Illumination

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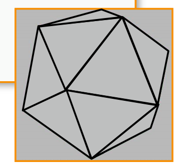
Princeton University

COS 426, Spring 2008

Ray Casting

```
Image RayCast(Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(scene.camera, i, j);
            Intersection hit = FindIntersection(ray, scene);
            image[i][j] = GetColor(scene, ray, hit);
        }
    }
    return image;
}
```



Wireframe

Ray Casting

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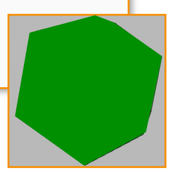

Without Illumination

Ray Casting

```
Image RayCast(Scene scene, int width, int height)
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            Intersection hit = FindIntersection(ray, scene);
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```
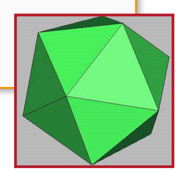
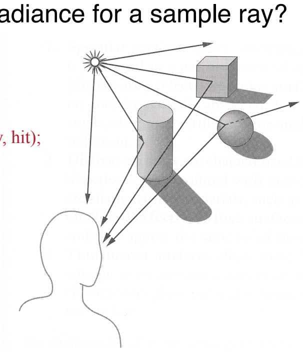



With Illumination

Illumination

- How do we compute radiance for a sample ray?

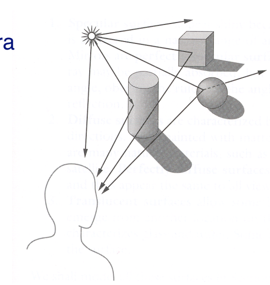
image[i][j] = GetColor(scene, ray, hit);



Angel Figure 6.2

Goal

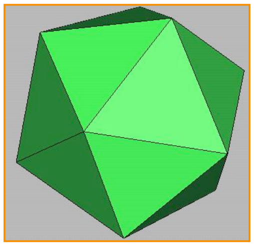
- Must derive computer models for ...
  - Emission at light sources
  - Scattering at surfaces
  - Reception at the camera



- Desirable features …
  - Concise
  - Efficient to compute
  - "Accurate"

Overview

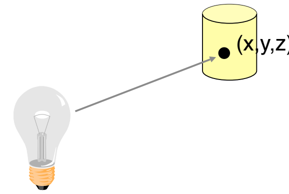
- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Refractions
  - Inter-object reflections



Direct Illumination

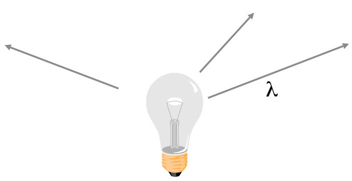
Emission at Light Sources

- $I_L(x,y,z,\theta,\phi,\lambda)$ ...
  - describes the intensity of energy,
  - leaving a light source, …
  - arriving at location(x,y,z), ...
  - from direction (θ,ϕ), ...
  - with wavelength λ



Empirical Models

- Ideally measure irradiant energy for “all” situations
  - Too much storage
  - Difficult in practice

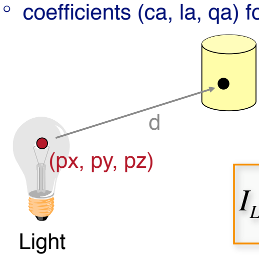


OpenGL Light Source Models

