COS 426
Computer Graphics
Princeton University

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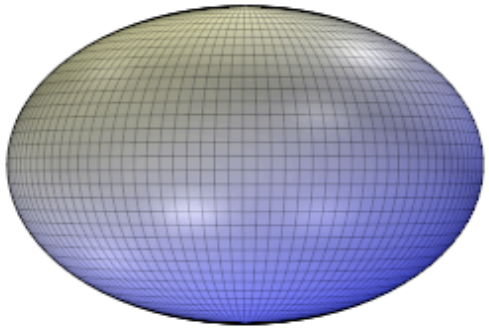
Feb 11, 2011

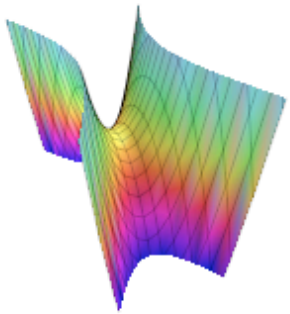
Ray Tracing

- **Ray/primitive intersection**
- Secondary rays – global illumination
- Acceleration

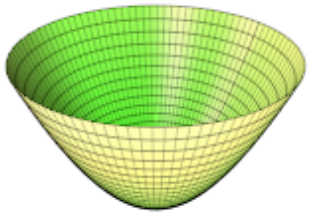
Quadrics



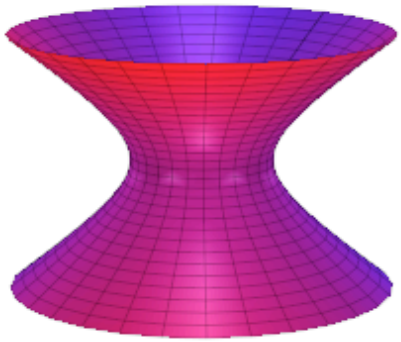
Ellipsoid



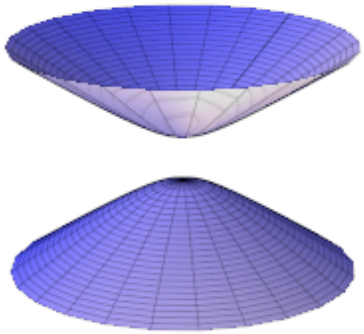
Hyperbolic paraboloid



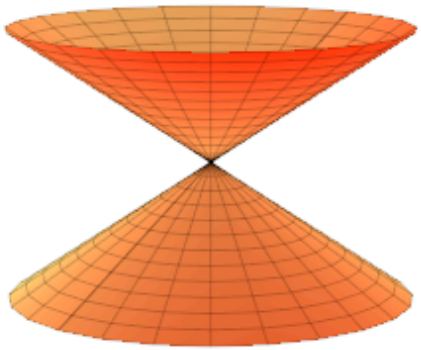
Elliptic paraboloid



Hyperboloid of one sheet

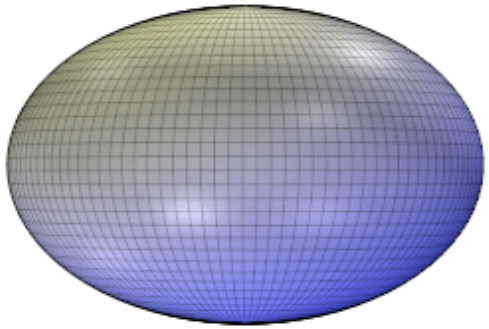


Hyperboloid of two sheets

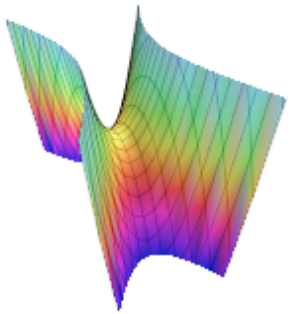


Cone

Quadrics



Ellipsoid



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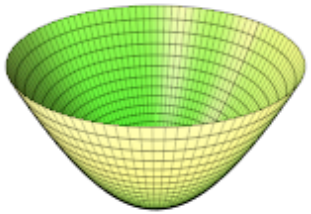

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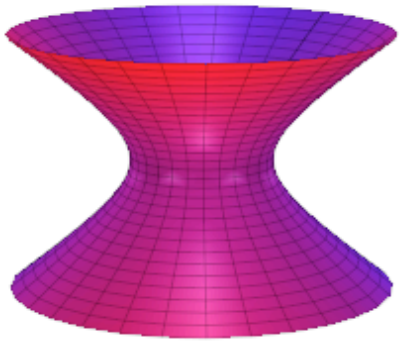

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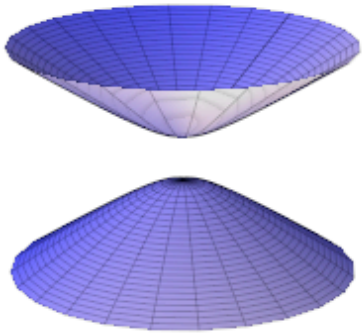
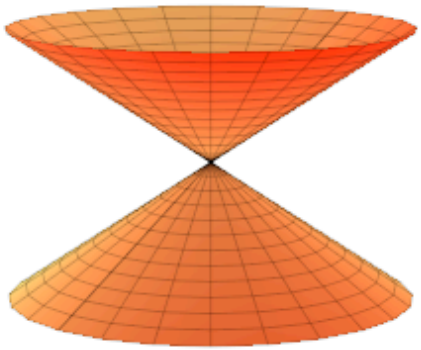


Hyperboloid of two sheets



Cone

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

Quadrics

- Ray/primitive intersection:
  - Write down all equations
  - Solve for intersection

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- Ray:

$$p = p_0 + t \cdot v$$

Quadrics

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v \begin{cases} x = x_0 + t \cdot v_x \\ y = y_0 + t \cdot v_y \\ z = z_0 + t \cdot v_z \end{cases}$$

In-class derivation:

Quadrics

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A positive real solution exists

Two complex solutions

Two real negative solution

Quadrics

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A positive real solution exists

