Passive Dynamics and Particle Systems

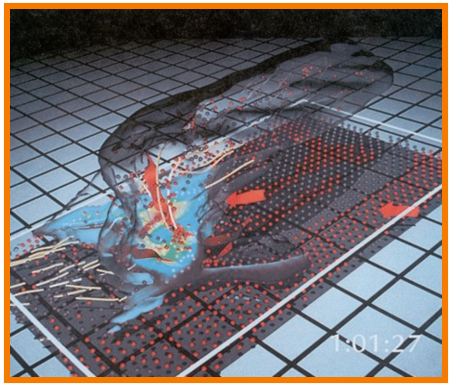
COS 426, Spring 2017
Princeton University

Animation & Simulation

- Animation
  - Make objects change over time according to scripted actions

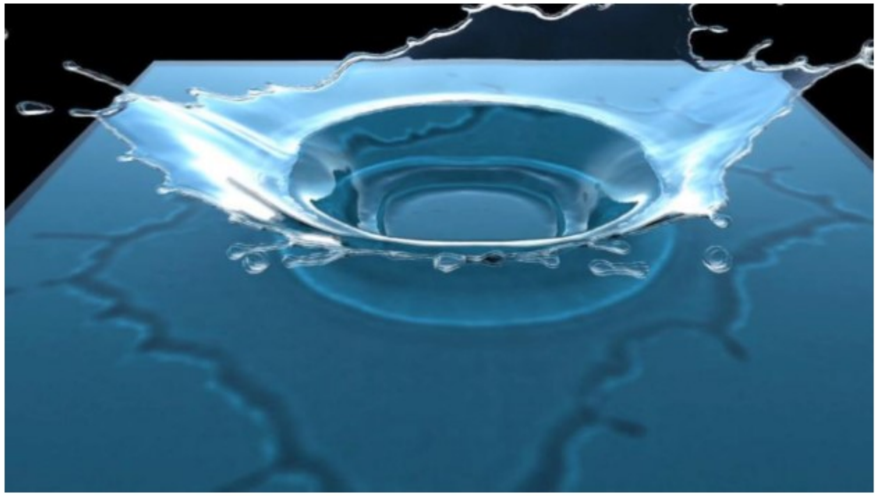
Pixar

- Simulation / dynamics
  - Predict how objects change over time according to physical laws



University of Illinois

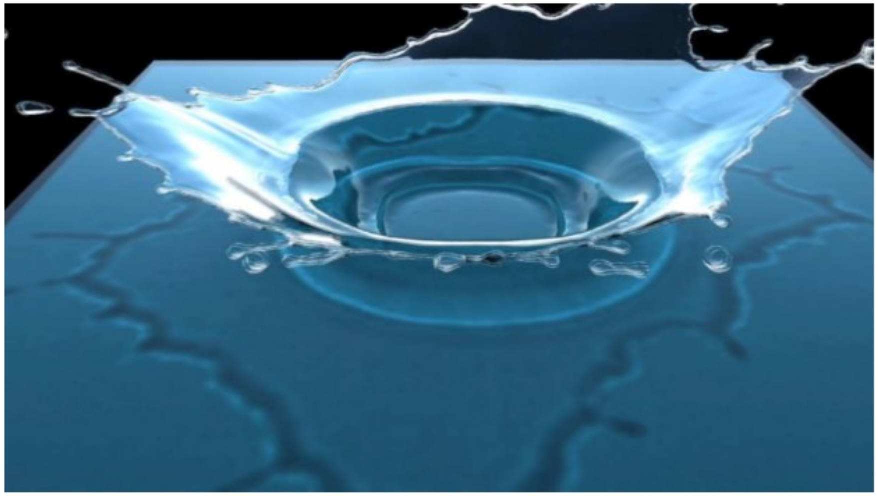
Animation & Simulation



Keyframing

- good for characters and simple motion
- but many physical systems are too complex

Simulation



1. Identify/derive mathematical model (ODE, PDE)
2. Develop computer model
3. Simulate

Simulation

Equations known for a long time

- Motion (Newton, 1660)

$$d/dt(m\mathbf{v}) = \mathbf{f}$$

- Elasticity (Hooke, 1670)

$$\boldsymbol{\sigma} = \mathbf{E}\boldsymbol{\varepsilon}$$

- Fluids (Navier, Stokes, 1822)

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -k\nabla\rho + \rho\mathbf{g} + \mu\nabla^2\mathbf{v}$$

1938: Zuse Z1

0.2 ops

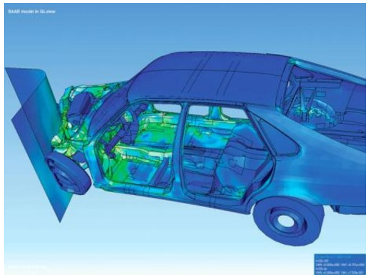
2014: Tianhe-2 @ NUDT (China)

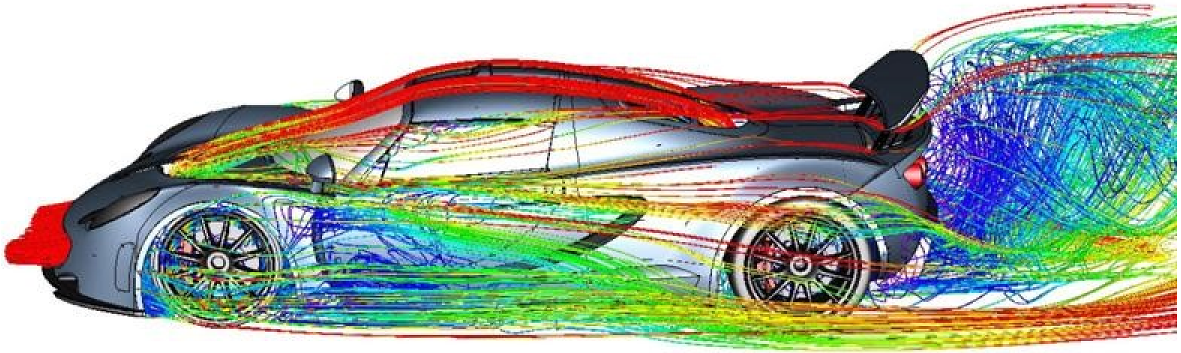
54,902 teraflops (3.12M cores)

Simulation

Physically-based simulation

- Computational Sciences
  - **Reproduction** of physical phenomena
  - Predictive capability (accuracy!)
  - Substitute for expensive experiments



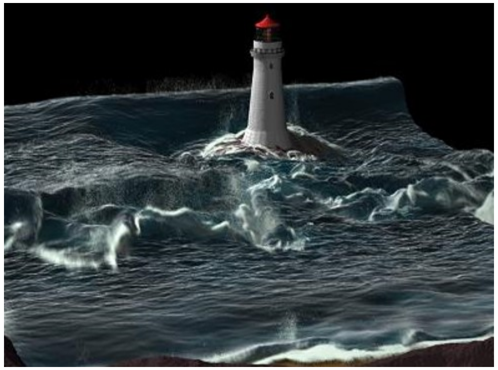


Simulation in Graphics

Physically-based simulation

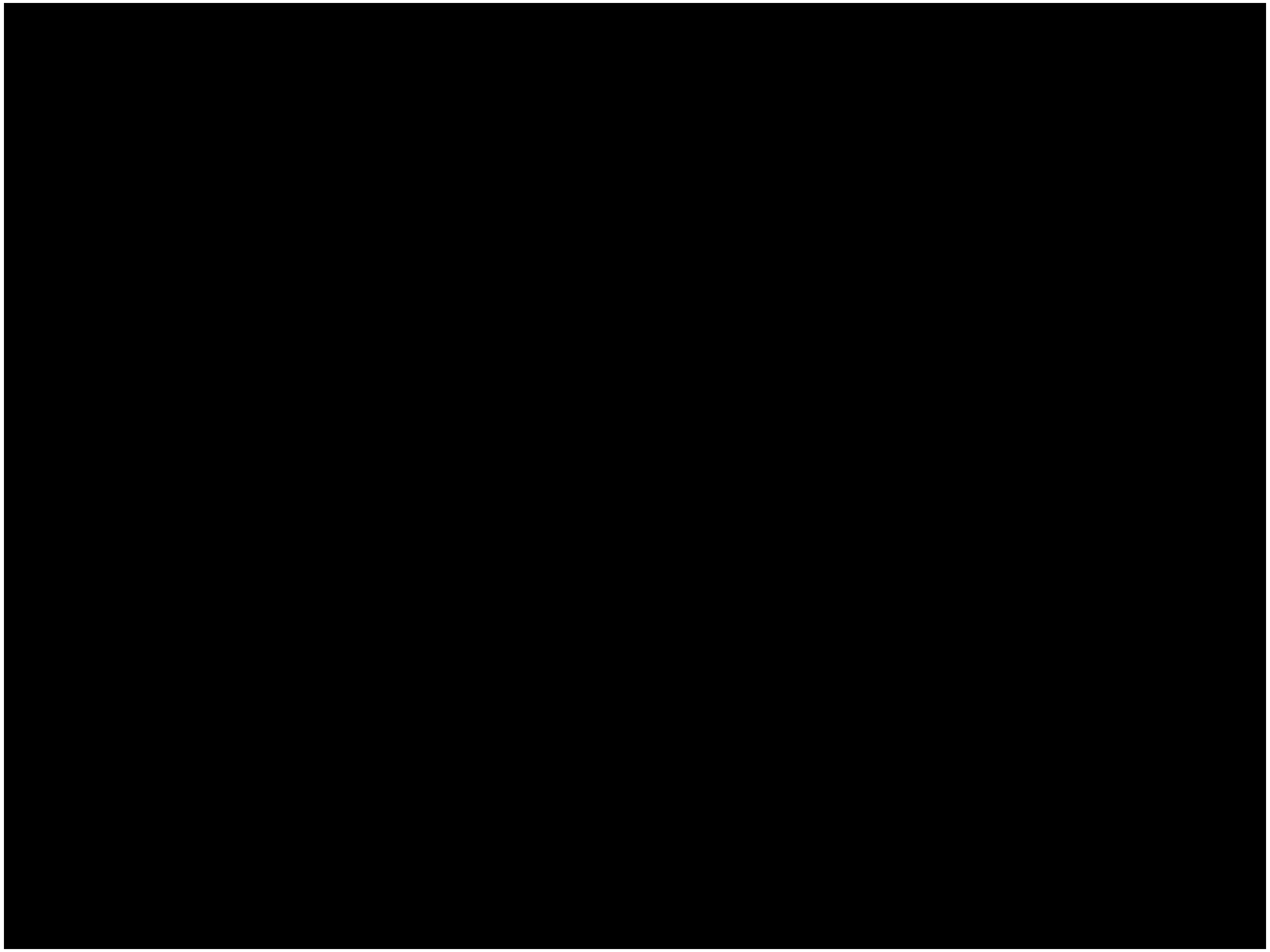
- Computational Sciences
  - **Reproduction** of physical phenomena
  - Predictive capability (accuracy!)
  - Substitute for expensive experiments

- Computer Graphics
  - **Imitation** of physical phenomena
  - Visually plausible behavior
  - Speed, stability, art-directability



Simulation in Graphics

- Art-directability



Simulation in Graphics

- Speed

https://www.youtube.com/watch?v=8jD1bz4N3_0

Simulation in Graphics

- Stability

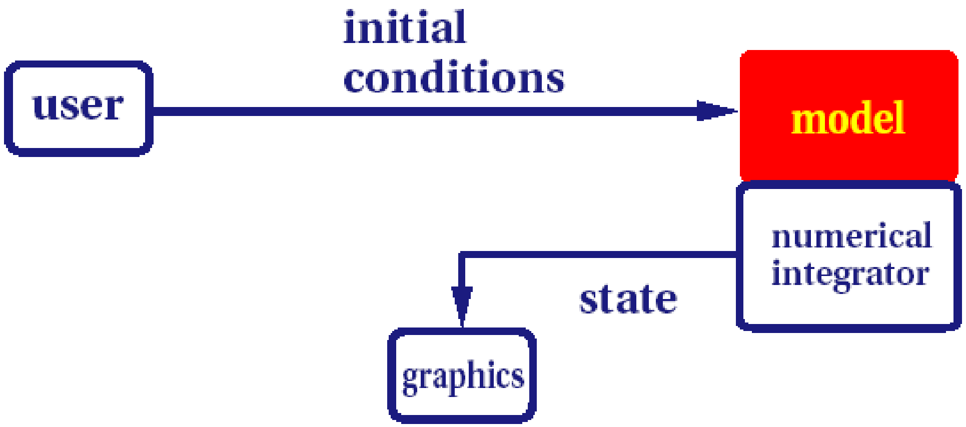
https://www.youtube.com/watch?v=tT81VPk_ukU