- Simple mathematical models:
  - Point light
  - Spot light
  - Directional light

Point Light Source 

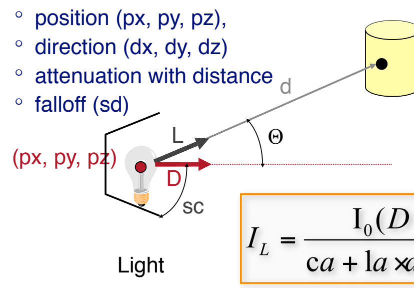
- Models omni-directional point source
  - intensity (I_0),
  - position (px, py, pz),
  - coefficients (ca, la, qa) for attenuation with distance (d)



$$I_L = \frac{I_0}{ca + la \cdot d + qa \cdot d^2}$$

Spot Light Source 

- Models point light source with direction
  - intensity (I_0),
  - position (px, py, pz),
  - direction (dx, dy, dz)
  - attenuation with distance
  - falloff (sd)

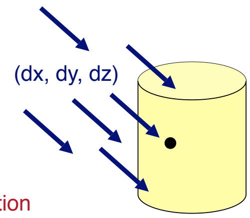


$$I_L = \frac{I_0 (D \bullet L)^{sd}}{ca + la \times d + qa \times d^2}$$

if ($\Theta > sc$) $I_L = 0$

Directional Light Source

- Models point light source at infinity
  - intensity (I_0),
  - direction (dx,dy,dz)

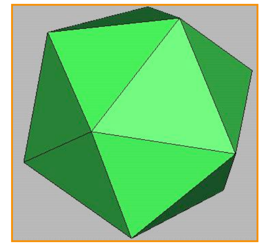


No attenuation with distance

$$I_L = I_0$$

Overview

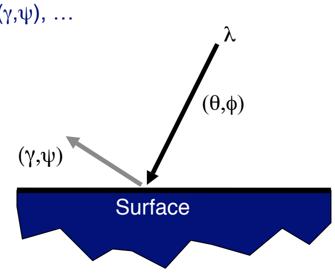
- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Refractions
  - Inter-object reflections



Direct Illumination

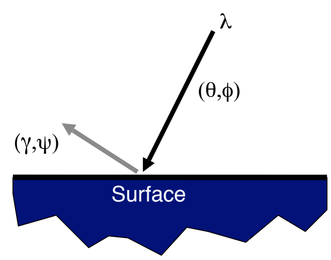
Scattering at Surfaces

- $R_s(\theta,\phi,\gamma,\psi,\lambda)$ ...
  - describes the amount of incident energy,
  - arriving from direction (θ,ϕ), ...
  - leaving in direction (γ,ψ), …
  - with wavelength λ



Empirical Models

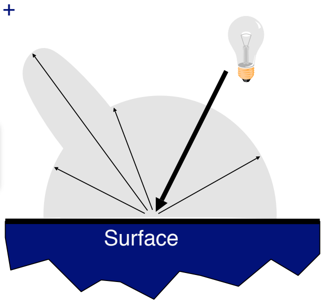
- Ideally measure radiant energy for “all” combinations of incident angles
  - Too much storage
  - Difficult in practice



OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”

Based on model proposed by Phong



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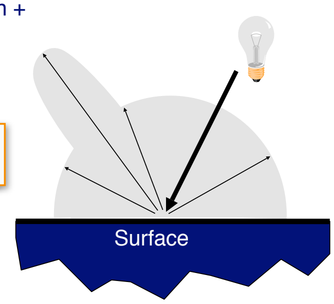
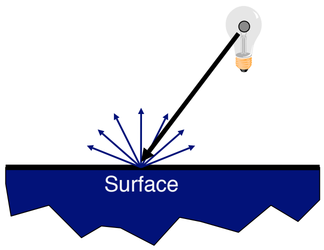


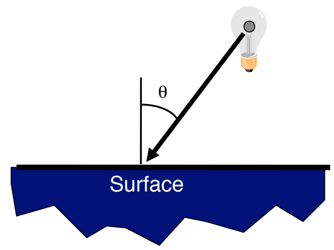
Diffuse Reflection

- Assume surface reflects equally in all directions
  - Examples: chalk, clay



Diffuse Reflection

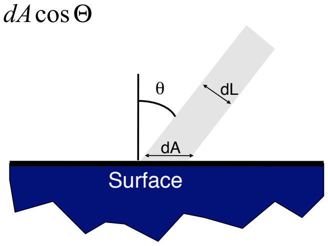
- How much light is reflected?