Pick smallest positive value to find intersection

Two complex solutions

Does not intersect

Two real negative solution

Does not intersect

Quadrics
Simpler Derivation

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Simpler Derivation

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$$p = p_0 + t \cdot v$$

$$\underset{3\times 3}{pQp^T} + \underset{1\times 3}{Pp^T} + \underset{1\times 1}{R} = 0$$

Quadrics
Simpler Derivation

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

$$p = p_0 + t \cdot v$$

$$\underset{3\times 3}{p}\underset{3\times 3}{Q}\underset{3\times 1}{p^T} + \underset{1\times 3}{P}\underset{3\times 1}{p^T} + \underset{1\times 1}{R} = 0$$

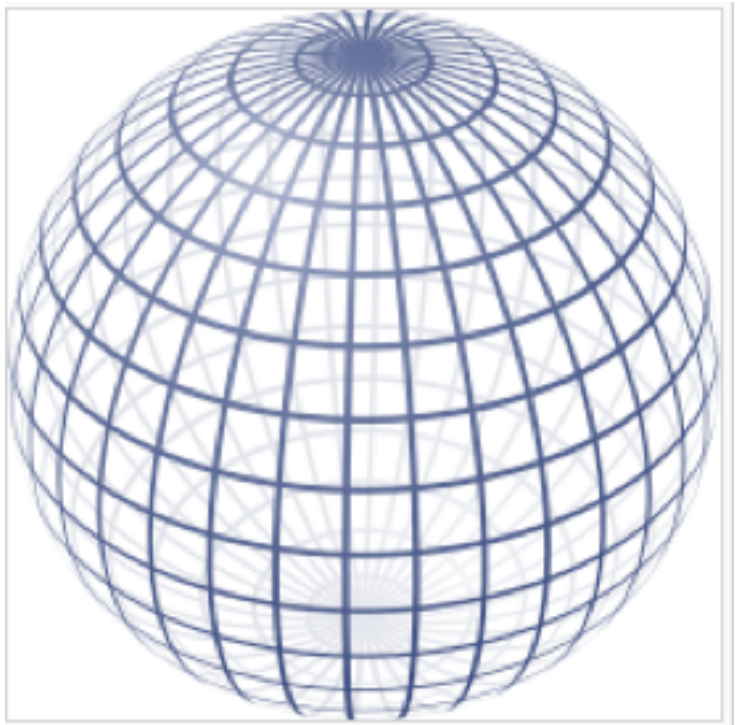
$$(p_0 + tv)Q(p_0 + tv)^T + P(p_0 + tv)^T + R = 0$$

In-class derivation:

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$



Quadrics

- If you use general quadric for sphere

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  - What do you need to define?

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$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Quadrics

- If you use general quadric for sphere

$$pQp^T + Pp^T + R = 0$$

  - What do you need to define?
  - Q, P, R

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R = -r^2$$

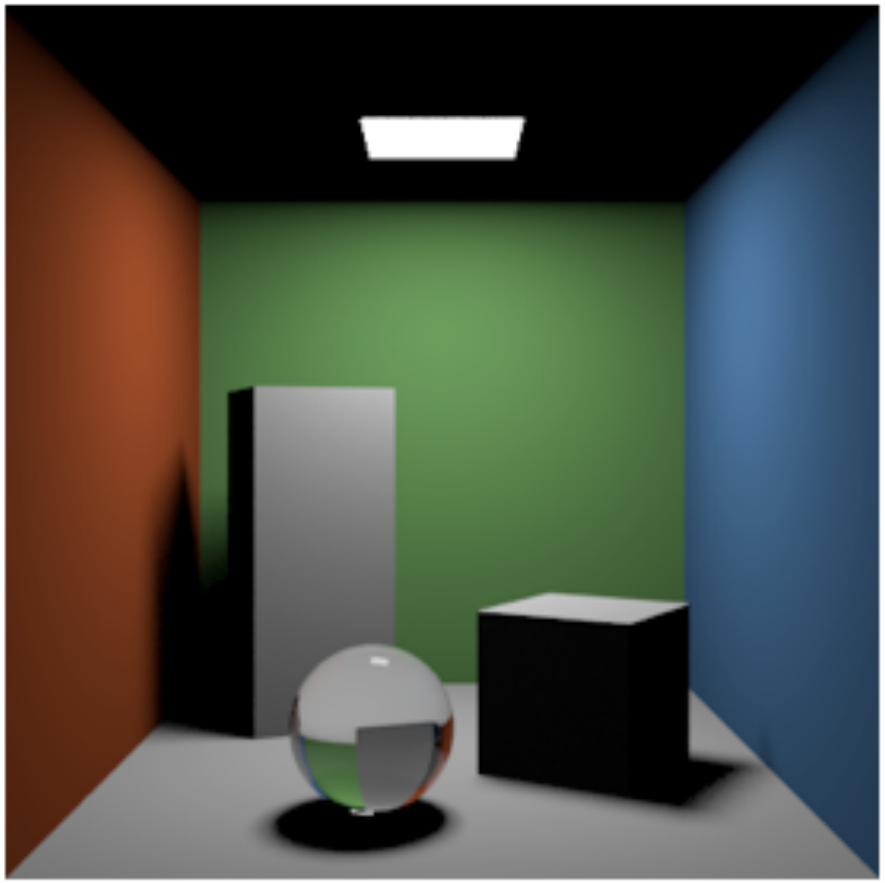
$$P = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

Ray Tracing

- Ray/primitive intersection
- **Secondary rays – global illumination**
- Acceleration

