



# Computer Animation

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Princeton University  
COS 426, Spring 2008



## Syllabus

I. Image processing

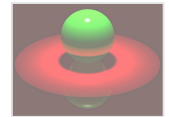
II. Modeling

III. Rendering

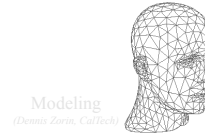
IV. Animation



Image Processing  
*(Rusty Coleman, CS426, Fall99)*



Rendering  
*(Michael Rosnick, CS426, Fall99)*



Modeling  
*(Dennis Zoran, CalTech)*



Animation  
*(Angel, Plate 1)*



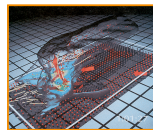
## Computer Animation

- What is animation?
  - Make objects change over time according to scripted actions



Pixar

- What is simulation?
  - Predict how objects change over time according to physical laws



University of Illinois



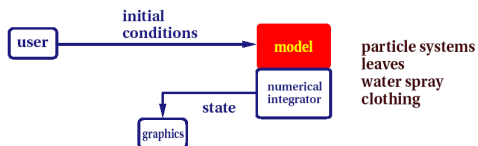
## Simulation

- Dynamics
  - Considers underlying forces
  - Compute motion from initial conditions and physics
- Kinematics
  - Considers only motion
  - Determined by positions, velocities, accelerations

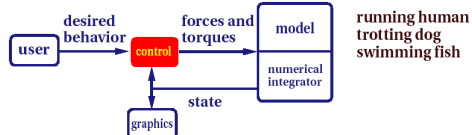


## Dynamics

### Passive--no muscles or motors



### Active--internal source of energy

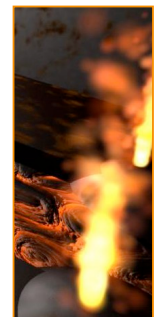


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## Passive Dynamics

- No muscles or motors
  - Smoke
  - Water
  - Cloth
  - Fire
  - Fireworks
  - Dice



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## Passive Dynamics



- Physical laws
  - Newton's laws
  - Hook's law
  - Etc.
- Physical phenomena
  - Gravity
  - Momentum
  - Friction
  - Collisions
  - Elasticity
  - Fracture



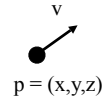
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## Particle Systems



- A particle is a point mass
  - Mass
  - Position
  - Velocity
  - Forces
  - Color
  - Lifetime



- Use lots of particles to model complex phenomena
  - Keep array of particles
  - Newton's laws

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## Particle Systems



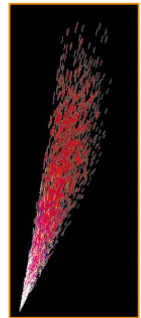
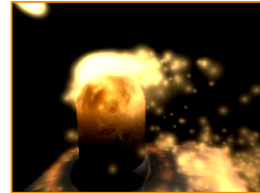
- For each frame:
  - Create new particles and assign attributes
  - Delete any expired particles
  - Update particles based on attributes and physics
  - Render particles

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## Creating Particles



- Where to create particles?
  - Predefined source
  - Surface of shape
  - Where particle density is low
  - etc.



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## Creating Particles



- Where to create particles?
  - Predefined source
  - Surface of shape
  - Where particle density is low
  - etc.

Reeves

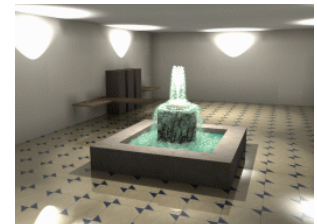


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## Deleting Particles



- When to delete particles?
  - Predefined sink
  - Surface of shape
  - Where density is high
  - Life span
  - Random



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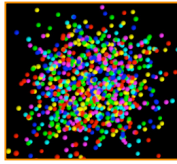
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## Rendering Particles



- Rendering styles

- Points
- Polygons
- Shapes
- Trails
- etc.