Simulation in Graphics

- Rigid bodies
  - Collision
  - Fracture
- Fluids
- Elasticity
  - Muscle + skin
  - Paper
  - Hair
  - Cloth
- …

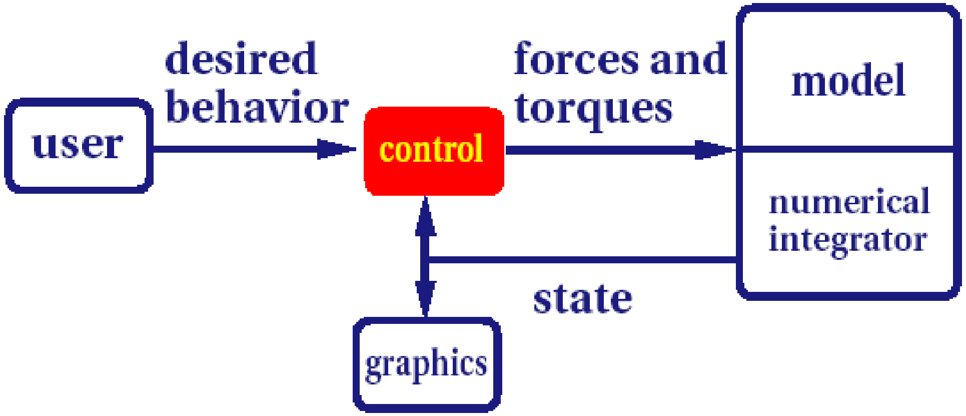
Dynamics

Passive--no muscles or motors



particle systems
leaves
water spray
clothing

Active--internal source of energy

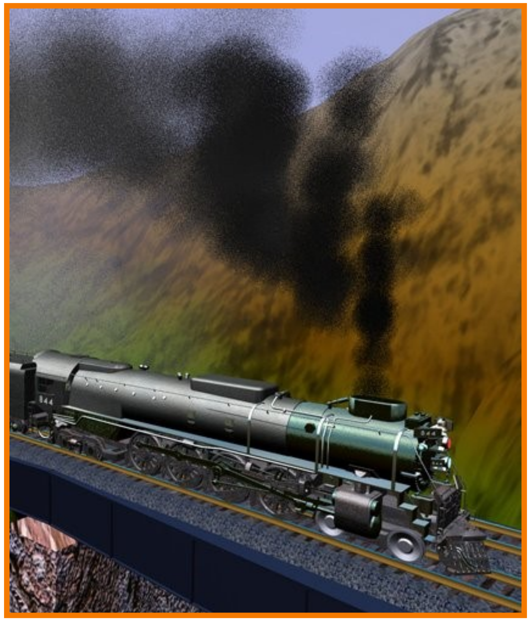


running human
trotting dog
swimming fish

Hodgins

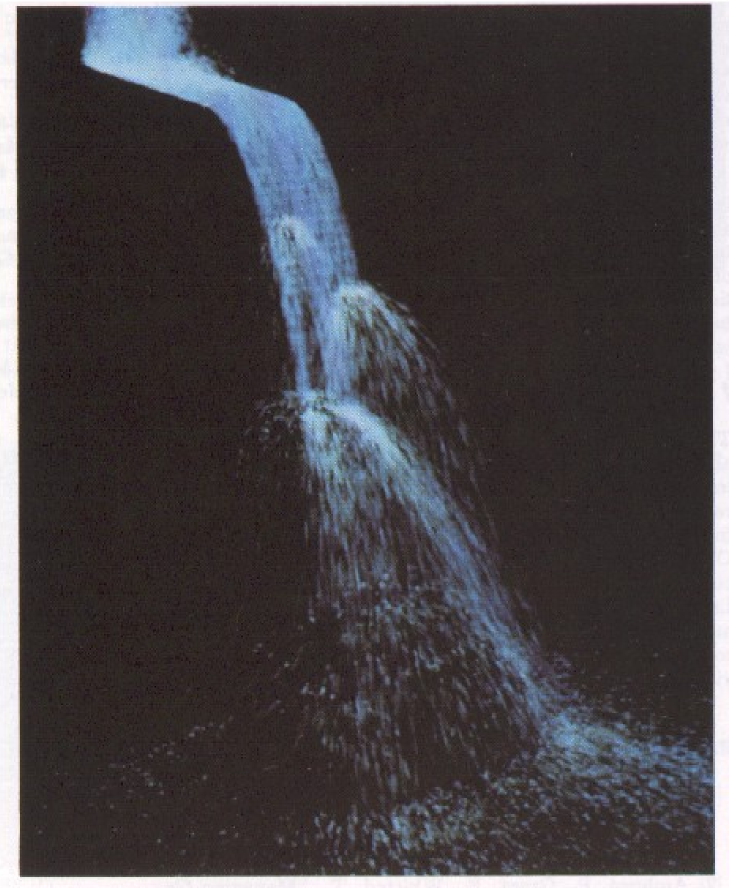
Passive Dynamics

- No muscles or motors
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice



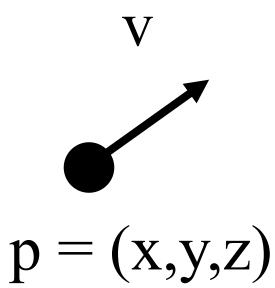
Passive Dynamics

- Physical laws
  - Newton's laws
  - Hooke's law
  - Etc.
- Physical phenomena
  - Gravity
  - Momentum
  - Friction
  - Collisions
  - Elasticity
  - Fracture



Particle Systems

- A particle is a point mass
  - Position
  - Velocity
  - Mass
  - Drag
  - Elasticity
  - Lifetime
  - Color



- Use many particles to model complex phenomena
  - Keep array of particles
  - Newton's laws

Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Creating Particles

- Where to create particles?
  - Predefined source
  - Where particle density is low
  - Surface of shape
  - etc.

Reeves

Creating Particles

- Where to create particles?
  - Predefined source
  - Where particle density is low
  - Surface of shape
  - etc.

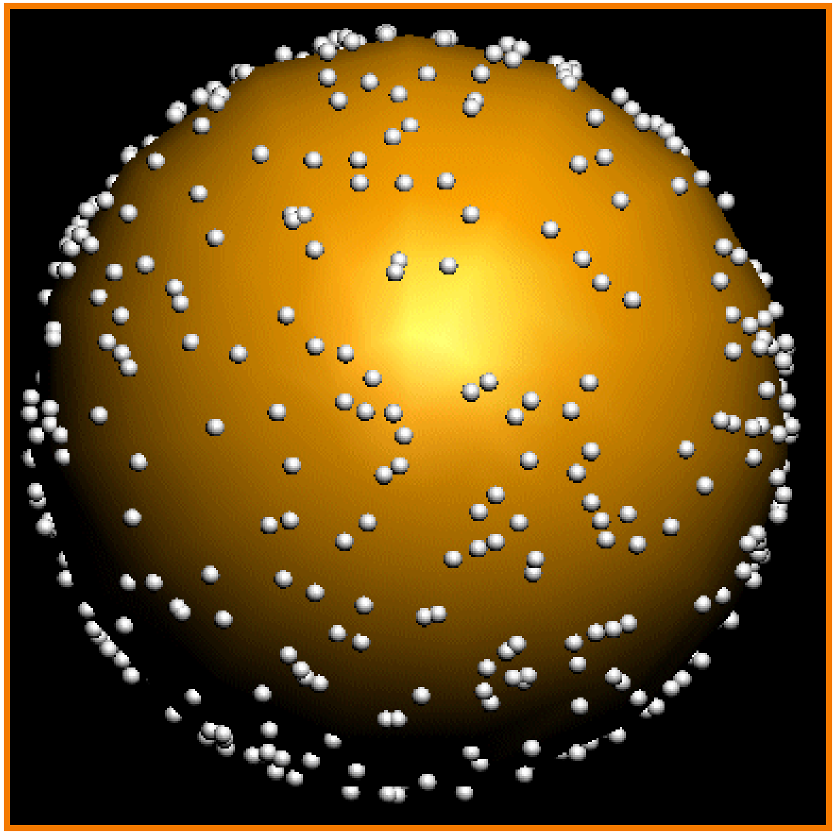
Reeves

Creating Particles

- Example: particles emanating from shape
  - Line
  - Box
  - **Circle**
  - **Sphere**
  - Cylinder
  - Cone
  - Mesh

Creating Particles

- Example: particles emanating from sphere

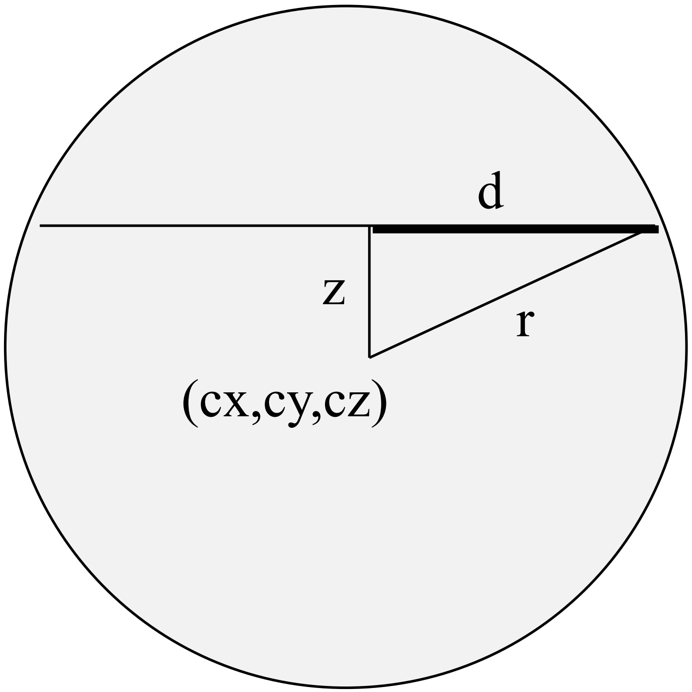


Creating Particles

- Example: particles emanating from sphere

Selecting random position on surface of sphere

1. z = random [−r, r]
2. phi = random [0, 2π)
3. d = sqrt($r^2 - z^2$)
4. px = cx + d * cos(phi)
5. py = cy + d * sin(phi)
6. pz = cz + z

