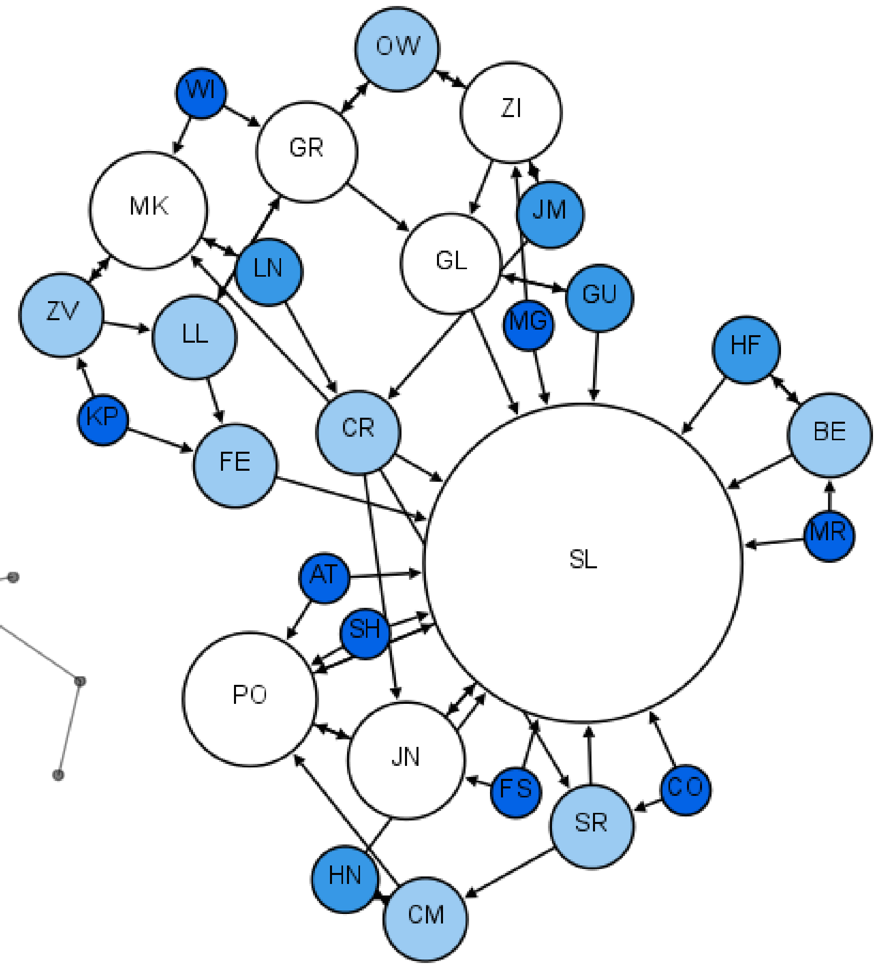
8 PUZZLE

KEY INGREDIENTS!

What is a graph?







Like a binary tree, except there can be cycles.

What is a Priority Queue?

- Comes in two flavors: MinPQ / MaxPQ

MinPQ

Key must be Comparable
(bounded type parameter)

public class MaxPQ<Key extends Comparable<Key>>

	MaxPQ()	MinPQ()	*create an empty priority queue*
	MaxPQ(Key[] a)		*create a priority queue with given keys*
void	insert(Key v)		*insert a key into the priority queue*
Key	delMax()	delMin()	*return and remove the largest key*
boolean	isEmpty()		*is the priority queue empty?*
Key	max()	min()	*return the largest key*
int	size()		*number of entries in the priority queue*

What is a Board?

.	1	3
4	2	5
7	8	6

- Immutable type (defensive copy)
- Knows how to compute neighbors
- Estimates how far from goal

```
public class Board {
    public Board(int[][] tiles)                // construct a board from an N-by-N array of tiles
                                               // (where tiles[i][j] = tile at row i, column j)
    public int tileAt(int i, int j)            // return tile at row i, column j (or 0 if blank)
    public int size()                          // board size N
    public int hamming()                       // number of tiles out of place
    public int manhattan()                     // sum of Manhattan distances between tiles and goal
    public boolean isGoal()                    // is this board the goal board?
    public boolean isSolvable()                // is this board solvable?
    public boolean equals(Object y)            // does this board equal y?
    public Iterable<Board> neighbors()         // all neighboring boards
    public String toString()                   // string representation of this board

    public static void main(String[] args) // unit testing (required)
}
```

STRAIGHTFORWARD

LESS STRAIGHTFORWARD

WHAT IS A* SEARCH?