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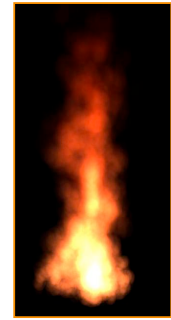
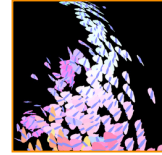
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## Rendering Particles



- Rendering styles

- Points
- Polygons
- Shapes
- Trails
- etc.



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## Rendering Particles



- Rendering styles

- Points
- Polygons
- Shapes
- Trails
- etc.



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## Rendering Particles



- Rendering styles

- Points
- Polygons
- Shapes
- Trails
- etc.



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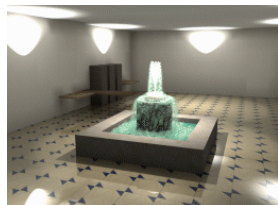
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## Particle Systems



- For each frame:

- Create new particles and assign attributes
- Delete any expired particles
- Update particles based on attributes and physics
- Render particles



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## Equations of Motion



- Newton's Law for a point mass

- $f = ma$

- Computing particle motion requires solving second-order differential equation

$$\ddot{x} = \frac{f(x, \dot{x}, t)}{m}$$

- Add variable  $v$  to form coupled first-order differential equations

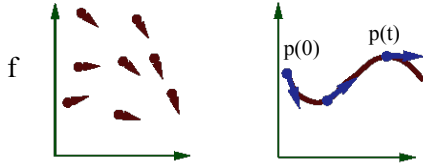
$$\begin{cases} \dot{x} = v \\ \dot{v} = \frac{f}{m} \end{cases}$$

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## Solving the Equations of Motion



- Initial value problem
  - Know  $p(0)$ ,  $v(0)$ ,  $a(0)$
  - Can compute force at any time and position
  - Compute  $p(t)$  by forward integration



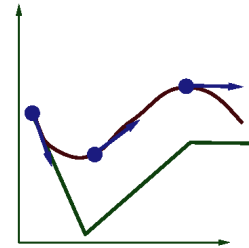
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## Solving the Equations of Motion



- Euler integration
  - $p(t+\Delta t) = p(t) + \Delta t v(t)$
  - $v(t+\Delta t) = v(t) + \Delta t f(x(t), t)/m$



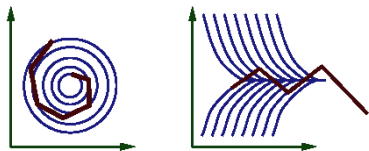
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## Solving the Equations of Motion



- Euler integration
  - $p(t+\Delta t) = p(t) + \Delta t v(t)$
  - $v(t+\Delta t) = v(t) + \Delta t f(p(t), t)/m$
- Problem:
  - Accuracy decreases as  $\Delta t$  gets bigger



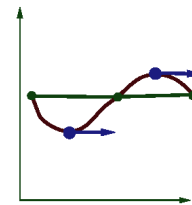
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## Solving the Equations of Motion



- Midpoint method (2<sup>nd</sup> order Runge-Kutta)
  - Compute an Euler step
  - Evaluate  $f$  at the midpoint
  - Take an Euler step using midpoint force
    - »  $v(t+\Delta t) = v(t) + \Delta t f(p(t) + 0.5 \Delta t v(t), t)$



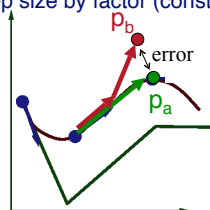
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## Solving the Equations of Motion



- Adapting step size
  - Compute  $p_a$  by taking one step of size  $h$
  - Compute  $p_b$  by taking 2 steps of size  $h/2$
  - Error =  $|p_a - p_b|$
  - Multiply step size by factor (constant/error)