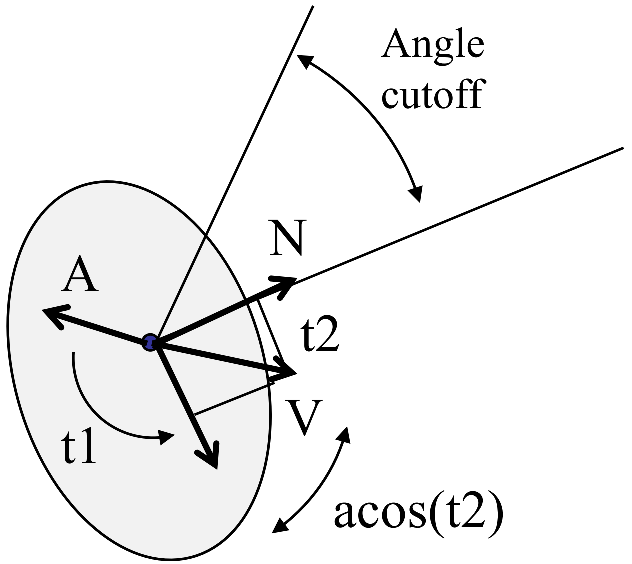


Creating Particles

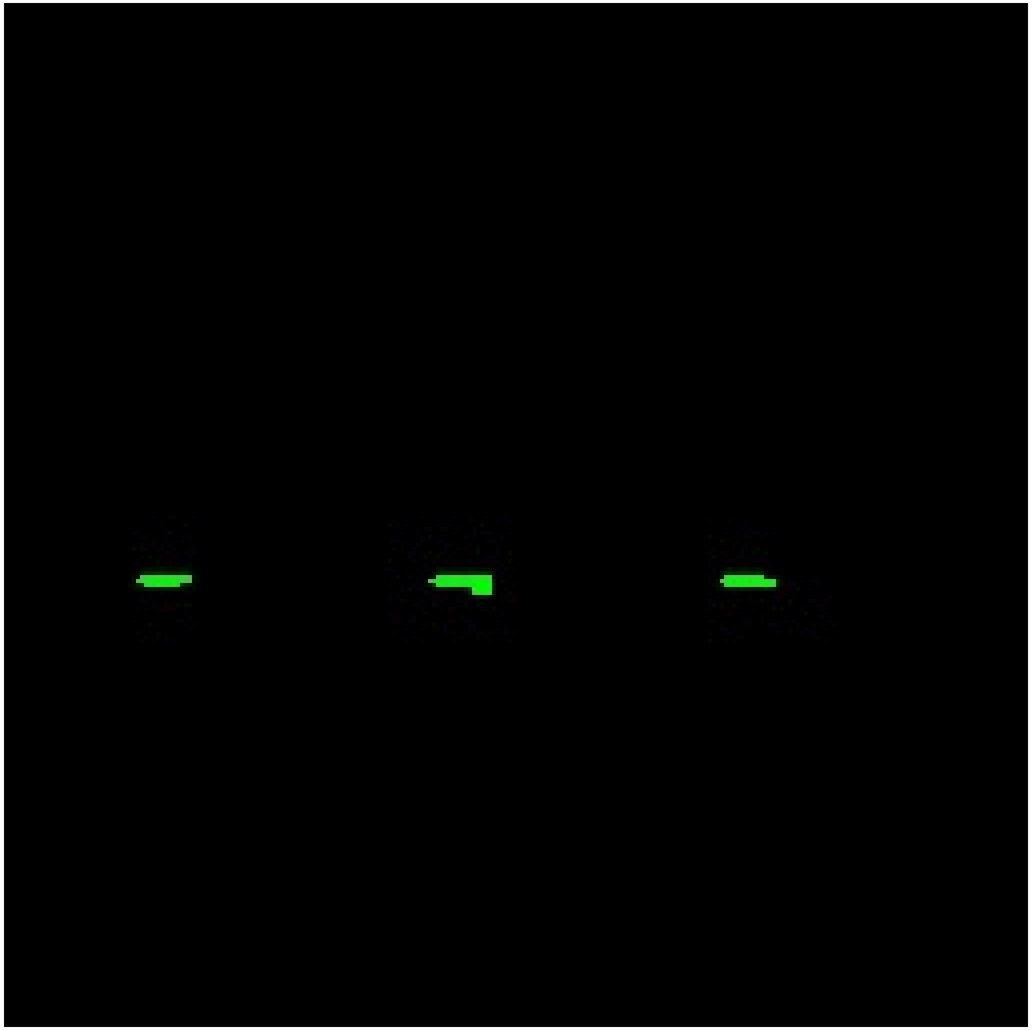
- Example: particles emanating from sphere

Selecting random direction within angle cutoff of normal

1. N = surface normal
2. A = any vector on tangent plane
3. t1 = random [0, 2π)
3. t2 = random [0, sin(angle cutoff))
4. V = rotate A around N by t1
5. V = rotate V around VxN by acos(t2)



Example: Fountains



Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Equations of Motion

- Newton's Law for a point mass
  - f = ma

- Computing particle motion requires solving second-order differential equation

$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$

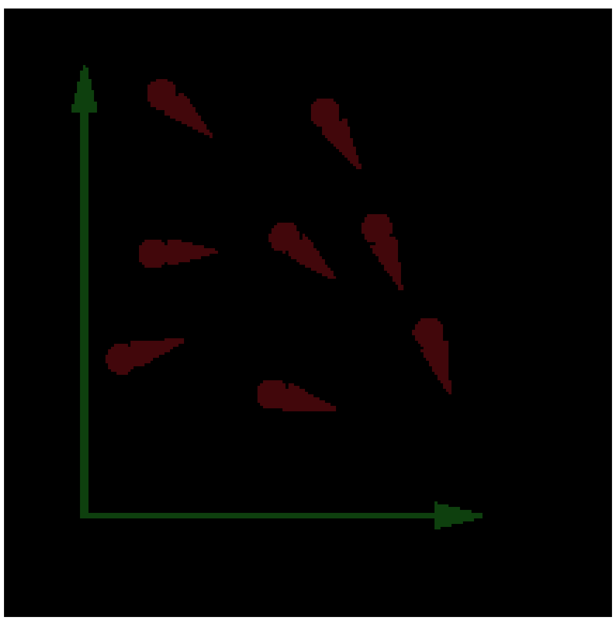
- Add variable v to form coupled first-order differential equations: "state-space form"

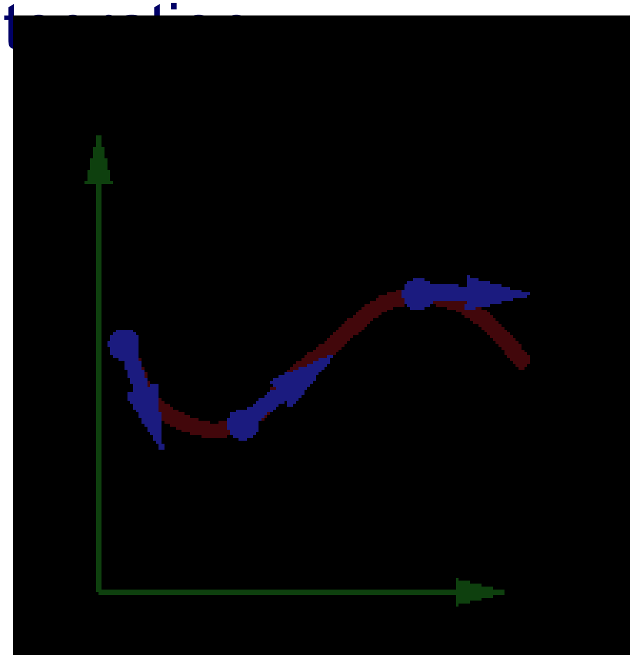
$$\begin{cases} \dot{x} = v \\ \dot{v} = \dfrac{f}{m} \end{cases}$$

Solving the Equations of Motion

- Initial value problem
  - Know x(0), v(0)
  - Can compute force (and therefore acceleration) for any position / velocity / time
  - Compute x(t) by forward int

f





Hodgins

Solving the Equations of Motion

- Forward (explicit) Euler integration

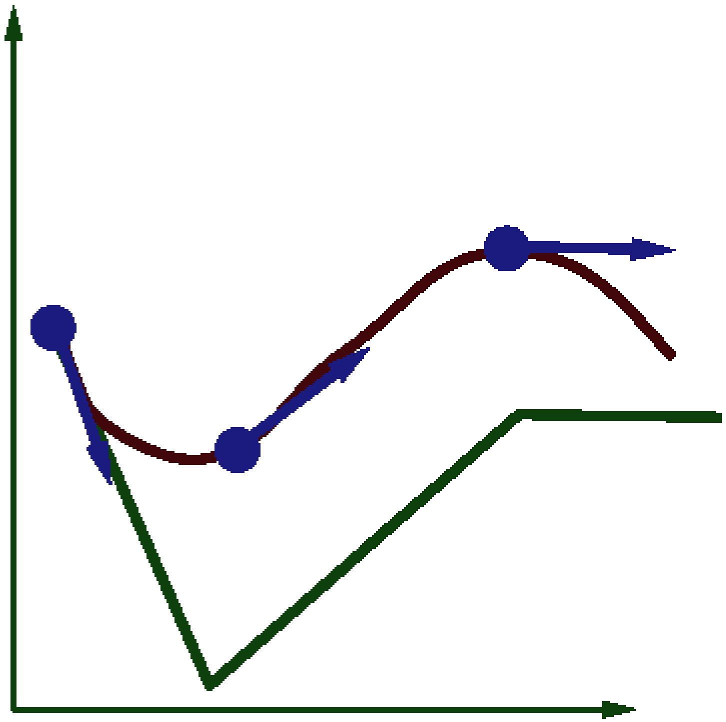
Euler Step (1768)

$$y_{n+1} = y_n + h \cdot f(t_n, y_n)$$

- **Idea**: start at initial condition and take a step into the direction of the tangent.
- Iteration scheme: $y_n \rightarrow f(t_n,y_n) \rightarrow y_{n+1} \rightarrow f(t_{n+1},y_{n+1}) \rightarrow \ldots$

Solving the Equations of Motion

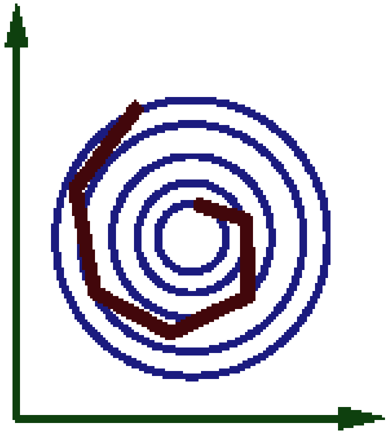
- Forward (explicit) Euler integration
  - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v(t)$
  - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f\big(x(t), v(t), t\big) / m$

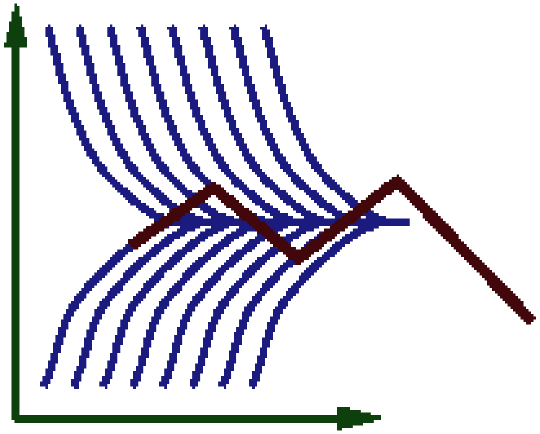


Hodgins

Solving the Equations of Motion

- Forward (explicit) Euler integration
  - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v(t)$
  - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f\big(x(t), v(t), t\big) / m$
- Problem:
  - Accuracy decreases as Δt gets bigger



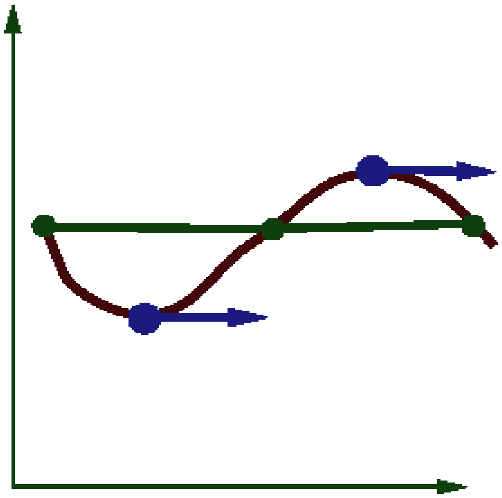


Hodgins

Solving the Equations of Motion

- Midpoint method (2nd-order Runge-Kutta)
  1. Compute an Euler step
  2. Evaluate f at the midpoint of Euler step
  3. Compute new position / velocity using midpoint velocity / acceleration