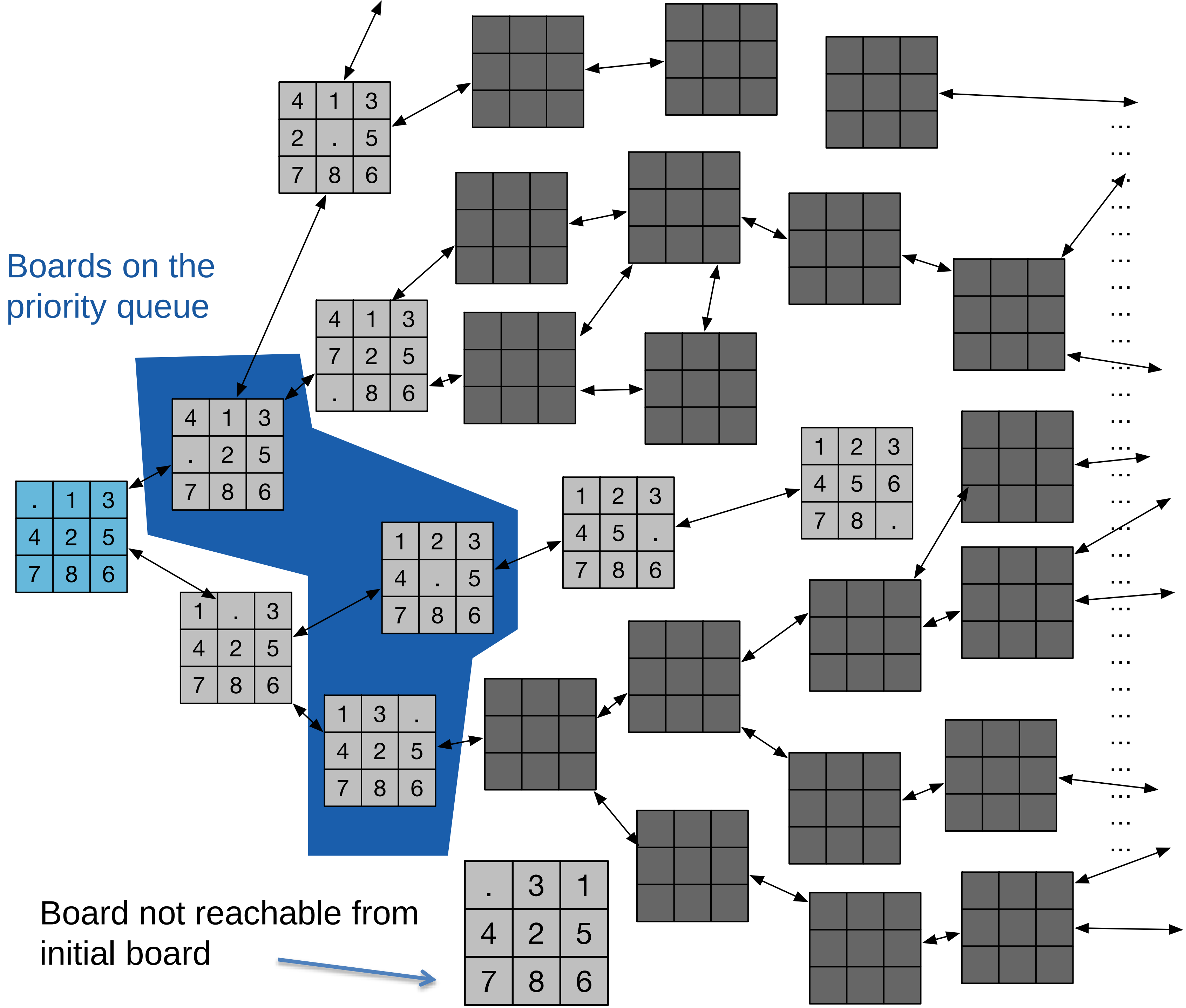
Example run

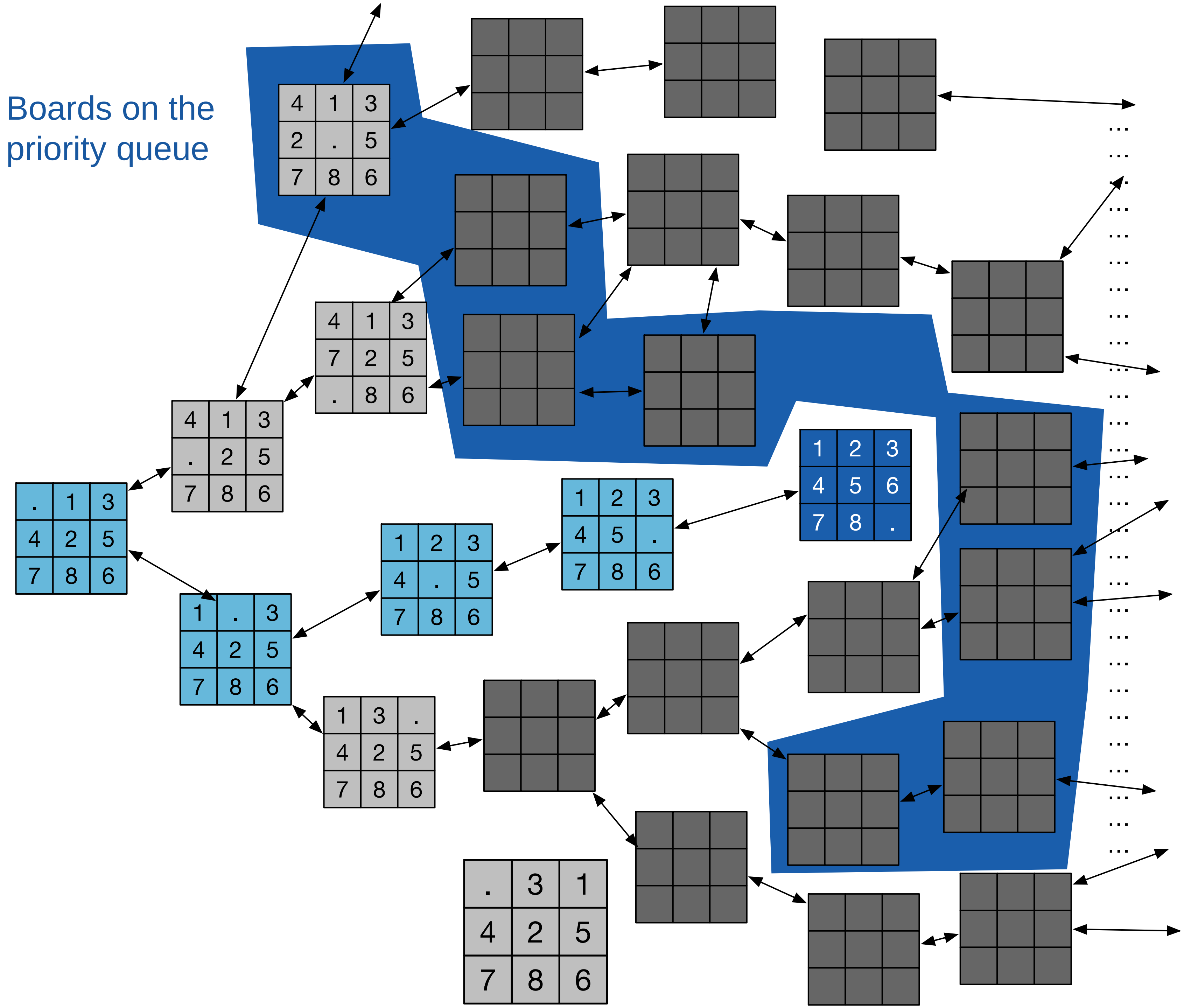
.	1	3
4	2	5
7	8	6

Initial board

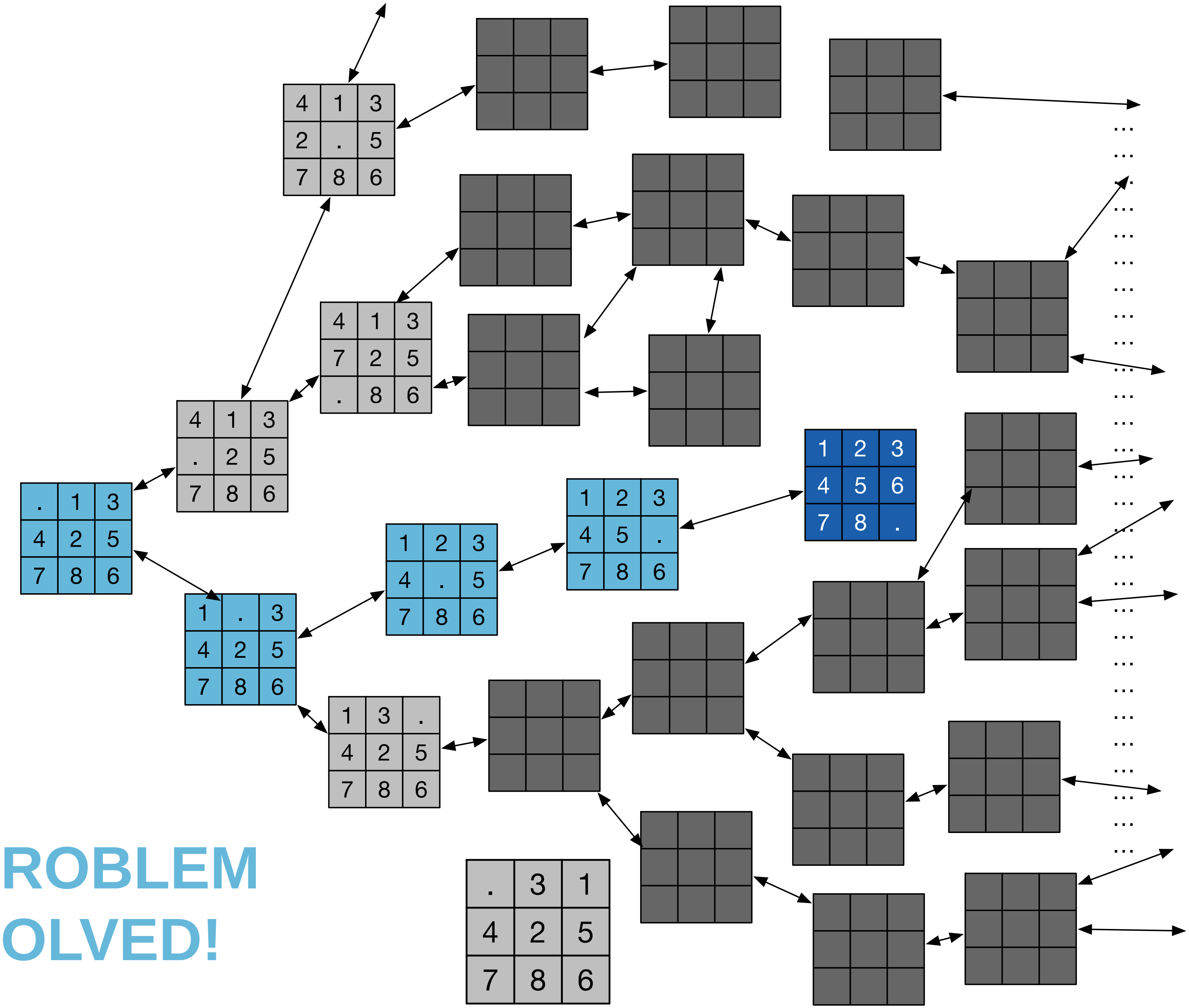
- Solve problem for board on left
- Draw graph of all boards
- Schematize search through graph of boards
- Show role of MinPQ

- This is `puzzle04.txt`


Boards on the priority queue
Board not reachable from initial board
. 1 3
4 2 5
7 8 6
4 1 3
. 2 5
7 8 6
4 1 3
2 . 5
7 8 6
4 1 3
7 2 5
. 8 6
1 . 3
4 2 5
7 8 6
1 2 3
4 . 5
7 8 6
1 3 .
4 2 5
7 8 6
1 2 3
4 5 .
7 8 6
1 2 3
4 5 6
7 8 .
. 3 1
4 2 5
7 8 6