Global Illumination

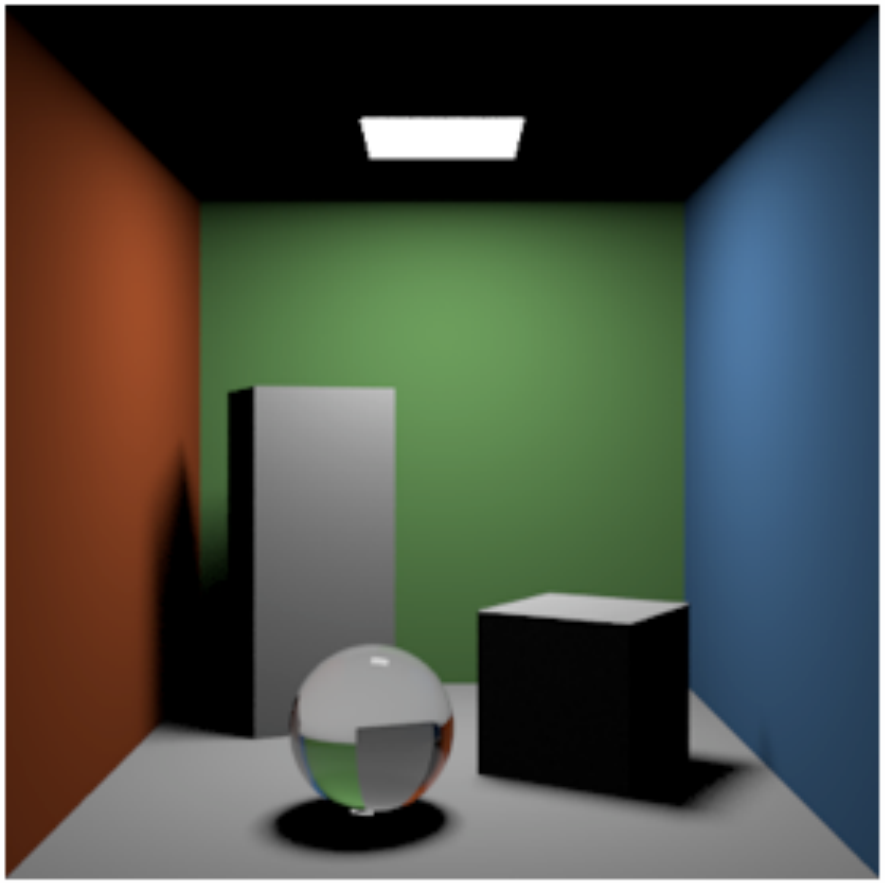
What you get with ray tracing



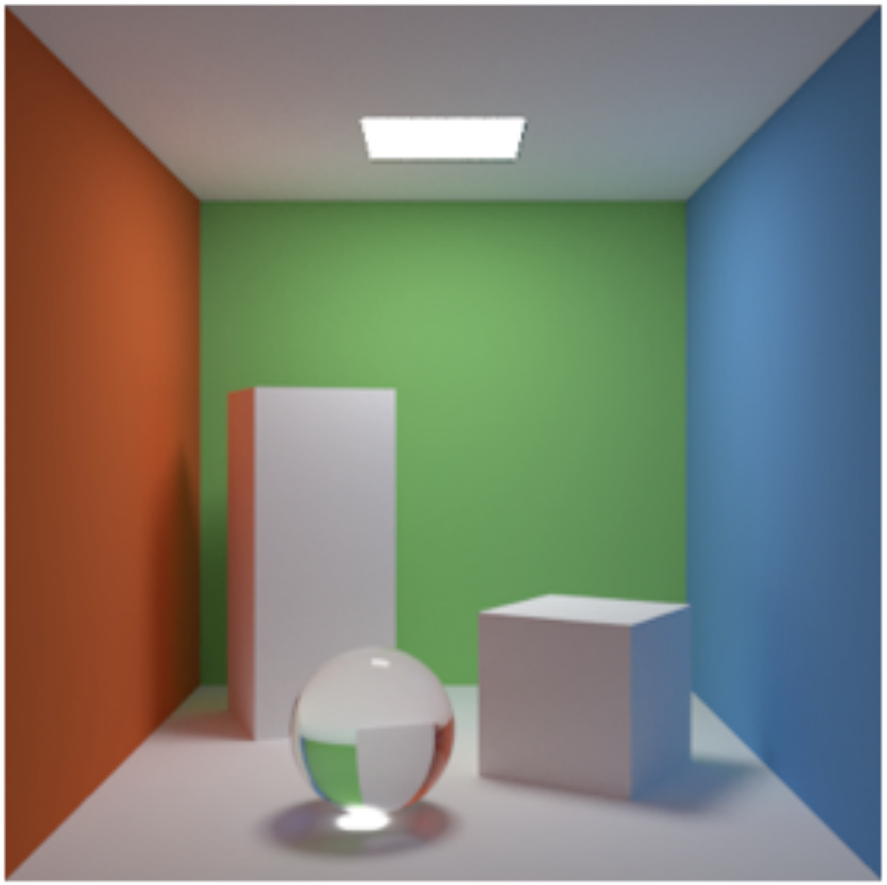
Problems?