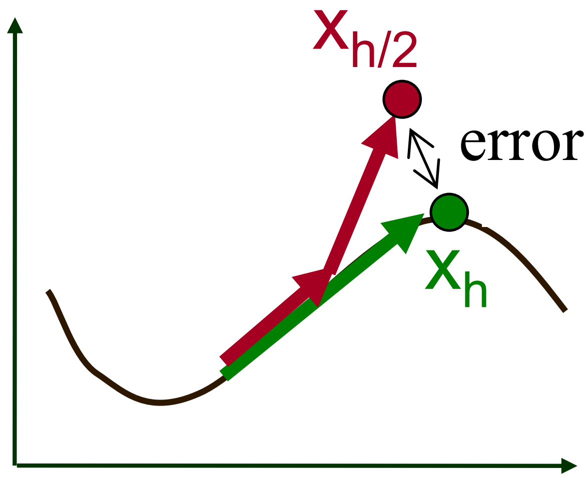
  - $x_{mid} \leftarrow x(t) + \Delta t / 2 * v(t)$
  - $v_{mid} \leftarrow v(t) + \Delta t / 2 * f(x(t), v(t), t) / m$
  - $x(t+\Delta t) \leftarrow x(t) + \Delta t\, v_{mid}$
  - $v(t+\Delta t) \leftarrow v(t) + \Delta t\, f(x_{mid}, v_{mid}, t) / m$

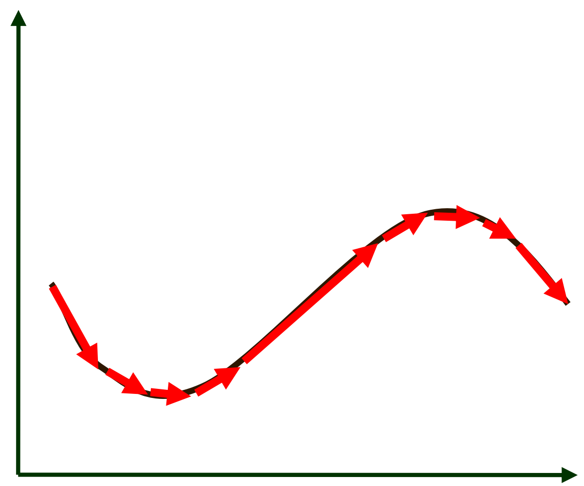


Hodgins

Solving the Equations of Motion

- Adaptive step size
  - Repeat until error is below threshold
    1. Compute x_h by taking one step of size h
    2. Compute $x_{h/2}$ by taking 2 steps of size h / 2
    3. Compute error = | x_h - $x_{h/2}$ |
    4. If (error < threshold) break
    5. Else, reduce step size and try again





Solving the Equations of Motion

Explicit Euler step

$$y_{n+1} = y_n + hf(t_n, y_n)$$

Implicit Euler step

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$$

Why are these methods called like this?

- **Explicit**: all quantities are known (*given explicitly*)
- **Implicit**: y_{n+1} is unknown (*given implicitly*)

→ solve (nonlinear) equation(s)!

Solving the Equations of Motion

Explicit Euler step

$$y_{n+1} = y_n + hf(t_n, y_n)$$

Implicit Euler step

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$$

Why are these methods called like this?

- **Explicit**: all quantities are known (*given explicitly*)
- **Implicit**: y_{n+1} is unknown (*given implicitly*)
- Stability conditions for Euler
  - Explicit $y_n = (1+h\lambda)^n y_0 < \infty \Leftrightarrow |1+h\lambda| < 1$
  - Implicit $y_n = (1-h\lambda)^{-n} y_0 < \infty \Leftrightarrow |1-h\lambda|^{-1} < 1$

⇨ **Implicit Euler is stable for all $h > 0$!**

Particle System Forces

- Force fields
  - Gravity, wind, pressure
- Viscosity/damping
  - Drag, friction
- Collisions
  - Static objects in scene
  - Other particles
- Attraction and repulsion
  - Springs between neighboring particles (mesh)
  - Gravitational pull, charge

Particle System Forces

- Gravity
  - Force due to gravitational pull (of earth)
  - g = acceleration due to gravity (m/s^2)

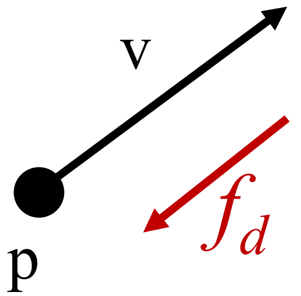
$$f_g = mg$$

g = (0, -9.80665, 0)

Particle System Forces

- Drag
  - Force due to resistance of medium
  - k_{drag} = drag coefficient (kg/s)

$$f_d = -k_{drag} v$$

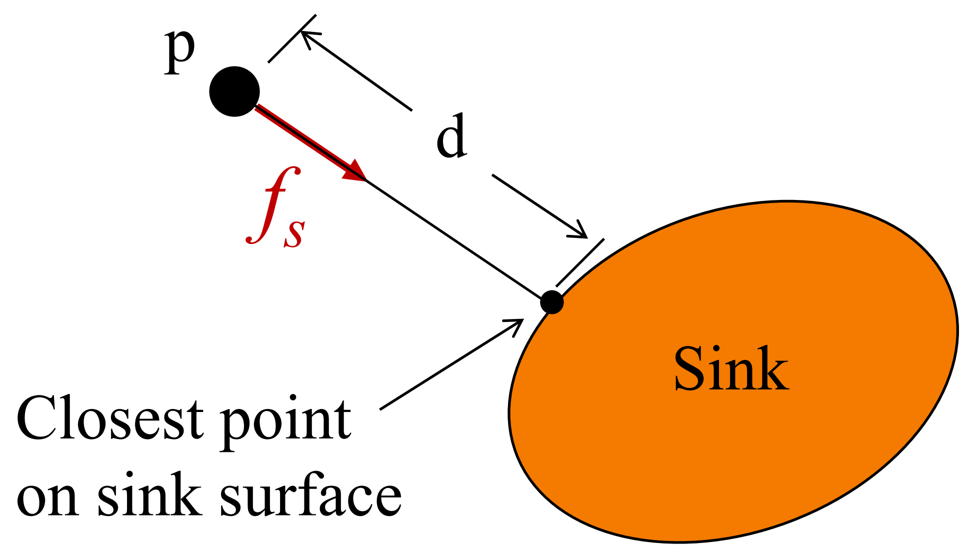


  - Air resistance sometimes taken as proportional to v^2

Particle System Forces

- Sinks
  - Force due to attractor in scene

$$f_s = \frac{\text{intensity}}{c_a + l_a \cdot d + q_a \cdot d^2}$$

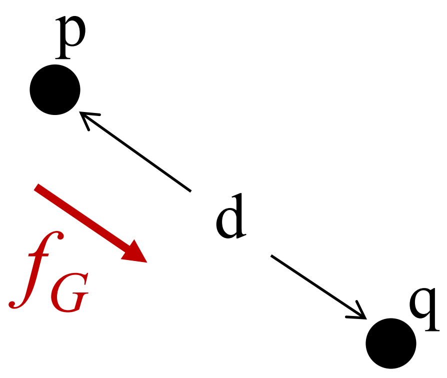


Particle System Forces

- Gravitational pull of other particles
  - Newton's universal law of gravitation

$$f_G = G\frac{m_1 \cdot m_2}{d^2}$$

$$G = 6.67428 \times 10^{-11} \ \text{N m}^2 \text{kg}^{-2}$$



Particle System Forces

- Springs
  - Hooke's law

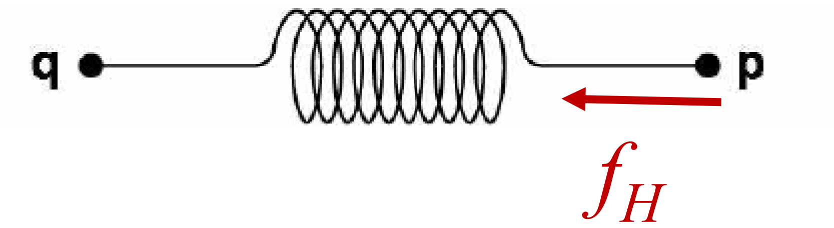
$$f_H(p) = k_s\left(d(p,q) - s\right)\ D$$

$$D = (q-p)/\|q-p\|$$

$$d(p,q) = \|q-p\|$$

s = resting length

k_s = spring coefficient



Particle System Forces

- Springs
  - Hooke's law with damping

$$f_H(p) = \left[k_s\left(d(p,q) - s\right) + k_d\left(v(q) - v(p)\right) \cdot D\right] D$$