  - Depends on angle of incident light



Diffuse Reflection

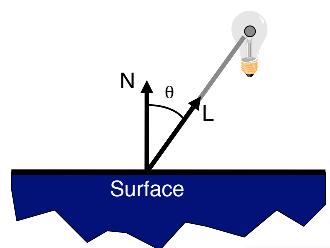
- How much light is reflected?
  - Depends on angle of incident light

$$dL = dA \cos \Theta$$



Diffuse Reflection

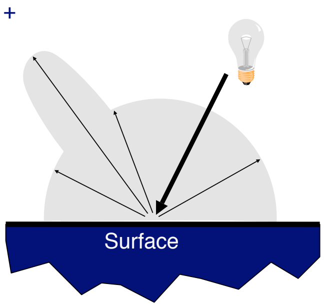
- Lambertian model
  - cosine law (dot product)



$$\boxed{I_D = K_D (N \bullet L) I_L}$$

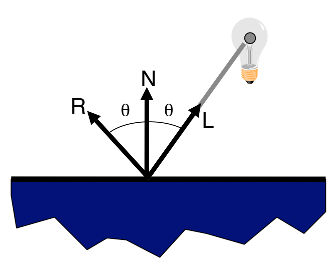
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - "ambient"



Specular Reflection

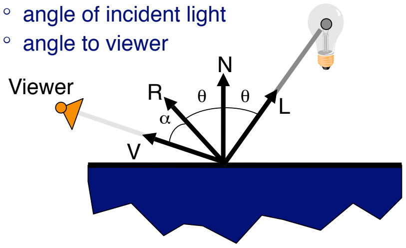
- Reflection is strongest near mirror angle
  - Examples: mirrors, metals



Specular Reflection

How much light is seen?

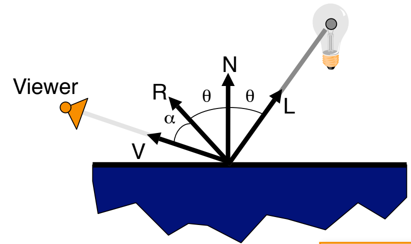
Depends on:
- angle of incident light
- angle to viewer



Specular Reflection

- Phong Model
  - $\cos(\alpha)^n$

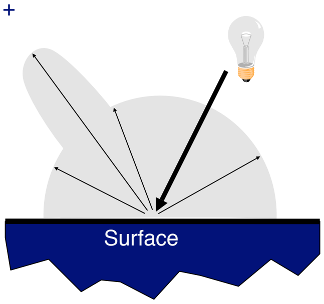
This is a physically-motivated hack!



$$I_S = K_S (V \bullet R)^n I_L$$

OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - "ambient"



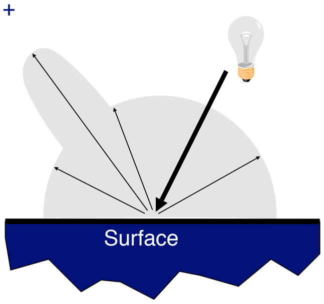
Emission

- Represents light eminating directly from polygon

Emission ≠ 0

OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - "ambient"



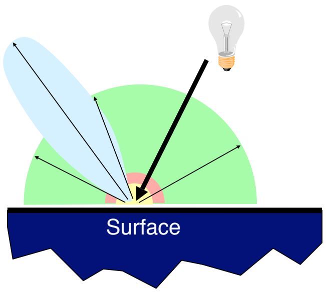
Ambient Term

- Represents reflection of all indirect illumination

This is a total hack (avoids complexity of global illumination)!

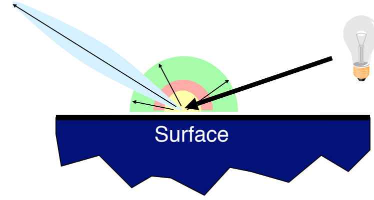
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”



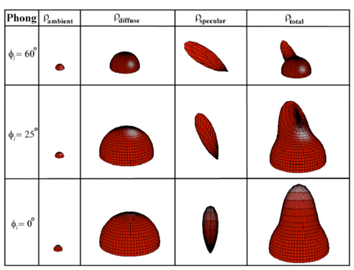
OpenGL Reflectance Model

- Simple analytic model:
  - diffuse reflection +
  - specular reflection +
  - emission +
  - “ambient”



OpenGL Reflectance Model

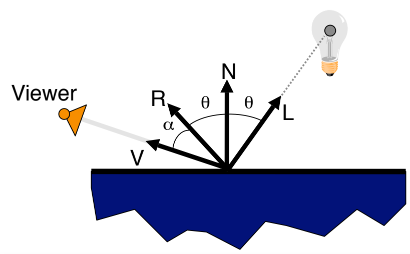
- Sum diffuse, specular, emission, and ambient



Leonard McMillan, MIT

Direct Illumination Calculation

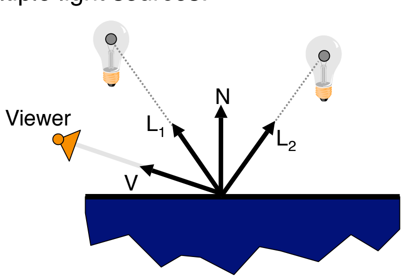
- Single light source:



$$I = I_E + K_A I_{AL} + K_D (N \bullet L) I_L + K_S (V \bullet R)^n I_L$$

Direct Illumination Calculation