Global Illumination

What you get with ray tracing



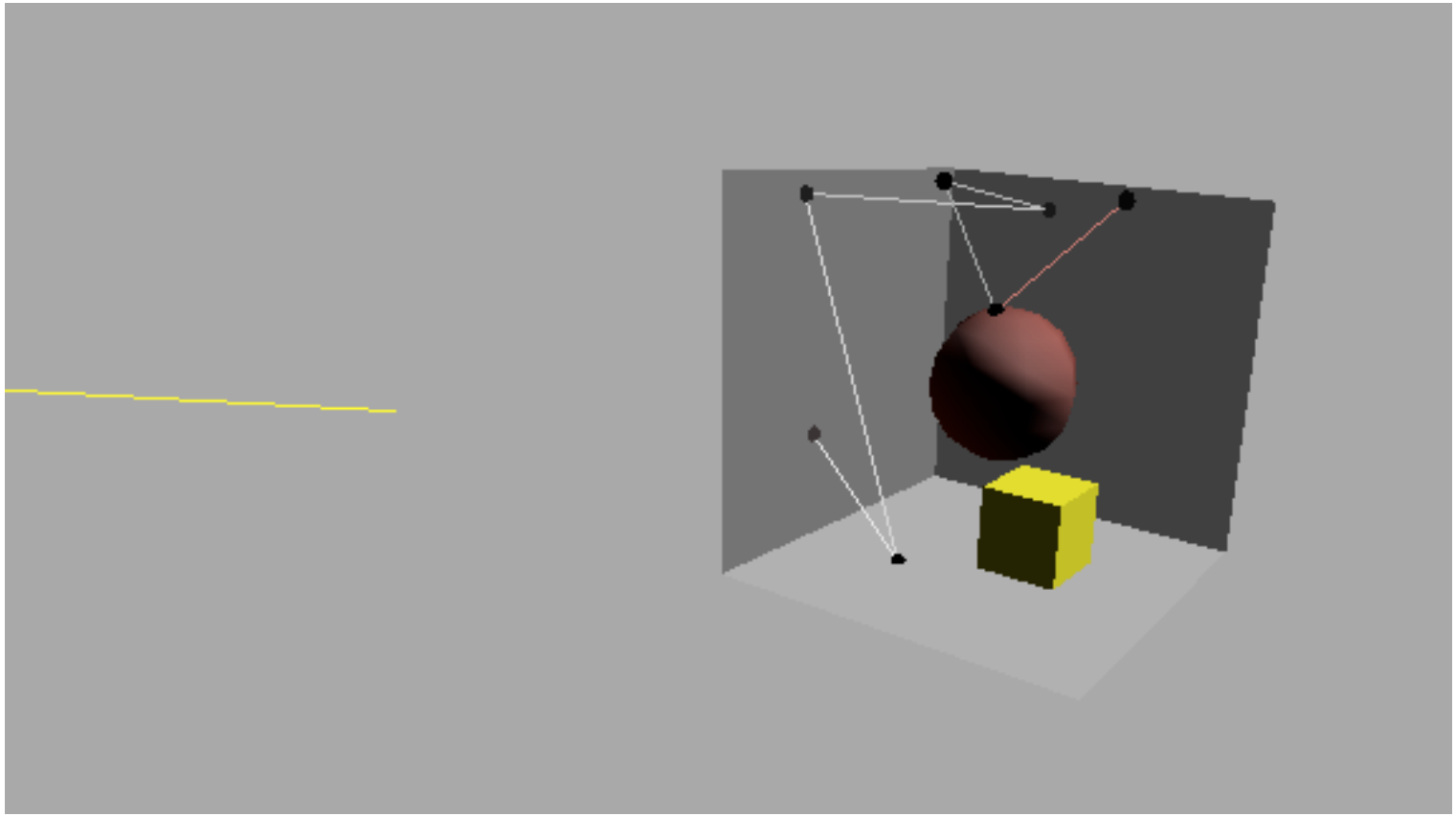
What you get with tracing secondary rays



* ‘Distributed ray tracing’ in your assignment suggests using Phong Shading to calculate final radiance – this will not look as nice

Global Illumination

DEMO (Path Tracer example):



Ray Tracing

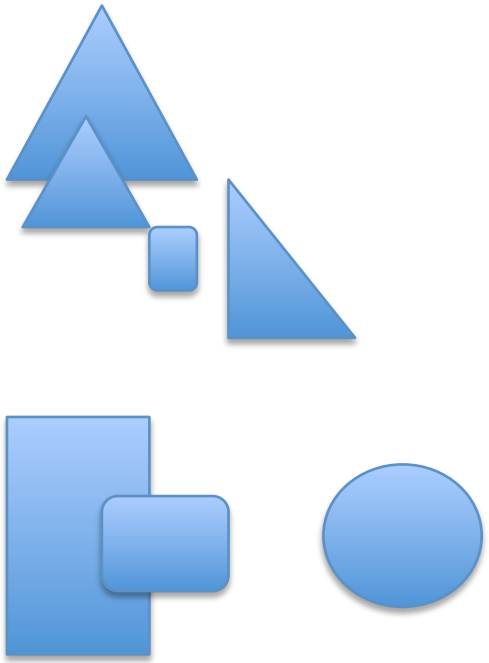
- Ray/primitive intersection
- Secondary rays – global illumination
- **Acceleration**