$$D = (q - p) / \|q - p\|$$

$$d(p,q) = \|q - p\|$$

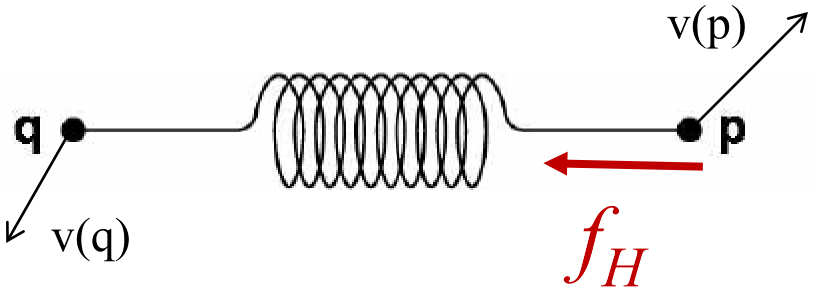
s = resting length

k_s = spring coefficient

k_d = damping coefficient

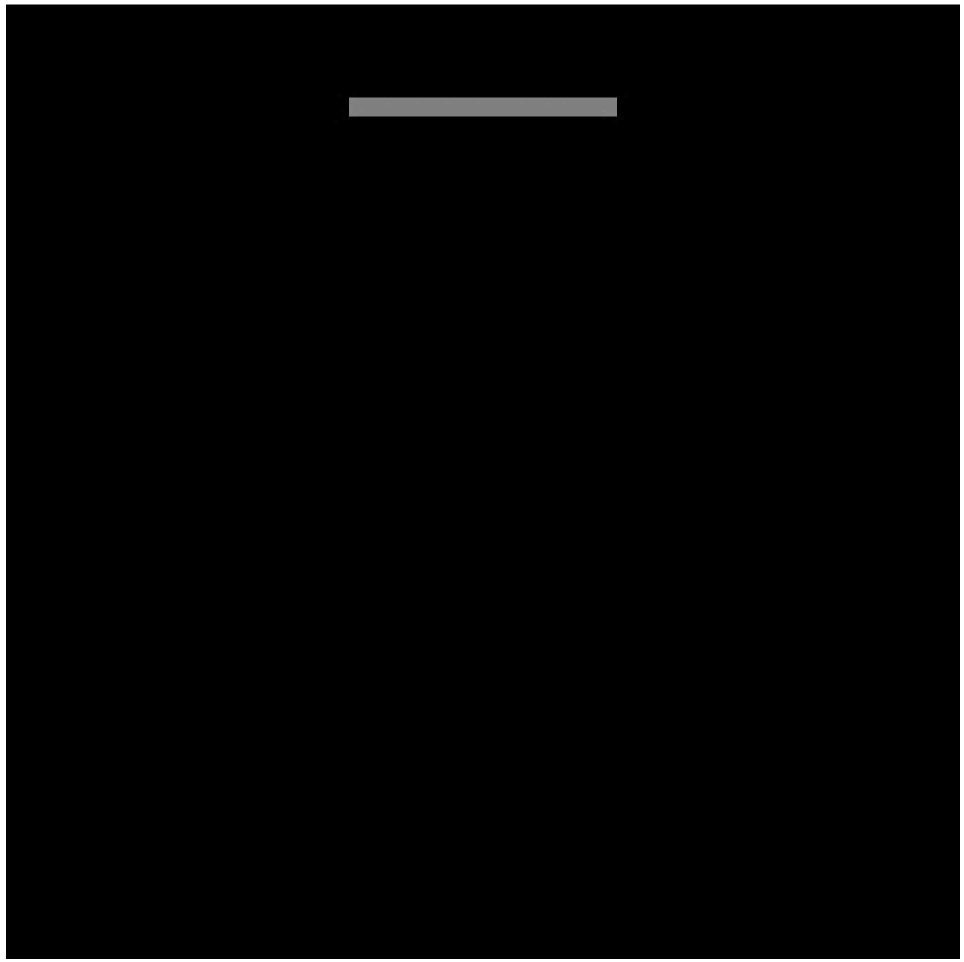
$v(p)$ = velocity of p

$v(q)$ = velocity of q



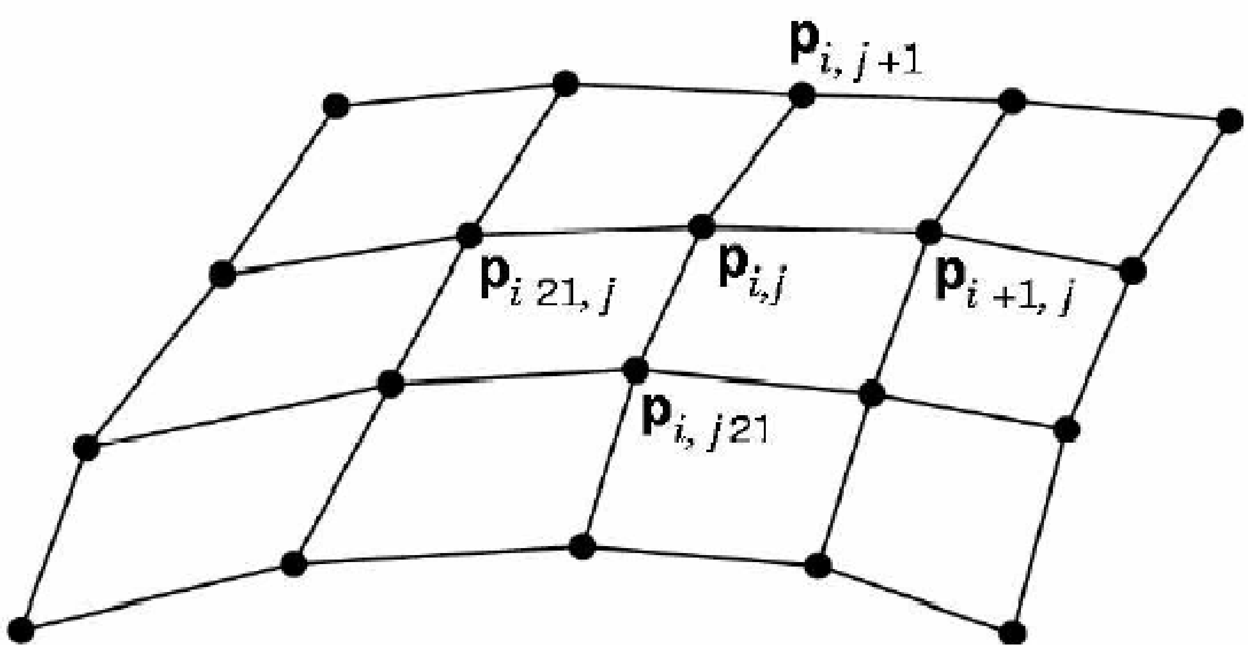
$$k_d \sim 2\sqrt{mk_s}$$

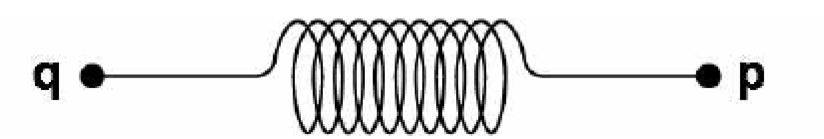
Example: Rope



Particle System Forces

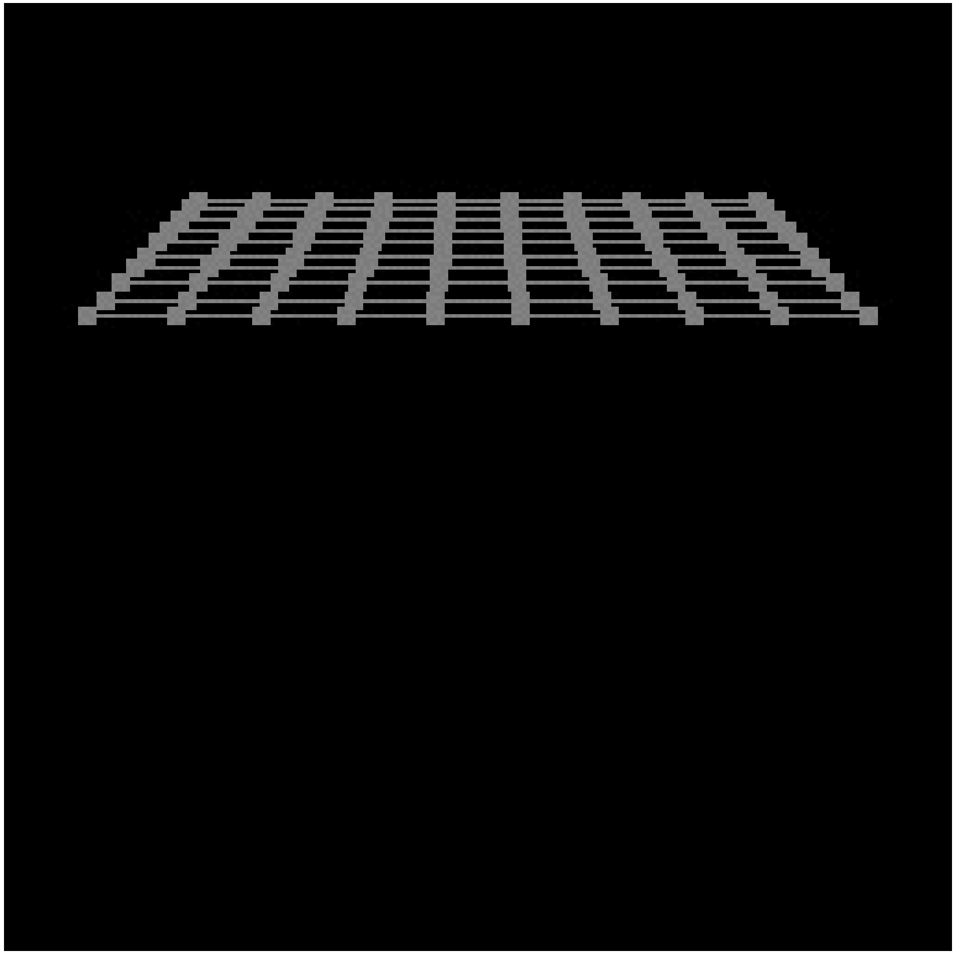
- Spring-mass mesh





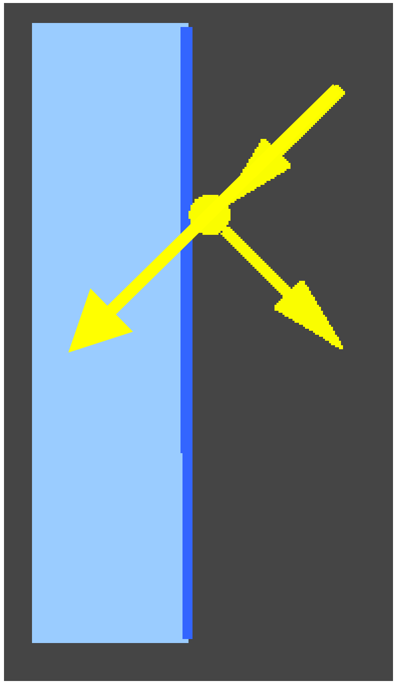
Hodgins

Example: Cloth



Particle System Forces

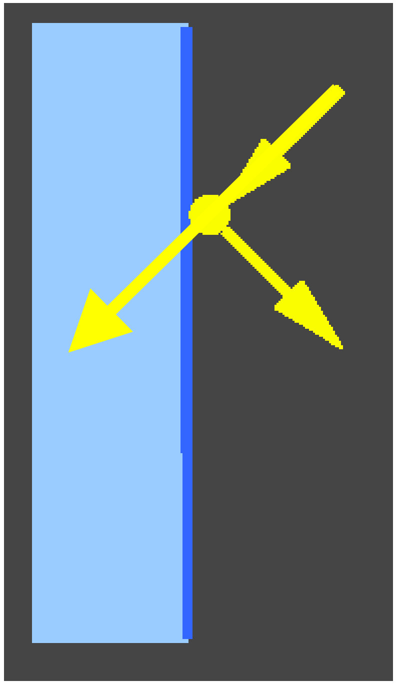
- Collisions
  - Collision detection
  - Collision response



Witkin

Particle System Forces

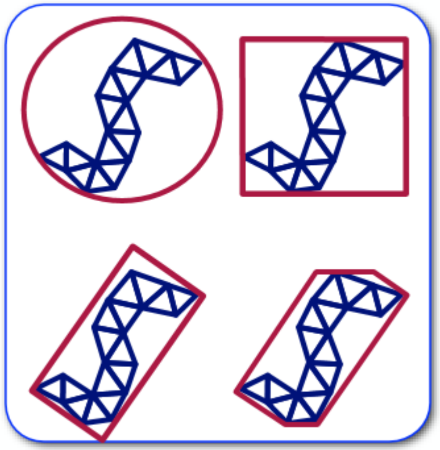
- Collision detection
  - Intersect ray with scene
  - Compute up to Δt at time of first collision, and then continue from there