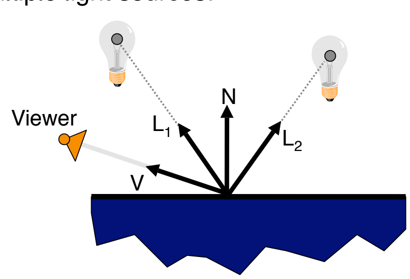
- Multiple light sources:



$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

Direct Illumination Calculation

- Multiple light sources:



Note: this is shorthand for (I_r, I_g, I_b)

$$I = I_E + K_A I_{AL} + \sum_i (K_D (N \bullet L_i) I_i + K_S (V \bullet R_i)^n I_i)$$

Example

Tin Toy (Pixar Animation Studios)

Overview

- Direct Illumination
  - Emission at light sources
  - Scattering at surfaces
- Global illumination
  - Shadows
  - Transmissions
  - Inter-object reflections

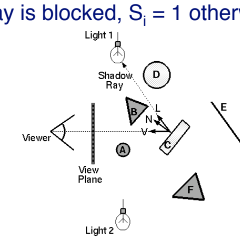
Global Illumination

Global Illumination

Greg Ward

Shadows

- Shadow term tells if light sources are blocked
  - Cast ray towards each light source L_i
  - $S_i = 0$ if ray is blocked, $S_i = 1$ otherwise

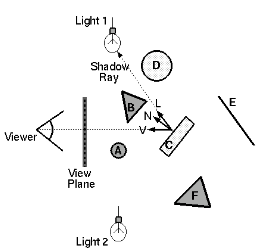


Shadow Term

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L$$

Ray Casting (last lecture)

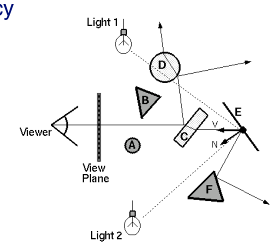
- Trace primary rays from camera
  - Direct illumination from unblocked lights only



$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L$$

Recursive Ray Tracing

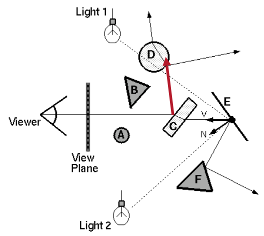
- Also trace secondary rays from hit surfaces
  - Global illumination from mirror reflection and transparency



$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Mirror reflections

- Trace secondary ray in mirror direction
  - Evaluate radiance along secondary ray and include it into illumination model

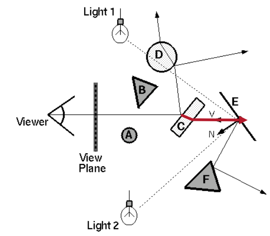


Radiance for mirror reflection ray

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Transparency

- Trace secondary ray in direction of refraction
  - Evaluate radiance along secondary ray and include it into illumination model

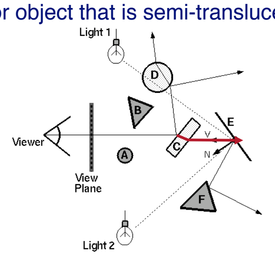


Radiance for refraction ray

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Transparency

- Transparency coefficient is fraction transmitted