Acceleration

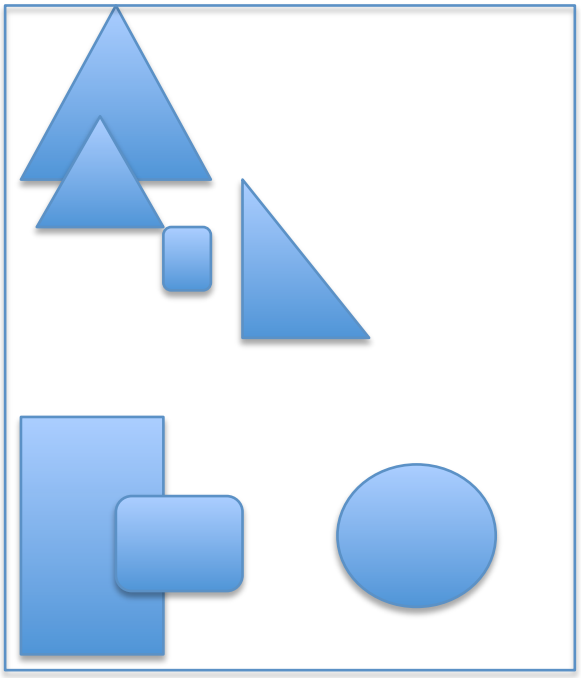
- **Generate Structure (e.g. octree)**

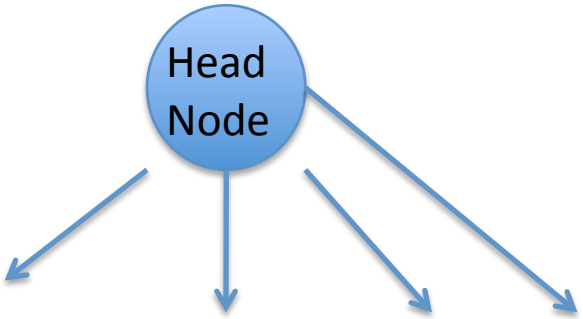
- Traverse Strucuture

Acceleration



Acceleration





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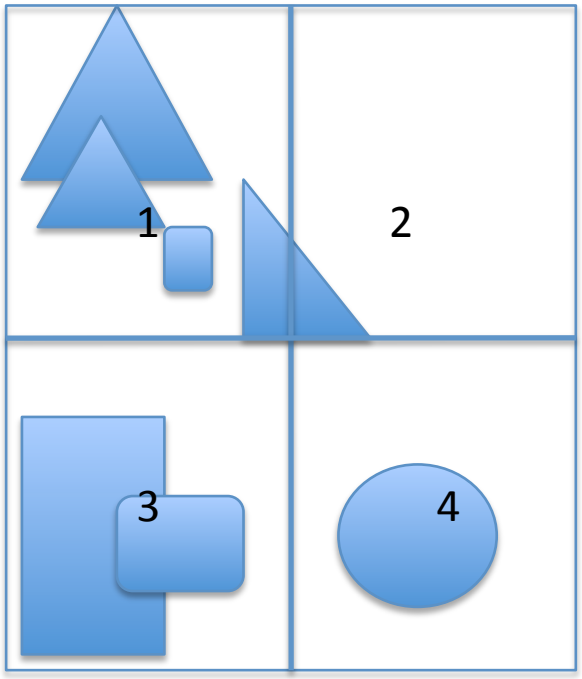

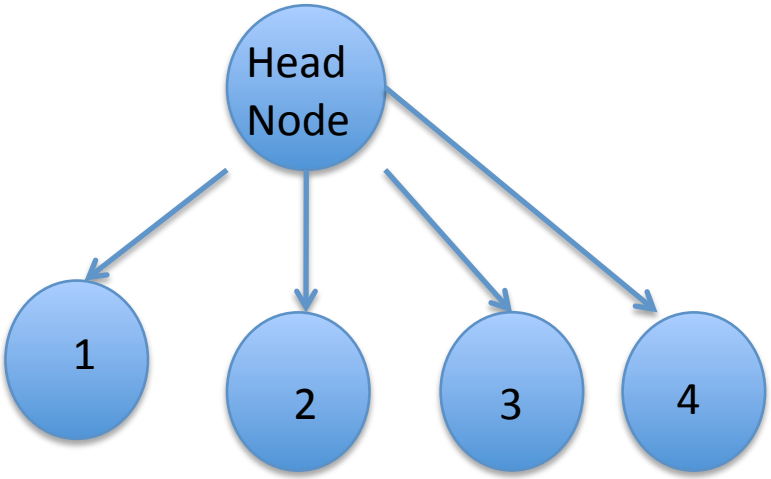

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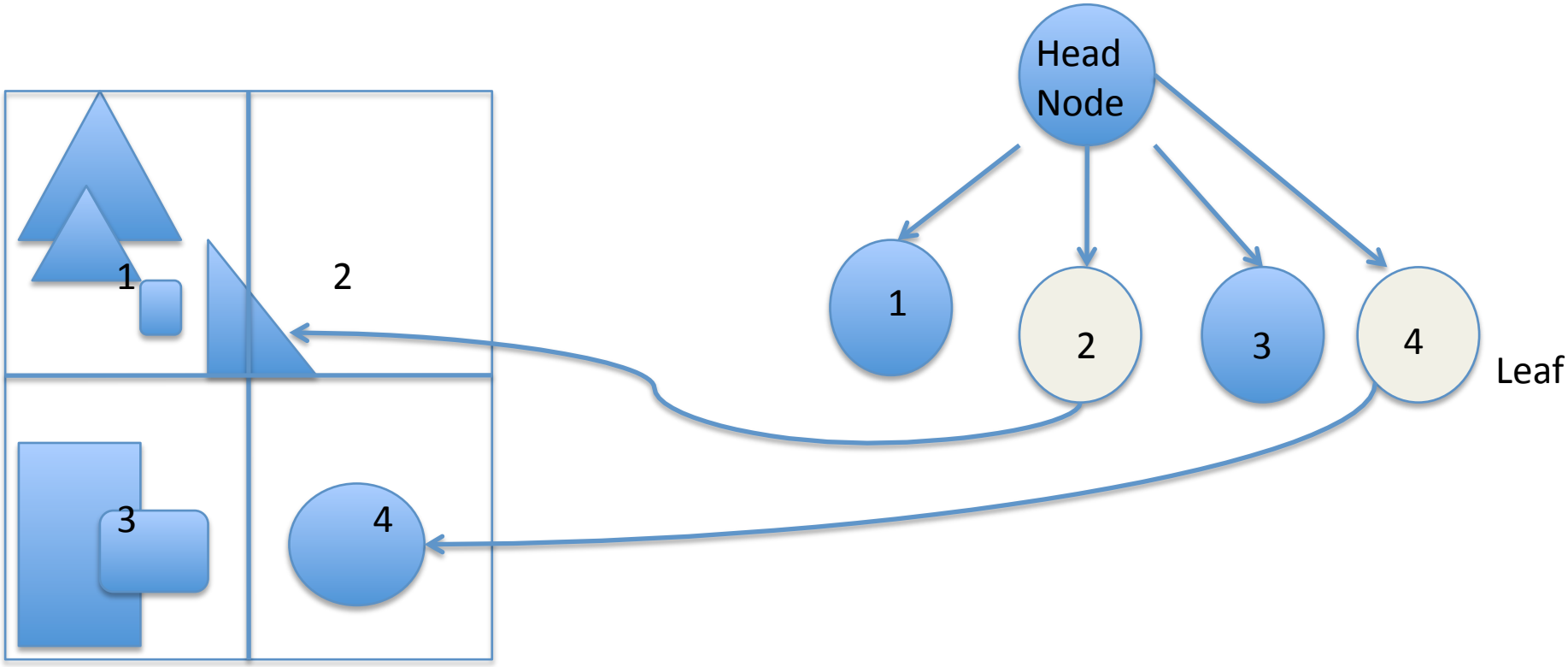