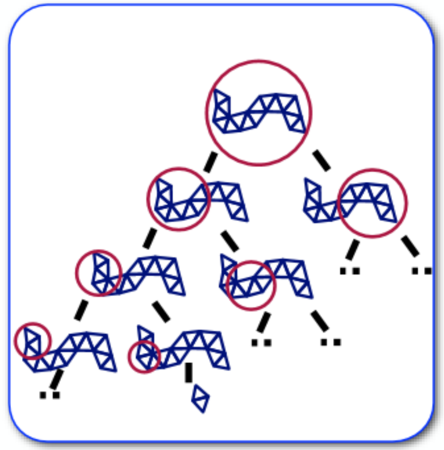


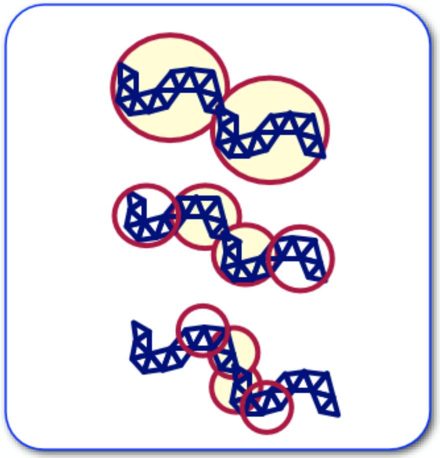
Witkin

Collision Detection

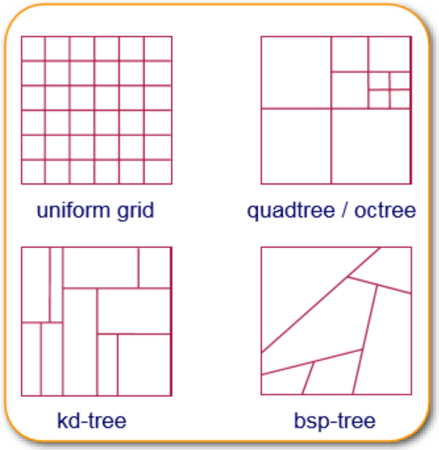
Bounding Volumes

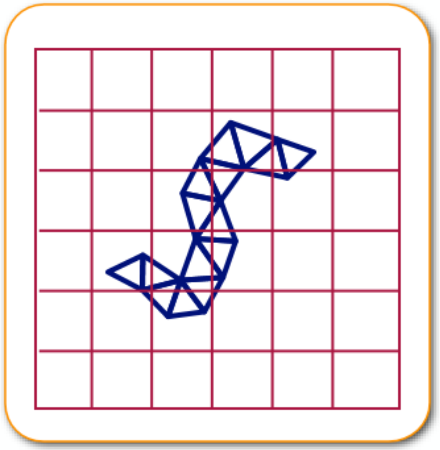


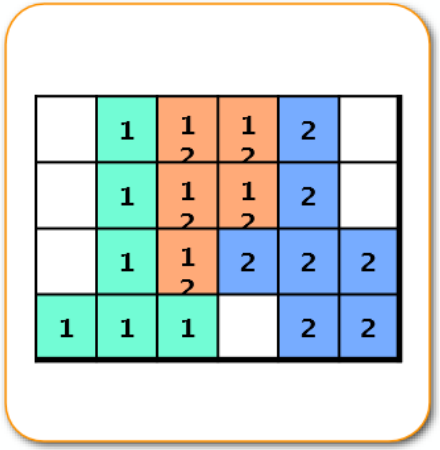




Spatial Partitioning

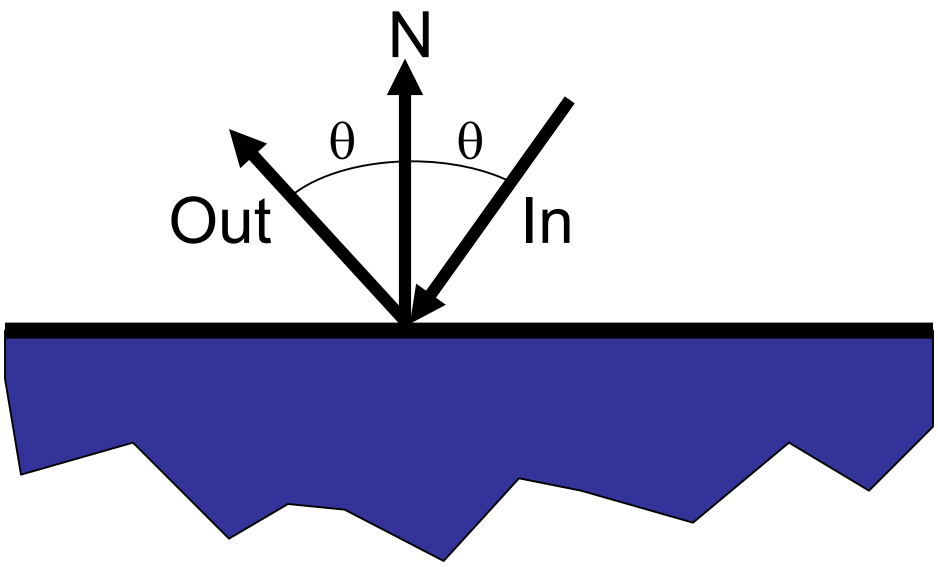






Particle System Forces

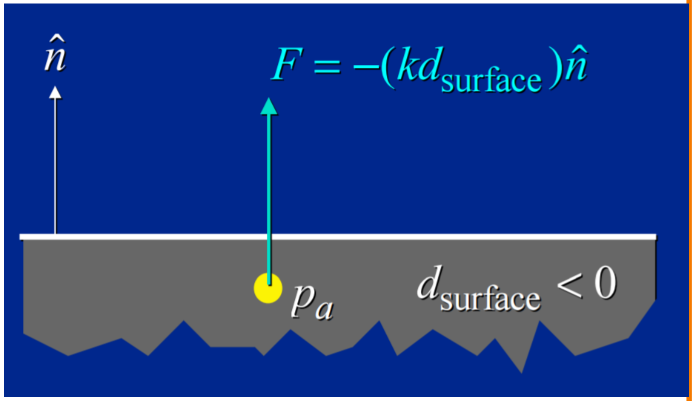
- Collision response
  - No friction: elastic collision (for $m_{target} >> m_{particle}$: specular reflection)



  - Otherwise, total momentum conserved, energy dissipated if inelastic

Particle System Forces

- Impulse driven
  - Manipulation of velocities
  - Fast, more difficult to compute
- Force driven
  - Penetration induces forces
  - Slow, easy to compute
- Position based response
  - Approximate, non physical
  - Lightweight



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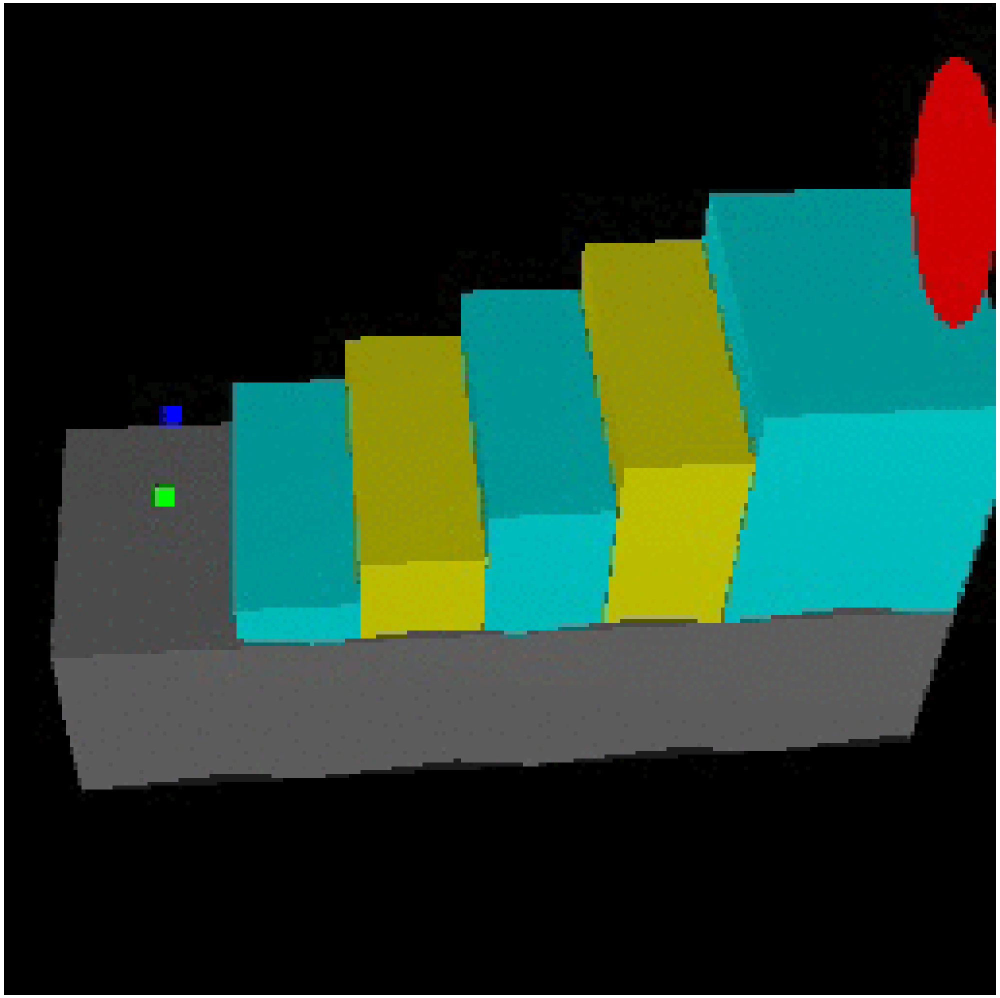
Example: Bouncing



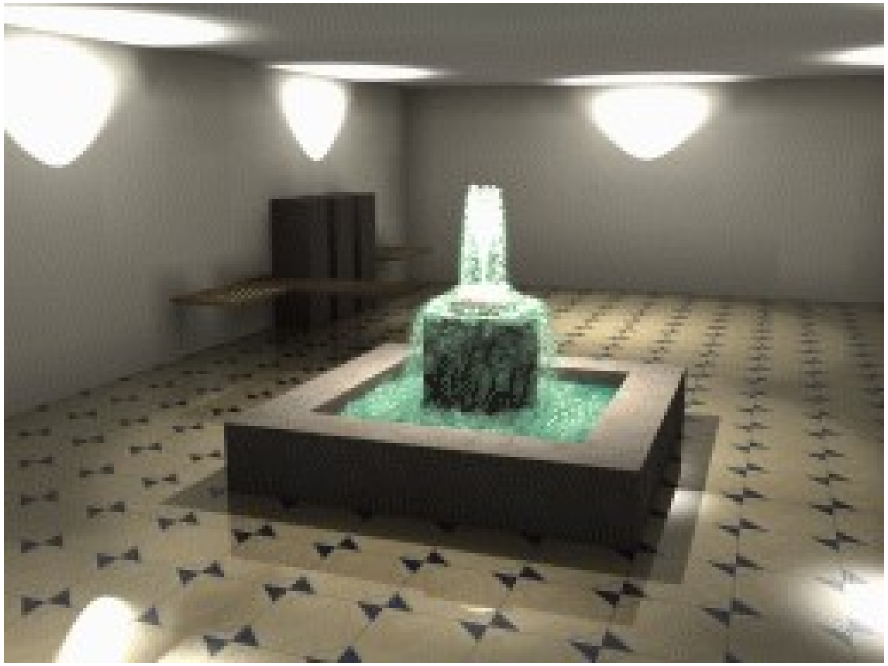
Ning Jin
COS 426, 2013

Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

Deleting Particles

- When to delete particles?
  - When life span expires
  - When intersect predefined sink surface
  - Where density is high
  - Random



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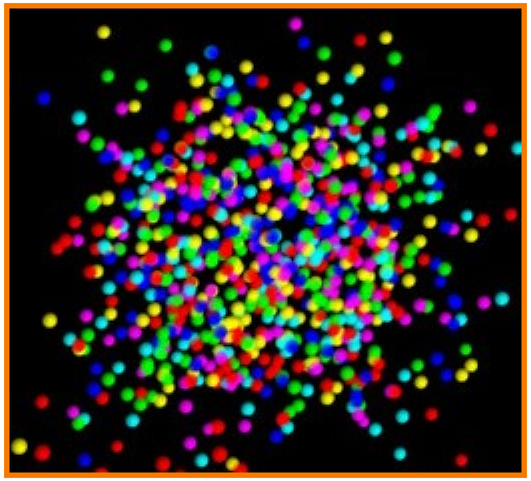
Particle Systems

- For each frame:
  - For each simulation step (Δt)
    - Create new particles and assign attributes
    - Update particles based on attributes and physics
    - Delete any expired particles
  - Render particles

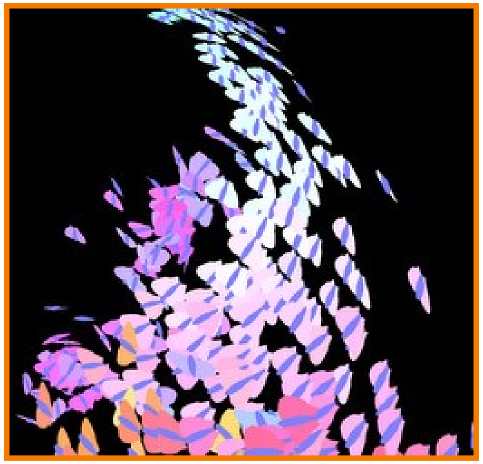
Rendering Particles

- Rendering styles
  - ➢ Points
  - ○ Polygons
  - ○ Shapes
  - ○ Trails
  - ○ etc.



Rendering Particles

- Rendering styles
  - Points
  - ➢ Textured polygons: sprites
  - Shapes
  - Trails
  - etc.



Rendering Particles

- Rendering styles
  - Points
  - Polygons
  - ➢ Shapes
  - Trails
  - etc.

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Rendering Particles

- Rendering styles
  - Points
  - Polygons
  - Shapes
  - Trails
  - etc.