Boards on the priority queue
4 1 3
2 . 5
7 8 6
4 1 3
7 2 5
. 8 6
4 1 3
. 2 5
7 8 6
. 1 3
4 2 5
7 8 6
1 . 3
4 2 5
7 8 6
1 2 3
4 . 5
7 8 6
1 2 3
4 5 .
7 8 6
1 2 3
4 5 6
7 8 .
1 3 .
4 2 5
7 8 6
. 3 1
4 2 5
7 8 6



PROBLEM SOLVED!

```
% more puzzle04.txt
3
 0  1  3
 4  2  5
 7  8  6

% java-algs4 Solver puzzle04.txt
Minimum number of moves = 4
3
 0  1  3
 4  2  5
 7  8  6

3
 1  0  3
 4  2  5
 7  8  6
```

```
3
 1  2  3
 4  0  5
 7  8  6

3
 1  2  3
 4  5  0
 7  8  6

3
 1  2  3
 4  5  6
 7  8  0
```

Observations

- The graph is MUCH TOO BIG
- Some boards are not reachable from start

.	1	3
4	2	5
7	8	6

.	3	1
4	2	5
7	8	6

(carefully read part about “unsolvable puzzles”)

- The MinPQ (Priority Queue) always contains a fringe of boards that we should look at next

A* search

- Use a “priority function” to try to guide the search through the large graph
- Some conditions on this priority function, but basically

  priority = estimated min. number of moves
- We give:
  - Hamming (number of misplaced squares + moves so far)
  - Manhattan (sum of distances to correct position)
  - other ideas?

TIPS

Tip #1: Avoid Dropbox Timeout

- Too much (Terminal) output
  - remove print out statements
  - or use assert / debugging that can be turned off easily
- Infinite loops
- Much more memory usage than predicted
  - it may be useful to test only one file at a time in Dropbox

Tip #2: Board before Solver

- Fully test Board.java before doing Solver.java
- If Board.java is not fully tested, things can go **very very very wrong** in Solver.java

Tip #3: Iterable neighbors

- You have to implement:

```
// return the neighboring board positions,
// as an Iterable
public Iterable<Board> neighbors() {
    ...
}
```

- **Idea:** create a Queue (or Stack), add boards to it, and return Queue (or Stack)
- Queue/Stack are Iterable objects

Tip #4: Class SearchNode

- In Solver.java, create a SearchNode
- This [immutable] SearchNode will wrap around a Board, and make it **Comparable** (by priority)
- Being **Comparable** is needed to use MinPQ

```
private static class SearchNode
    implements Comparable<SearchNode>
{
    // ...
}
```

- SearchNode should also have a pointer to the previous Node (so you can remember the solution)

Tip #5: Test Equality of Board

- The **critical optimization** is making sure we don't go back and forth between two boards (may cause infinite loop, or significantly delay search)
- To avoid this, **Board needs to implement equals**
- Tricky!

Equality test

All Java classes inherit a method `equals()`.

Useful for assignment

Java requirements. For any references `x`, `y` and `z`:

- Reflexive: `x.equals(x)` is `true`.
- Symmetric: `x.equals(y)` iff `y.equals(x)`.
- Transitive: if `x.equals(y)` and `y.equals(z)`, then `x.equals(z)`.
- Non-null: `x.equals(null)` is `false`.

equivalence relation

do x and y refer to the same object?

Default implementation. `(x == y)`

Customized implementations. `Integer`, `Double`, `String`, `java.io.File`, ...

User-defined implementations. Some care needed.

Implementing equals for user-defined types

Seems easy.

```
public       class Date implements Comparable<Date>
{
   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals(Date that)
   {









      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

check that all significant fields are the same

Implementing equals for user-defined types

Seems easy, but requires some care.

typically unsafe to use equals() with inheritance (would violate symmetry)

```
public final class Date implements Comparable<Date>
{
   private final int month;
   private final int day;
   private final int year;
   ...

   public boolean equals(Object y)
   {
      if (y == this) return true;

      if (y == null) return false;

      if (y.getClass() != this.getClass())
         return false;