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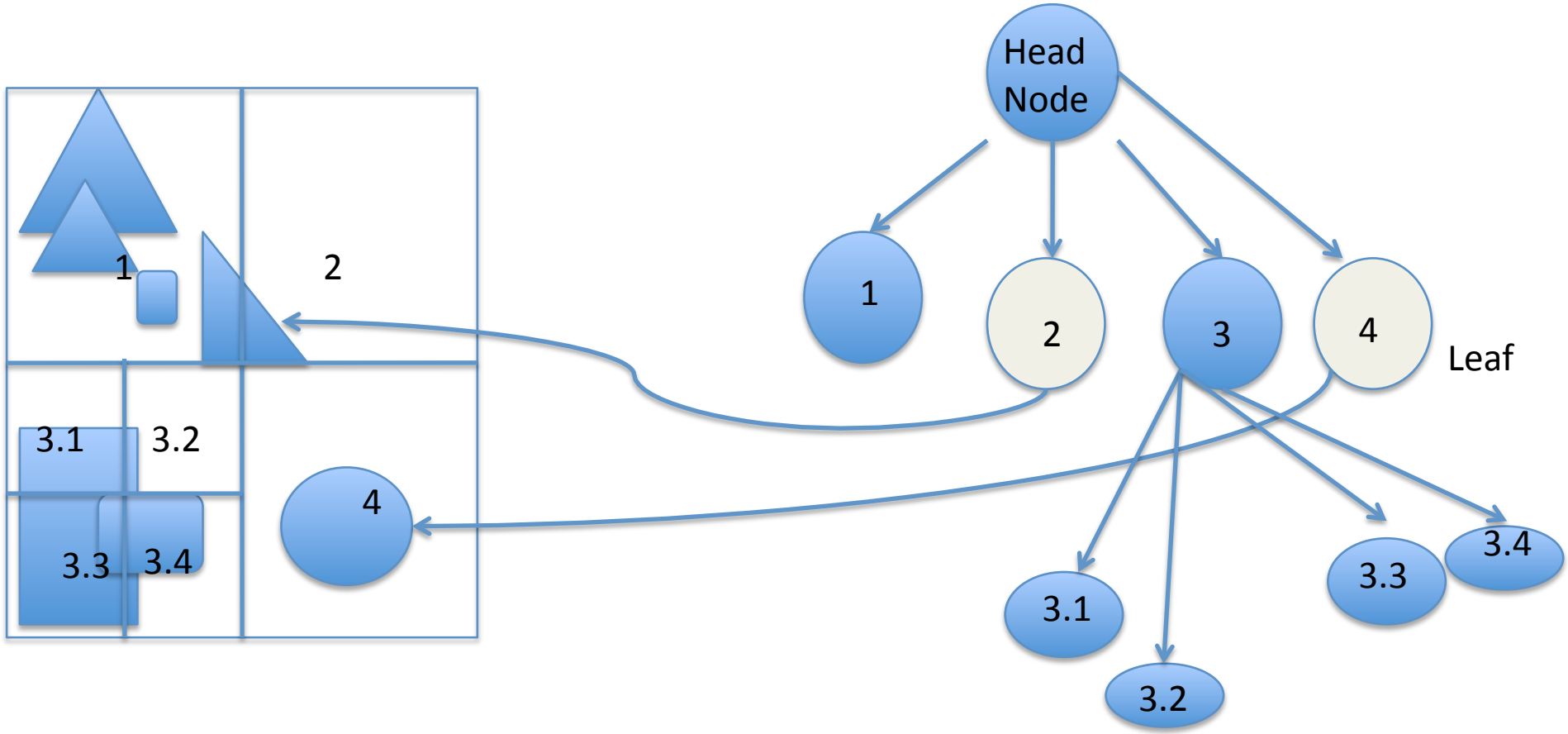

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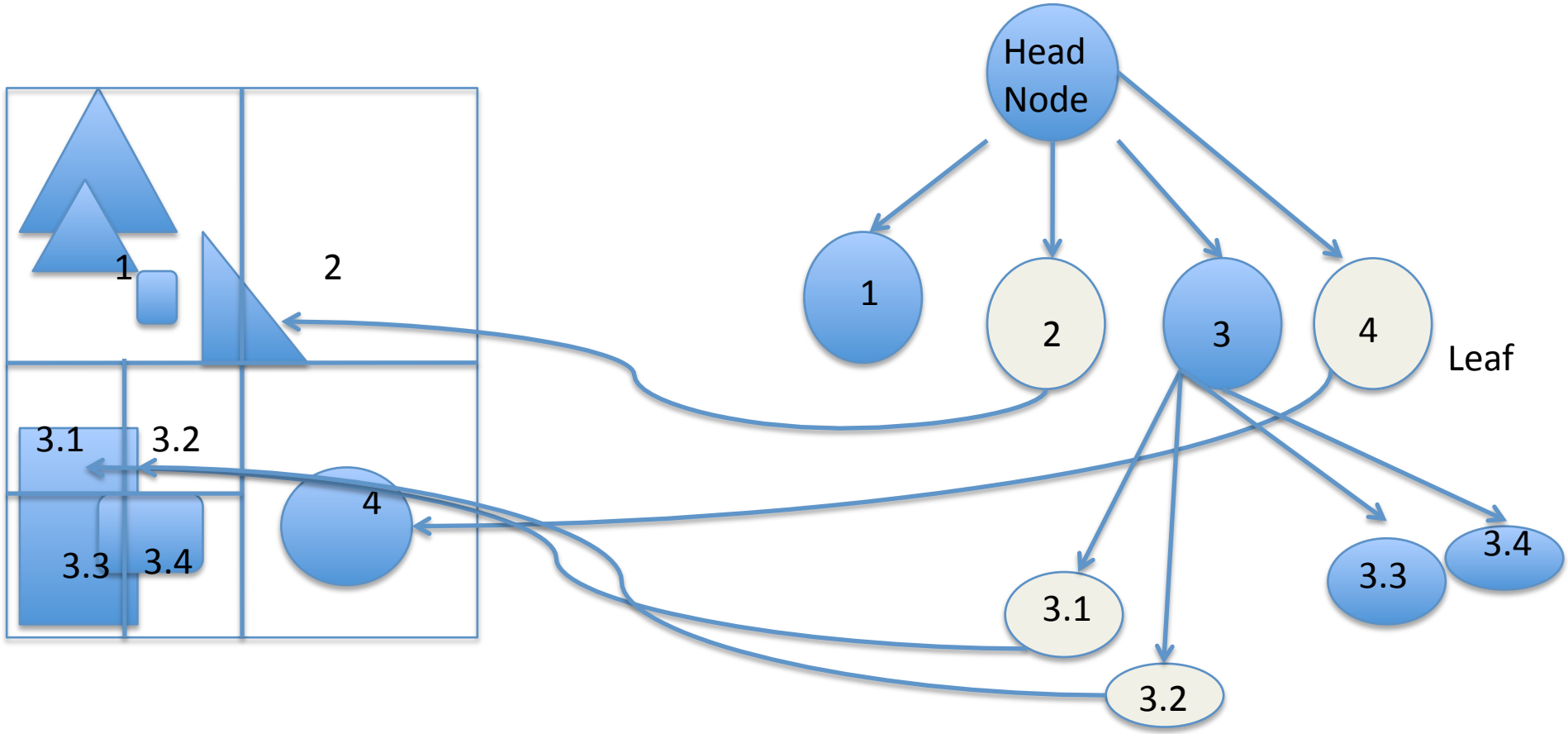