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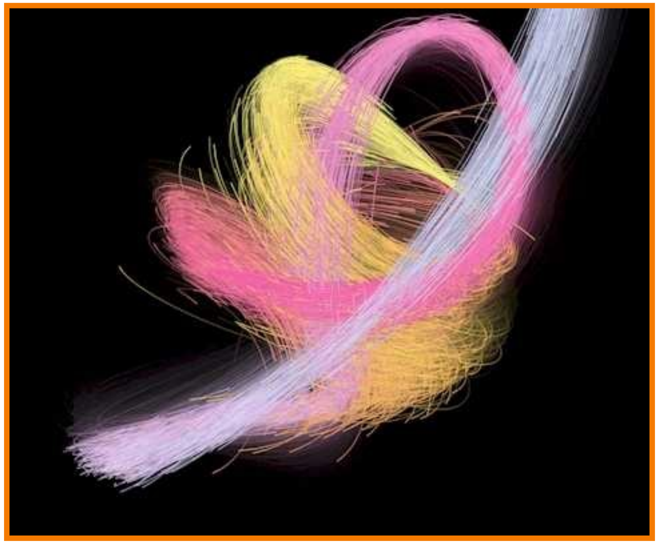
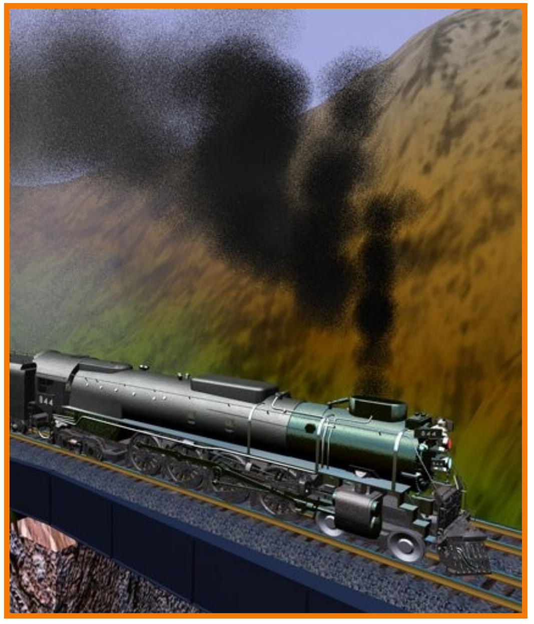
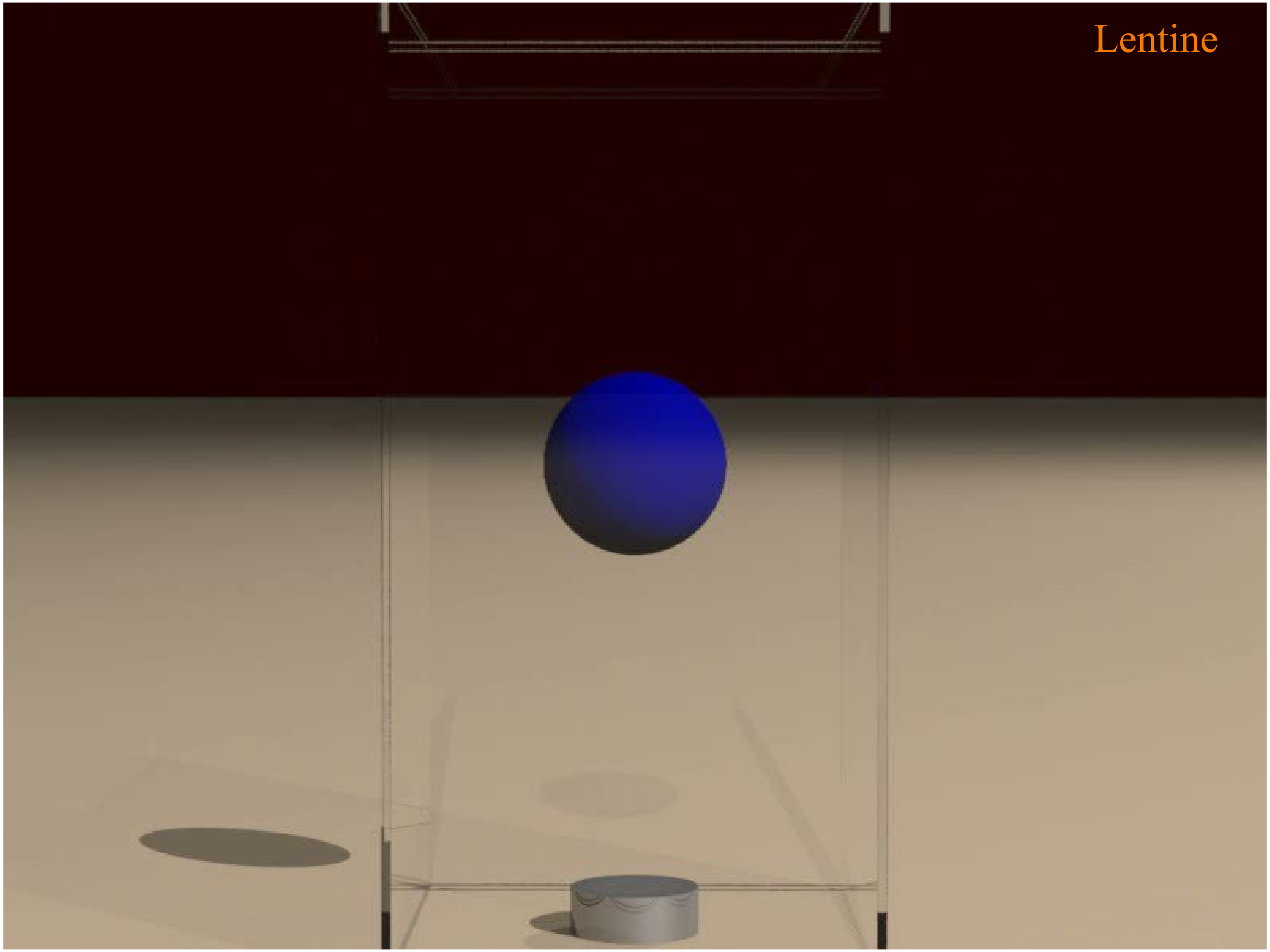


Putting it All Together

- Examples
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice

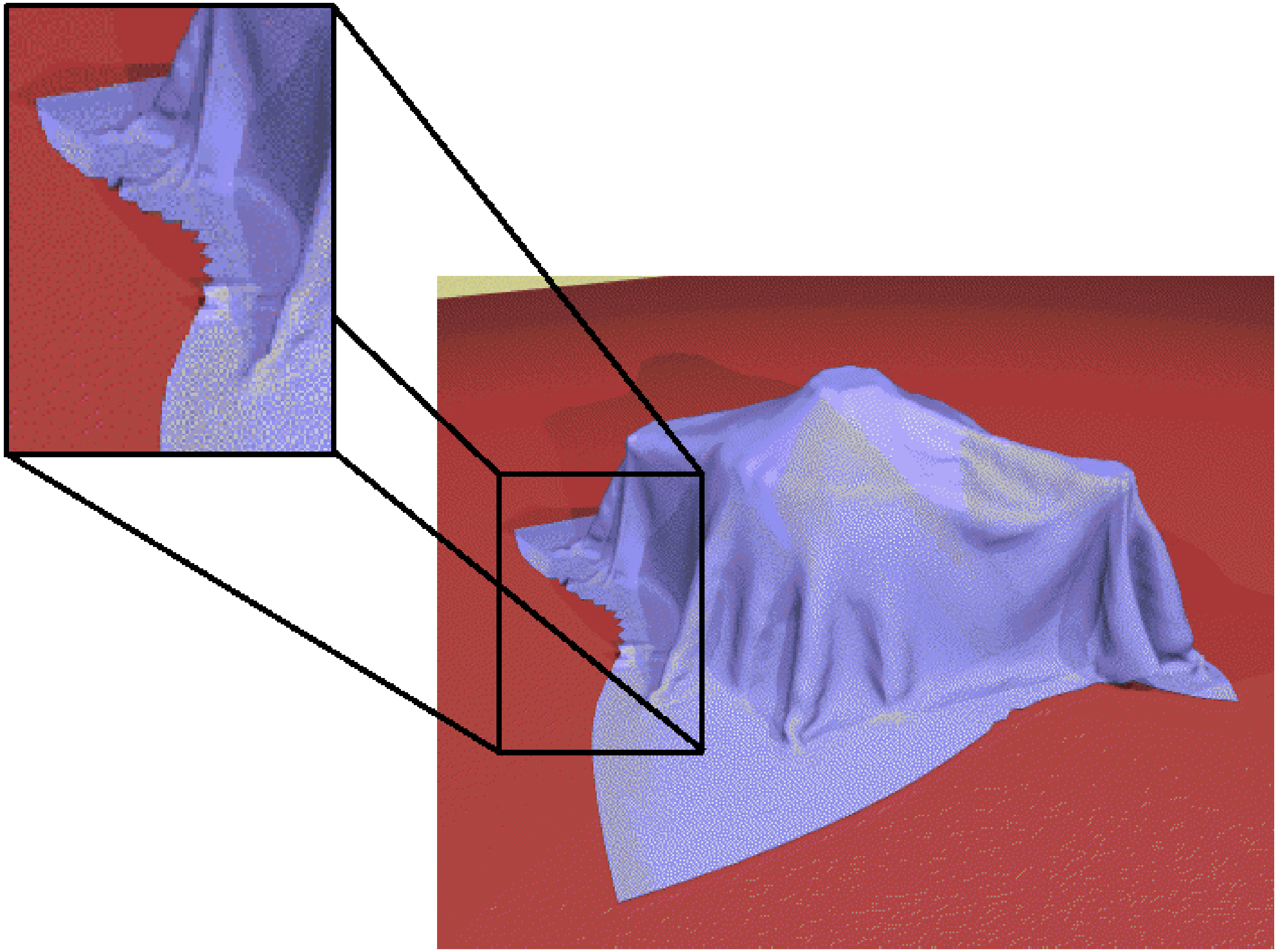


Example: "Smoke"