      Date that = (Date) y;
      if (this.day   != that.day  ) return false;
      if (this.month != that.month) return false;
      if (this.year  != that.year ) return false;
      return true;
   }
}
```

must be Object.
Why? Experts still debate.

optimize for true object equality

check for null

objects must be in the same class (religion: getClass() vs. instanceof)

cast is guaranteed to succeed

check that all significant fields are the same

Equals design

"Standard" recipe for user-defined types.

Useful for assignment

- Optimization for reference equality.
- Check against `null`.
- Check that two objects are of the same type; cast.
- Compare each significant field:
  - if field is a primitive type, use == ← but use Double.compare() with double (to deal with -0.0 and NaN)
  - if field is an object, use `equals()` ← apply rule recursively
  - if field is an array, apply to each entry ← can use `Arrays.deepEquals(a, b)` but not `a.equals(b)`

Best practices.

e.g., cached Manhattan distance

- No need to use calculated fields that depend on other fields.
- Compare fields mostly likely to differ first.
- Make `compareTo()` consistent with `equals()`.

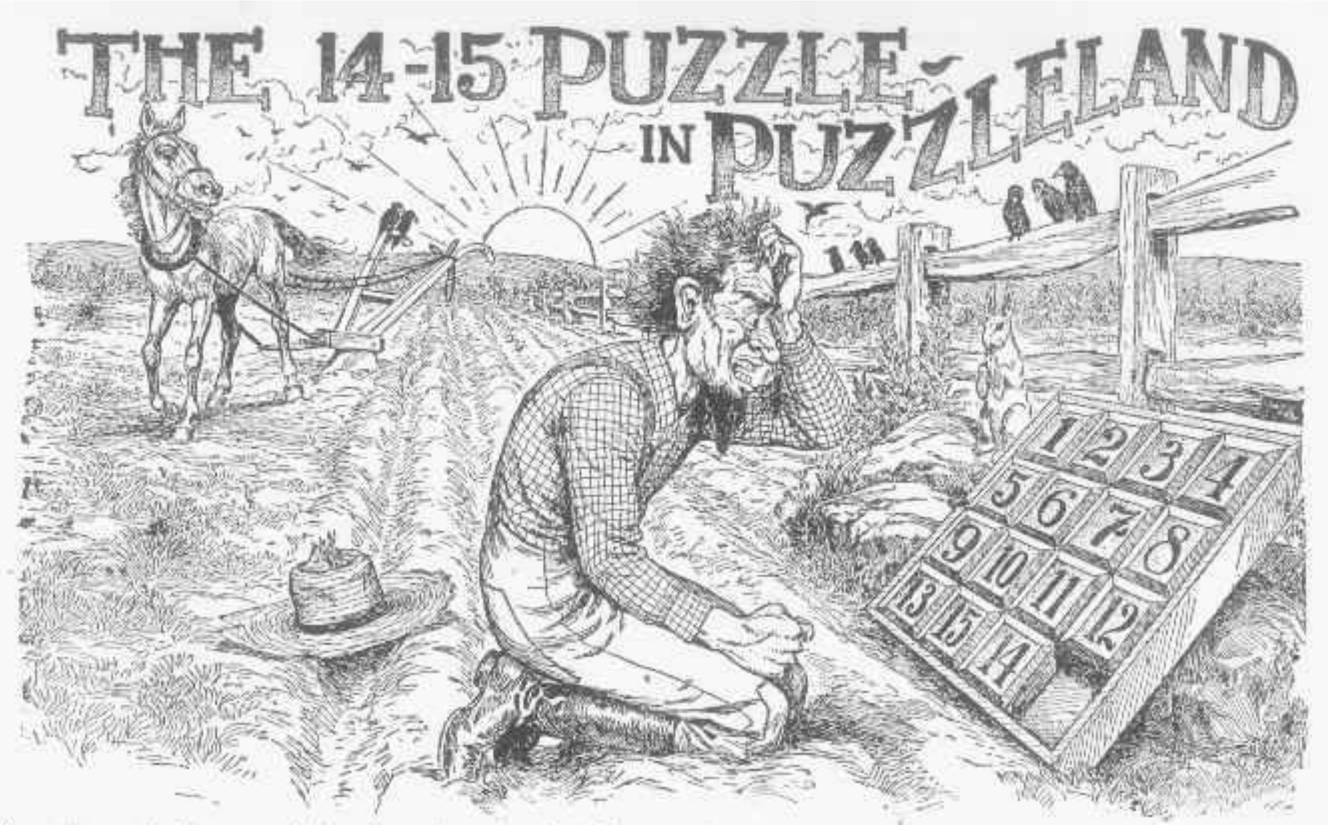
`x.equals(y)` if and only if `(x.compareTo(y) == 0)`

Two optimizations

- **Critical**: Avoid adding the neighbor “you just arrived from” to the priority queue:

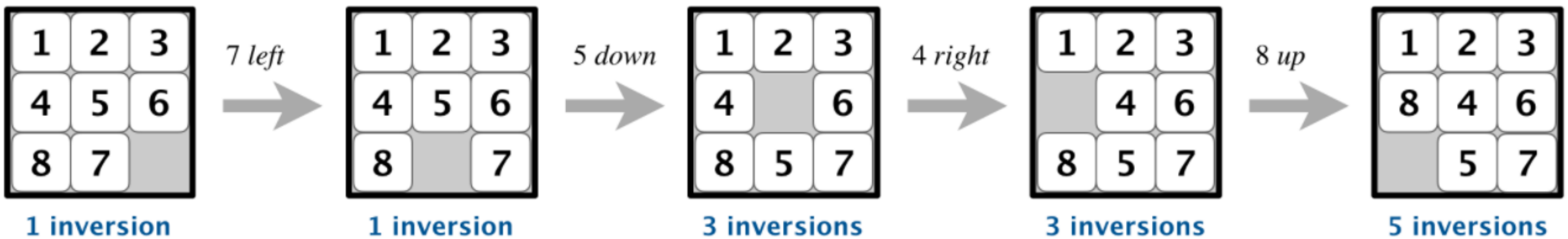
- Cache Manhattan distance inside the board as an instance variable and compute in the constructor (to avoid recomputing it)

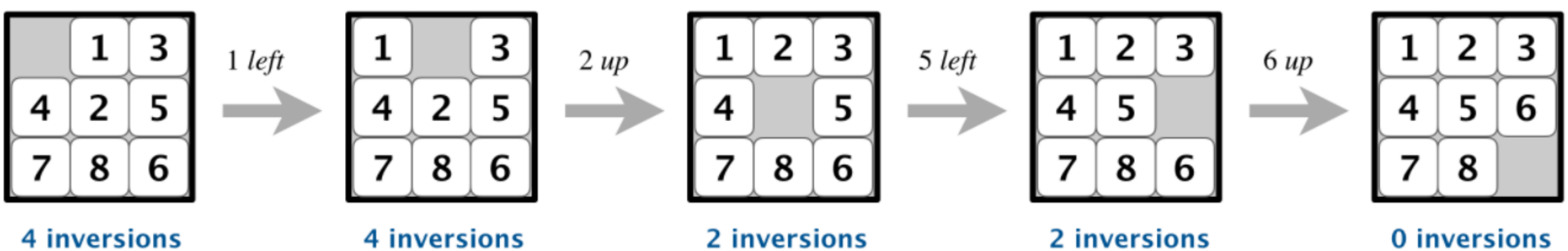
When is a board solvable?



- By Sam Loyd, scanned by Ed Pegg Jr, 2005 - Sam Loyd's Cyclopedia of Puzzles pp. 234–235, scanned image, on web page, linked from The Cyclopedia of Puzzles, page by Ed Pegg Jr., Public Domain, https://commons.wikimedia.org/w/index.php?curid=10520923

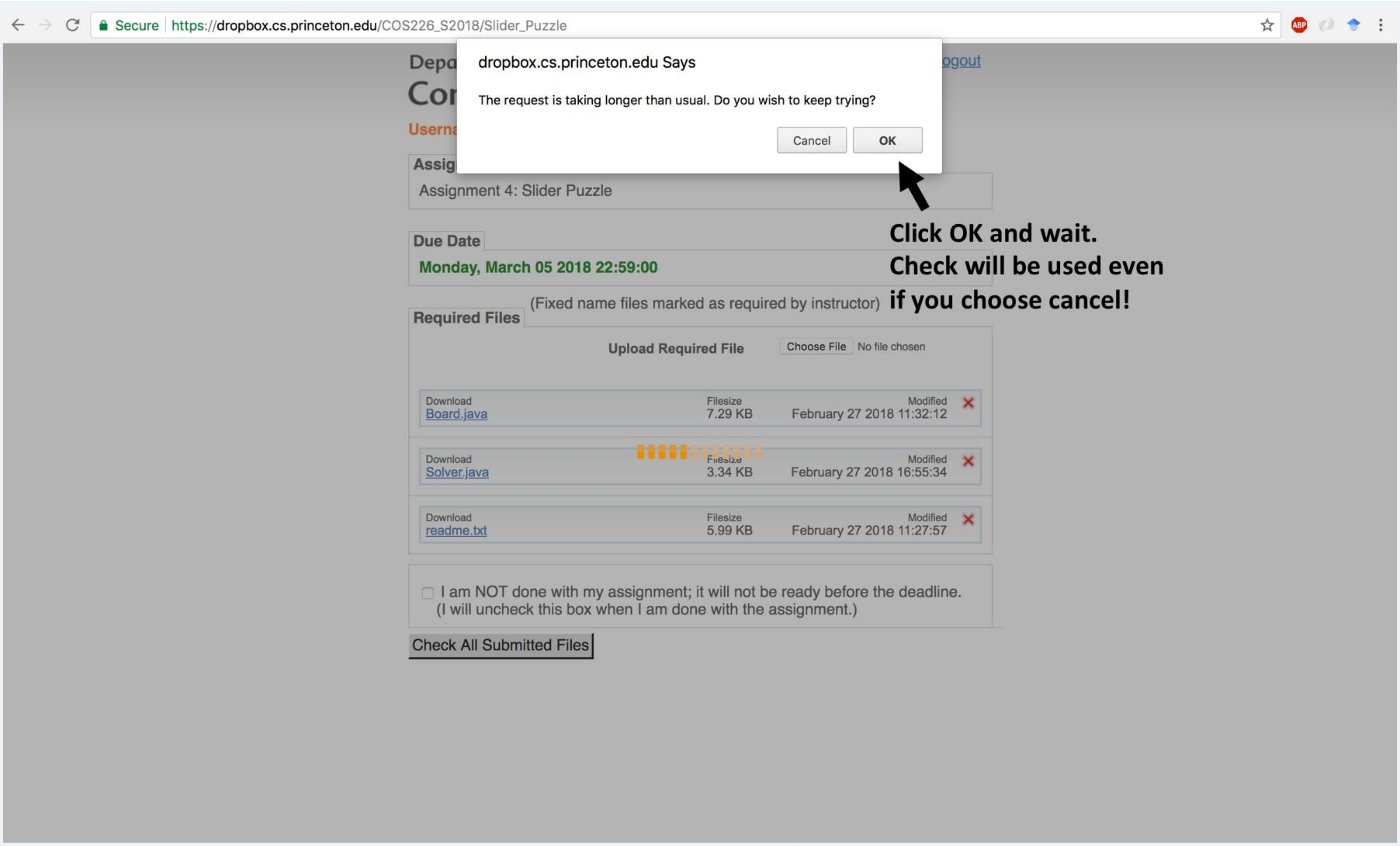
When is a board solvable?





When is a board solvable?

- An odd-size board is solvable if and only if the number of inversions is even.
- If n is even, the board is solvable if and only if the number of inversion plus the row of the blank square (counting from 0) is odd.


Secure https://dropbox.cs.princeton.edu/COS226_S2018/Slider_Puzzle
dropbox.cs.princeton.edu Says
The request is taking longer than usual. Do you wish to keep trying?
Cancel
OK
Logout
Assignment 4: Slider Puzzle
Due Date
Monday, March 05 2018 22:59:00
Required Files
(Fixed name files marked as required by instructor)
Upload Required File
Choose File No file chosen
Download Board.java Filesize 7.29 KB Modified February 27 2018 11:32:12
Download Solver.java Filesize 3.34 KB Modified February 27 2018 16:55:34
Download readme.txt Filesize 5.99 KB Modified February 27 2018 11:27:57
I am NOT done with my assignment; it will not be ready before the deadline.
(I will uncheck this box when I am done with the assignment.)
Check All Submitted Files
Click OK and wait.
Check will be used even
if you choose cancel!