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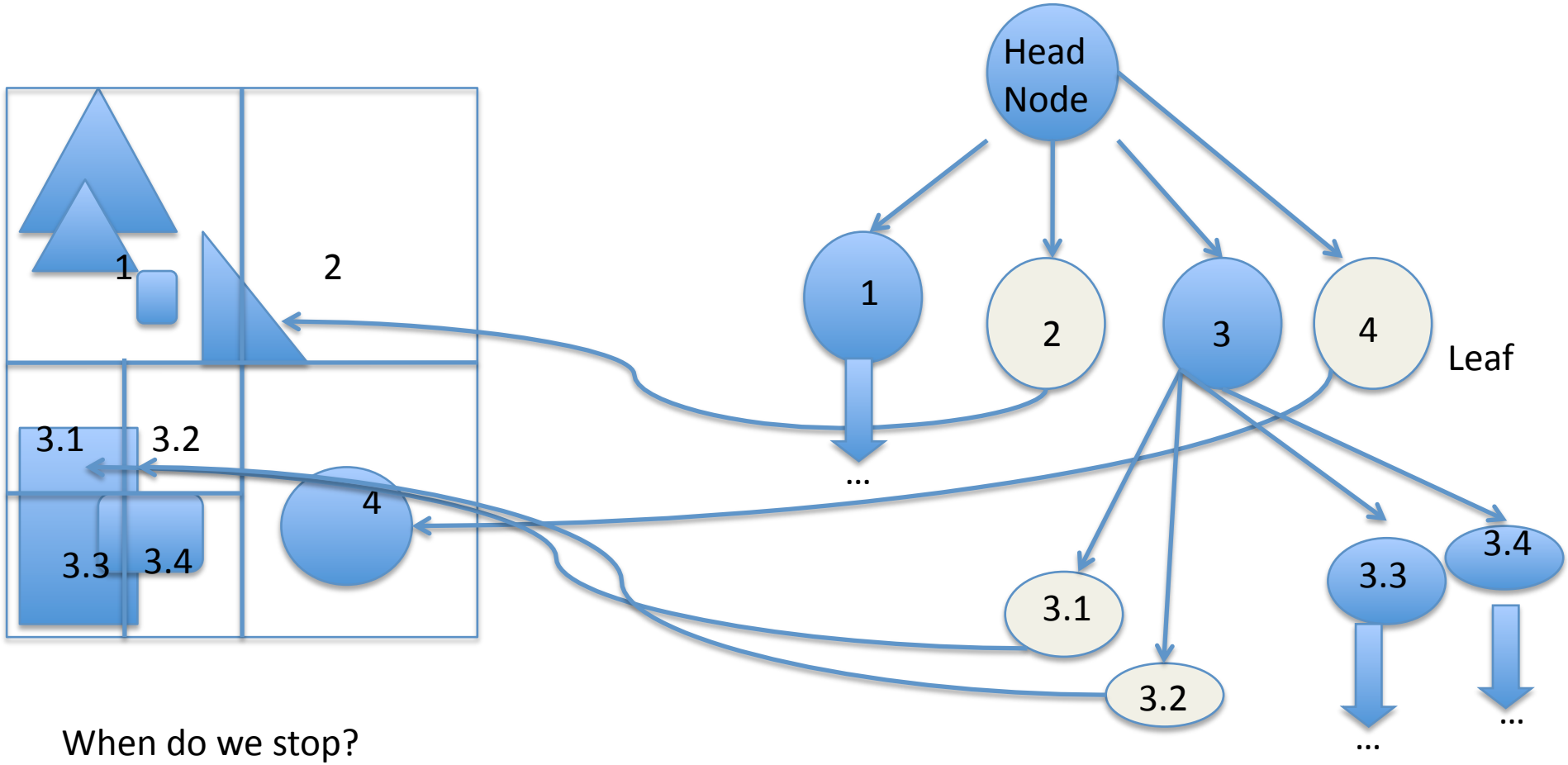