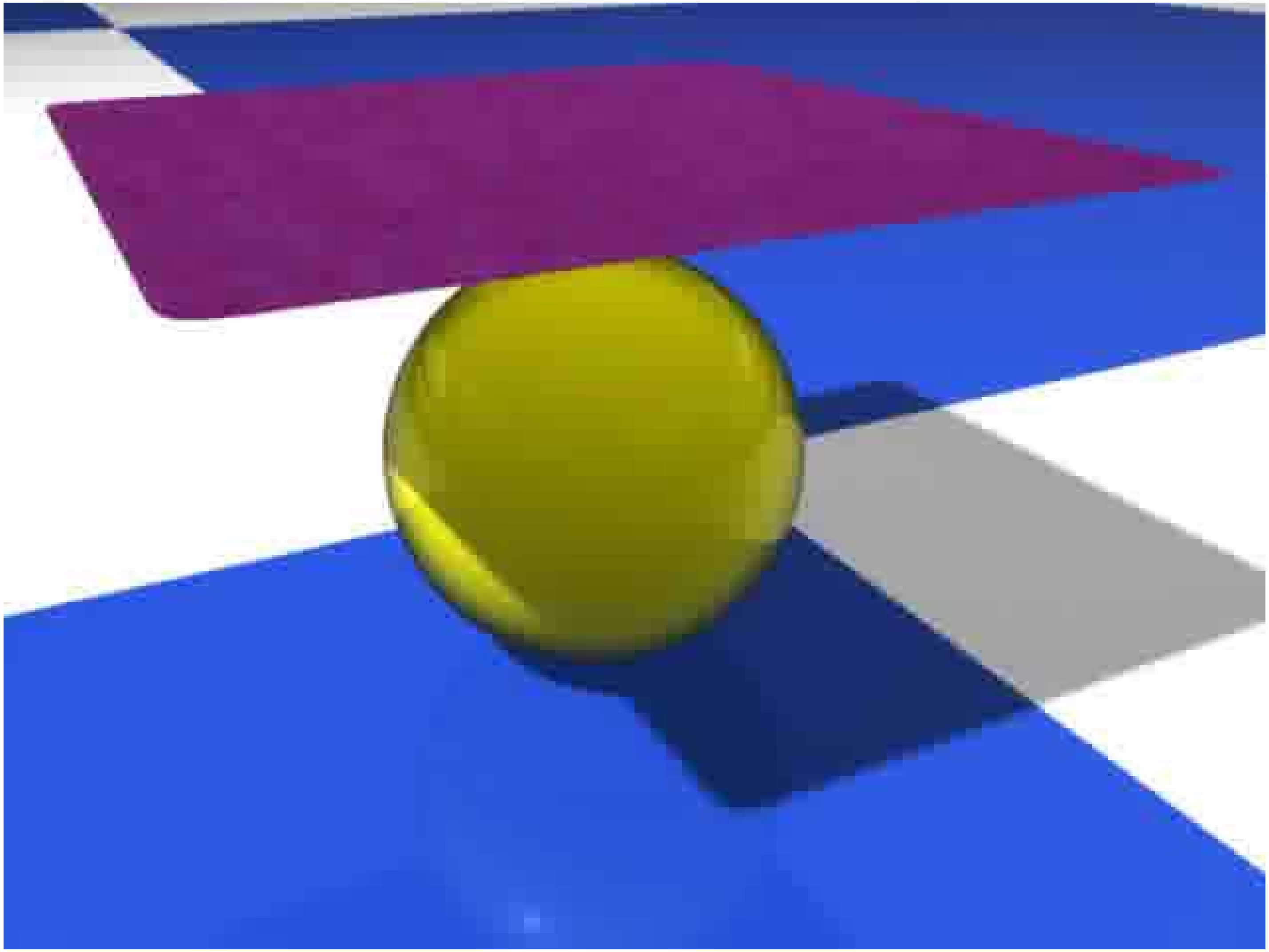
Example: Fire

Example: Cloth



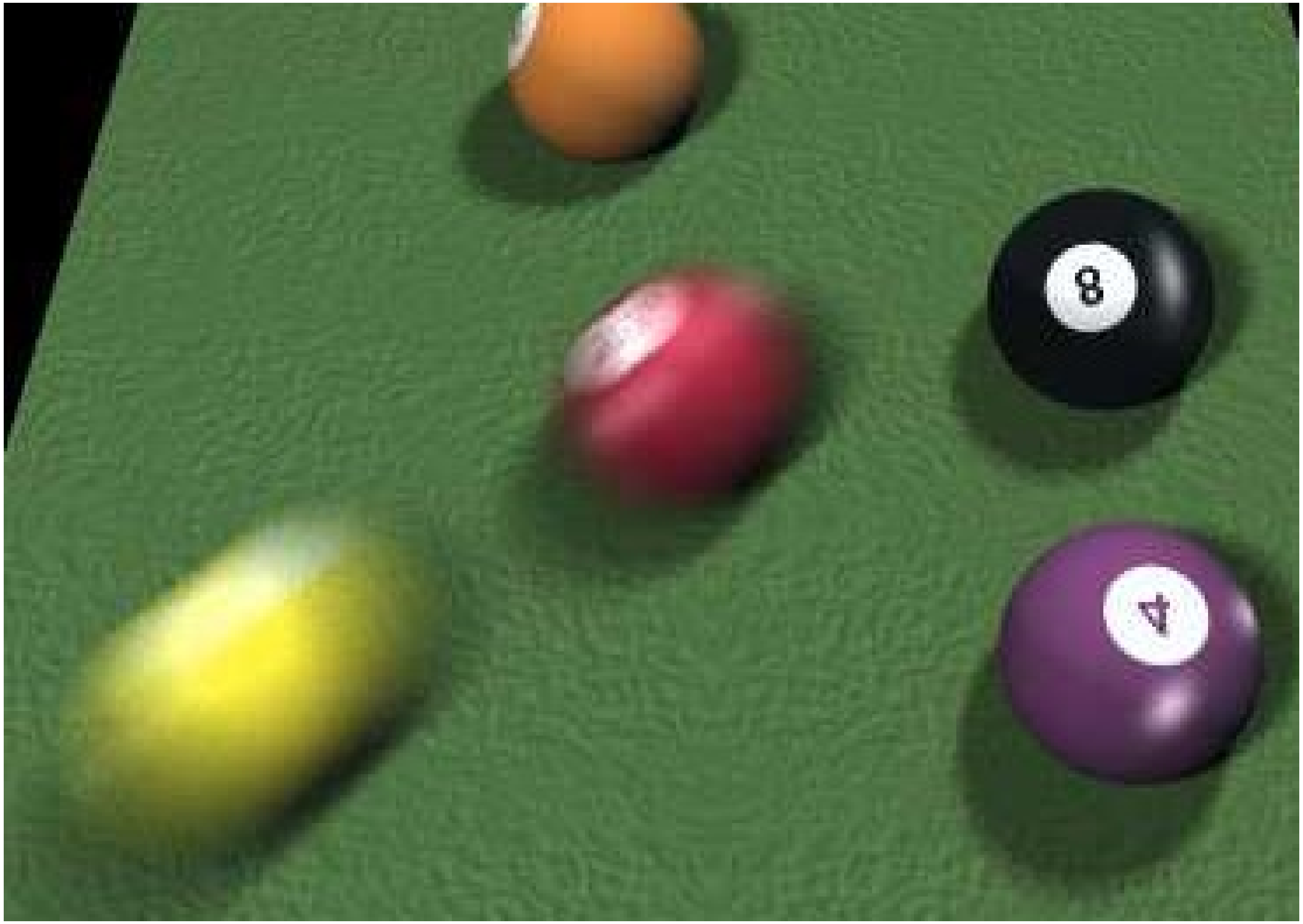
Breen

Example: Cloth

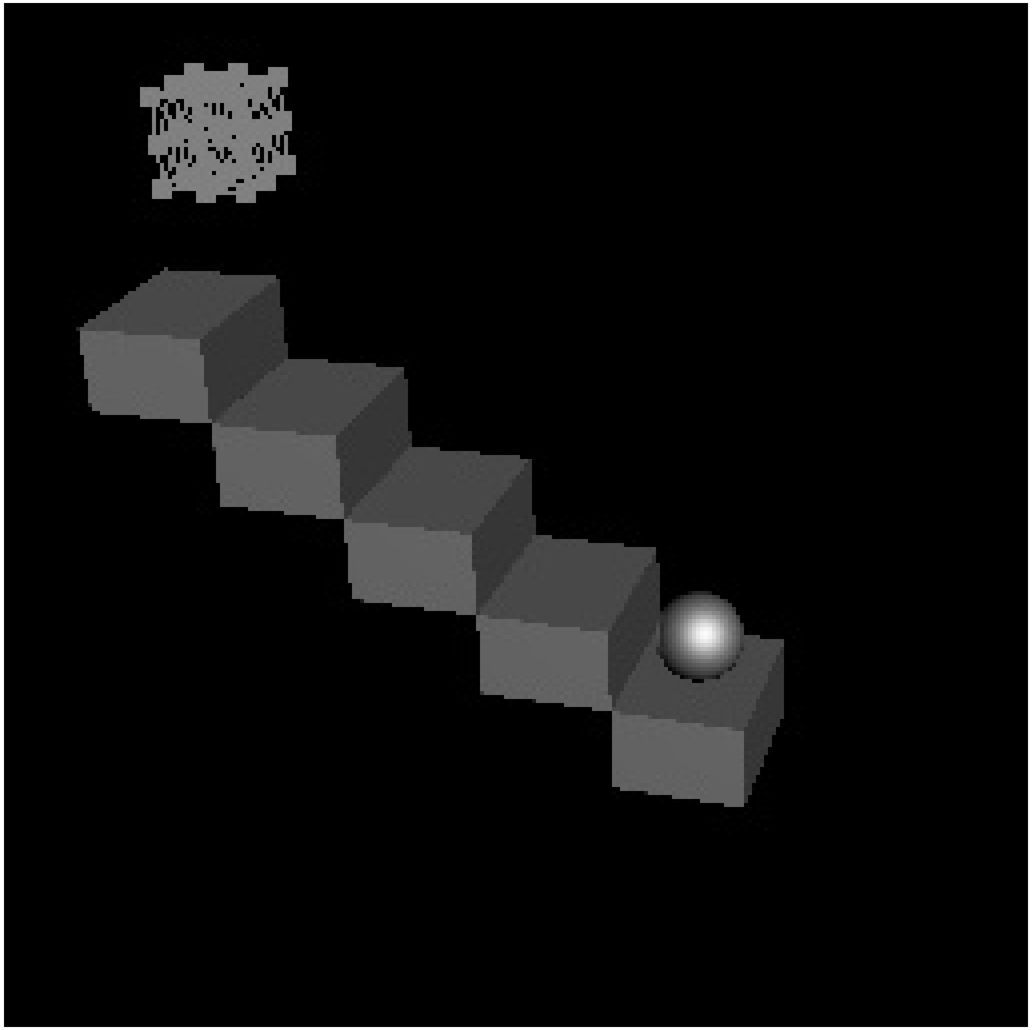


Bender

Example: Bouncing Particles

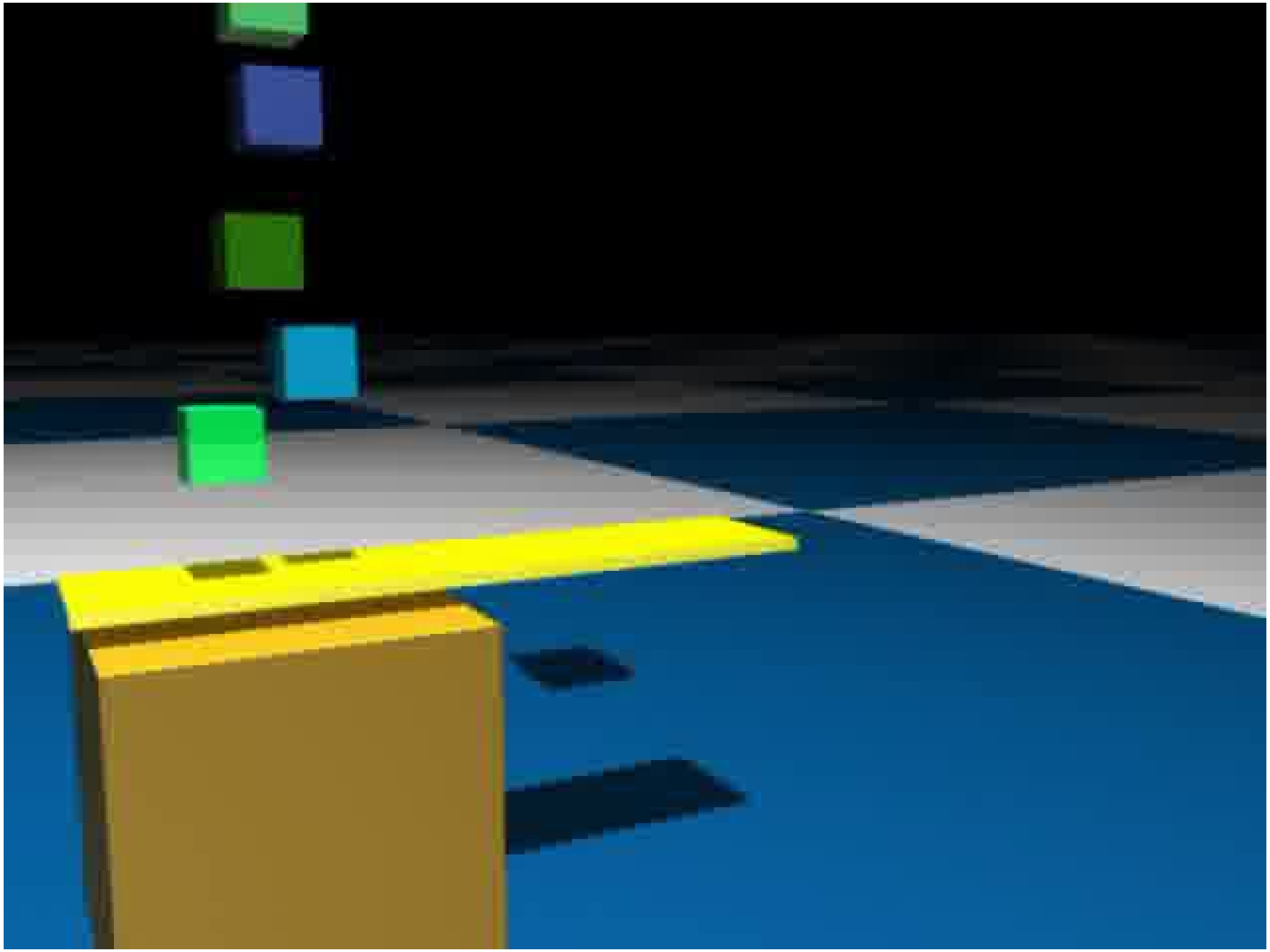


Example: Bouncing Particles



Zhaoyang Xu
COS 426, 2007

Example: More Bouncing



Bender

Example: Flocks & Herds

Reynolds

Summary

- Particle systems
  - Lots of particles
  - Simple physics
- Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks
- Solving motion equations
  - For each step, first sum forces, then update position and velocity