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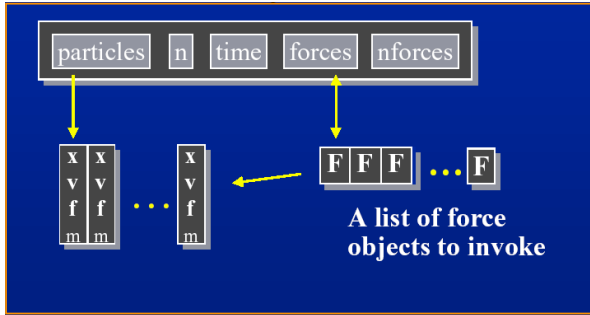
## Particle System Forces



- Force fields
  - Gravity, wind, pressure
- Viscosity/damping
  - Liquids, drag
- Collisions
  - Environment
  - Other particles
- Other particles
  - Springs between neighboring particles (mesh)
  - Useful for cloth

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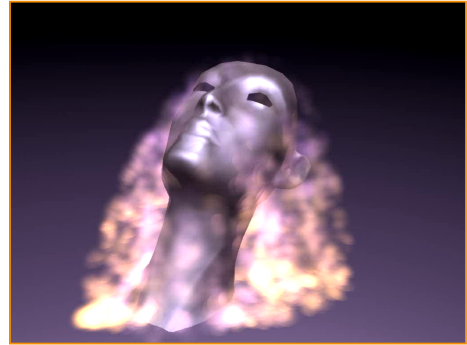
## Particle System Forces



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## Example: Gravity



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## Example: Fire

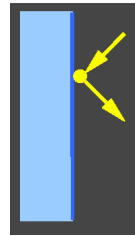


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## Example: Bouncing Off Wall



- Requires
  - Collision detection
  - Collision response (dynamic forces)



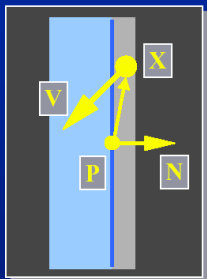
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## Example: Bouncing Off Wall



### Collision Detection



$$(X - P) \cdot N < \epsilon$$

$$N \cdot V < 0$$

- Within  $\epsilon$  of the wall.
- Heading in.

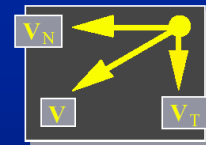
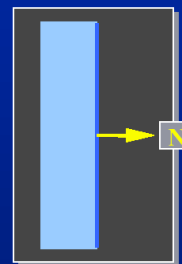
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## Example: Bouncing Off Wall



### Normal and Tangential Components



$$V_N = (N \cdot V)N$$

$$V_T = V - V_N$$

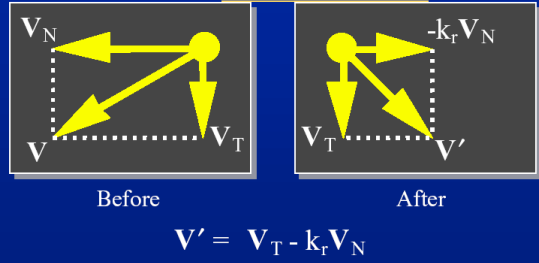
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### Example: Bouncing Off Wall



#### Collision Response



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### Example: Bouncing Off Wall

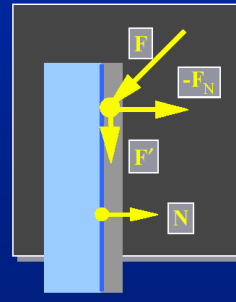


#### Contact Force

$$F' = F_T$$

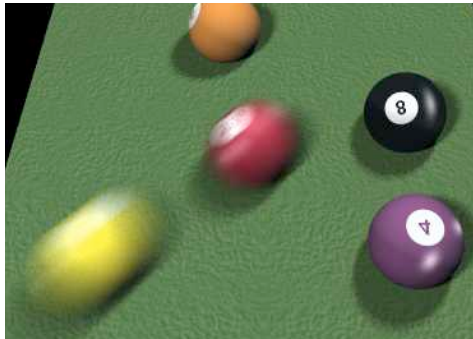
The wall pushes back, cancelling the normal component of  $F$ .