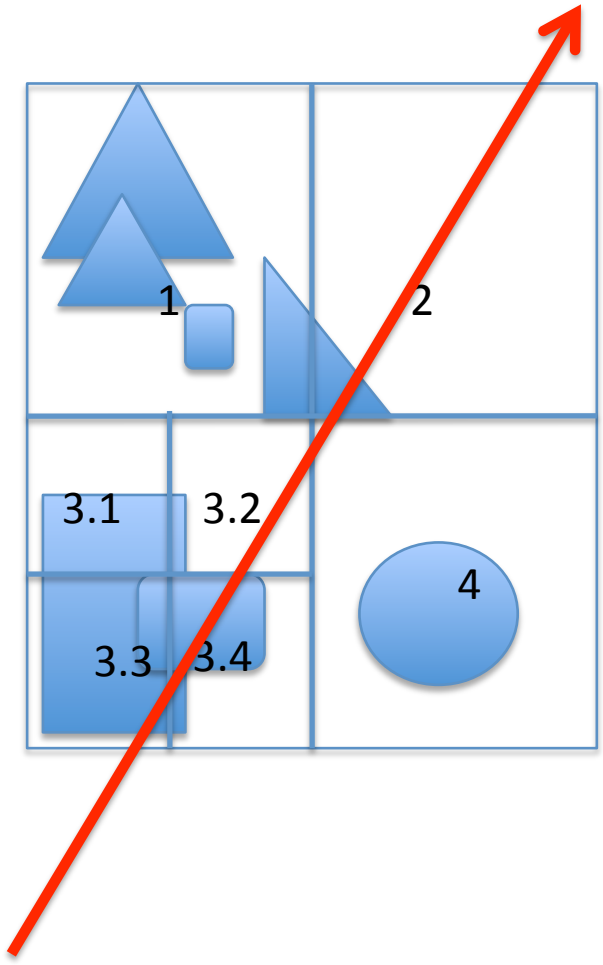
When do we stop?

Acceleration

- Generate Structure (e.g. octree)

- **Traverse Strucuture**

Acceleration



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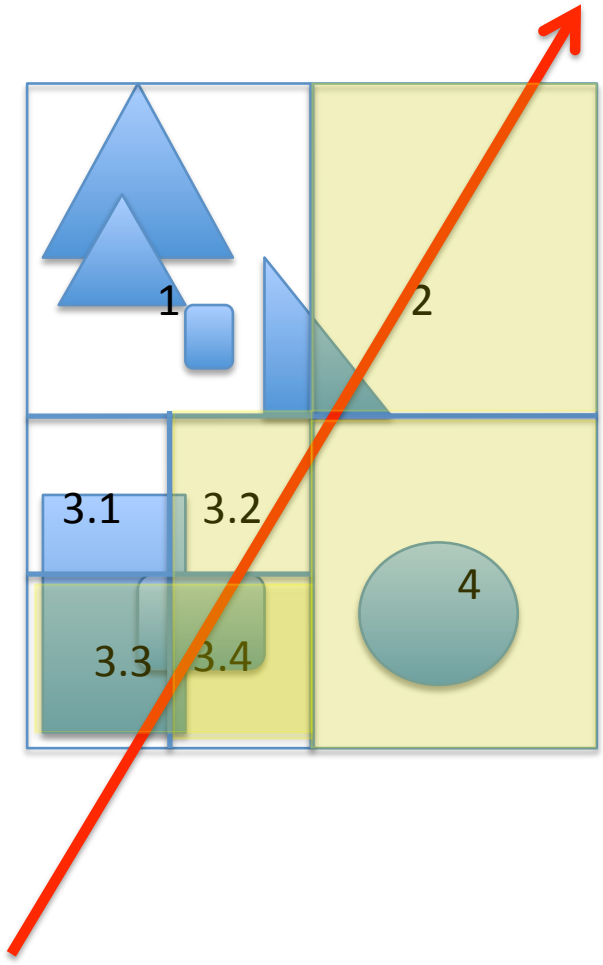
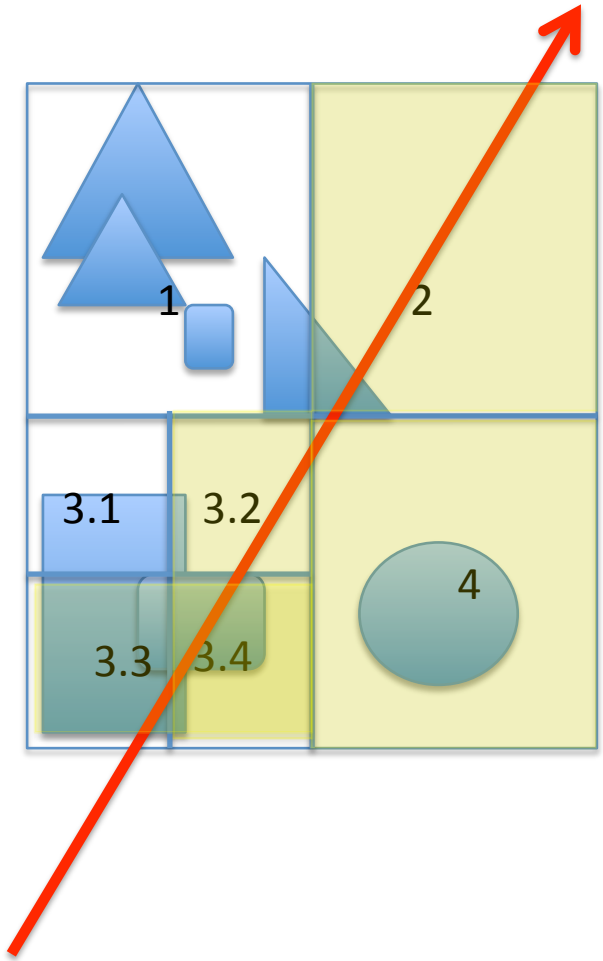


ORDER: 3.3, 3.4, 3.2, 2

If a ray intersected something in 3.3,
can it intersect something (with a smaller t)
in a later node?

Acceleration



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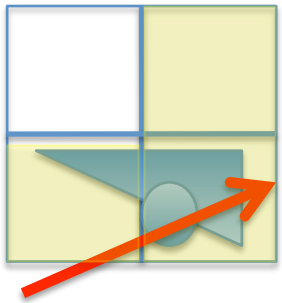

Think about objects on boundaries.

Example: should intersect the circle,
but the triangle is visited first

The End

Questions?