(An example of a *constraint force*.)

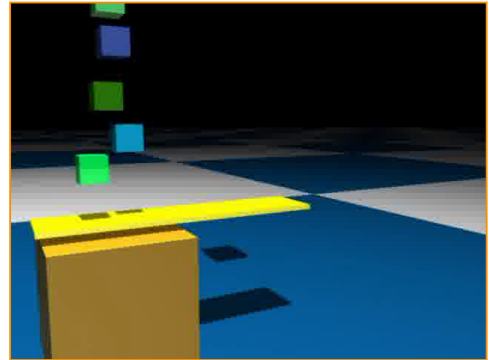


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### Example: Bouncing Off Particles

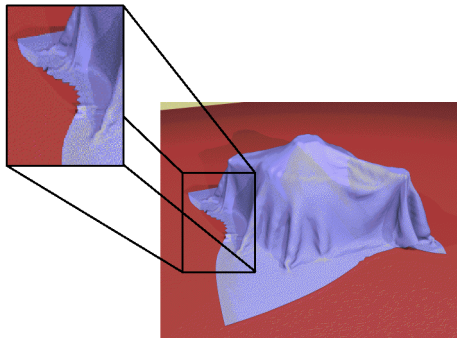


### Example: More Bouncing



Bender

### Example: Cloth



Breen

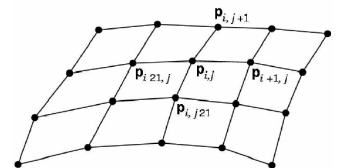
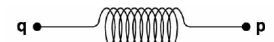
### Example: Cloth



- Spring-mass mesh
- Hooke's law

$$f = -k_s(|d| - s) \frac{d}{|d|}$$

- $f$  = force
- $k_s$  = spring constant
- $d = p - q$
- $s$  = resting length

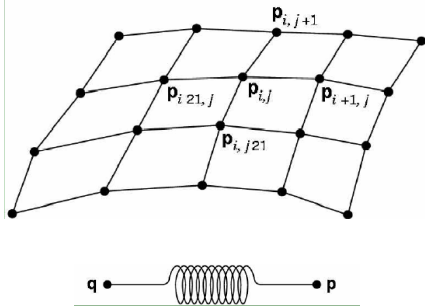


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## Example: Cloth



- Spring-mass mesh



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## Example: Cloth



- Hooke's law

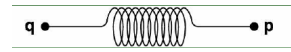
$$f = -k_s(|d| - s) \frac{d}{|d|}$$

f = force  
 $k_s$  = spring constant  
 $d = p - q$   
 $s$  = resting length

- Damping term

$$f = -\left(k_s(|d| - s) + k_d \frac{d \cdot \dot{d}}{|d|}\right) \frac{d}{|d|}$$

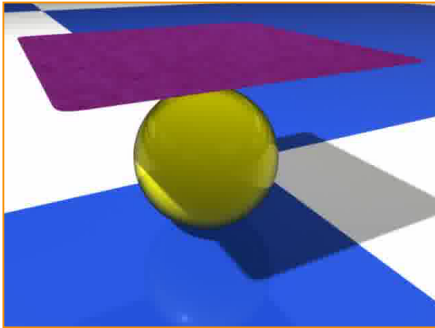
$$\dot{d} = \dot{p} - \dot{q}$$



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## Example: Cloth



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## Example: Flocks & Herds



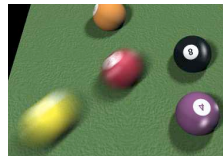
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## Summary



- Particle systems
  - Lots of particles
  - Simple physics
- Interesting behaviors
  - Waterfalls
  - Smoke
  - Cloth
  - Flocks
- Solving motion equations
  - Simplest method is Euler integration
  - Better to use adaptive step sizes



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