  - $K_T = 1$ for translucent object, $K_T = 0$ for opaque
  - $0 < K_T < 1$ for object that is semi-translucent

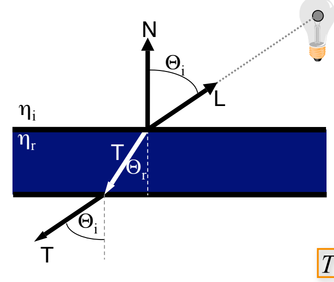


Transparency Coefficient

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Refractive Transparency

- For thin surfaces, can ignore change in direction
  - Assume light travels straight through surface

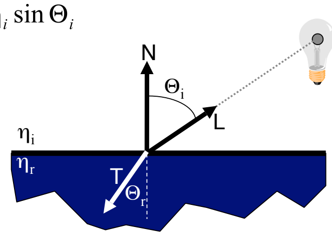


$$T \cong -L$$

Refractive Tranparency

For solid objects, apply Snell's law:

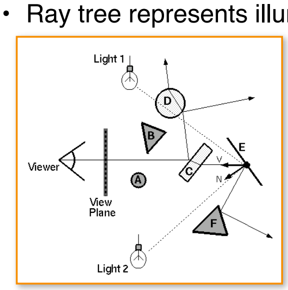
$$\eta_r \sin\Theta_r = \eta_i \sin\Theta_i$$



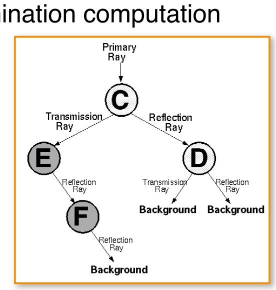
$$T = \left(\frac{\eta_i}{\eta_r}\cos\Theta_i - \cos\Theta_r\right)N - \frac{\eta_i}{\eta_r}L$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene

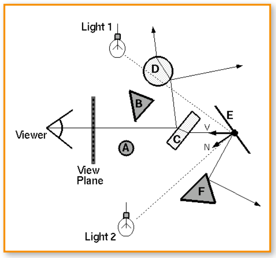


Ray tree

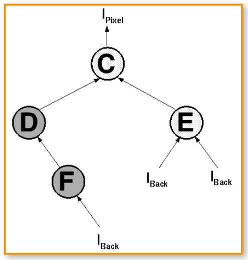
$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

- Ray tree represents illumination computation



Ray traced through scene



Ray tree

$$I = I_E + K_A I_A + \sum_L (K_D (N \bullet L) + K_S (V \bullet R)^n) S_L I_L + K_S I_R + K_T I_T$$

Recursive Ray Tracing

- GetColor is called recursively

```
Image RayTrace(Scene scene, int width, int height)
{
    Image image = new Image(width, height);
    for (int i = 0; i < width; i++) {
        for (int j = 0; j < height; j++) {
            Ray ray = ConstructRayThroughPixel(scene.camera, i, j);
            image[i][j] = GetColor(scene, ray);
        }
    }
    return image;
}
```


Recursive Ray Tracing

- GetColor is called recursively

```
Rgb GetColor(Scene scene, Ray ray)
{
    Intersection hit = FindIntersection(ray, scene);
    Ray specular_ray = SpecularRay(ray, hit);
    Ray refractive_ray = RefractiveRay(ray, hit);
    Color color = Phong(scene, ray, hit) +
                  Ks * GetColor(scene, specular_ray) +
                  Kt * GetColor(scene, refractive_ray);
    return color;
}
```


Example

Red's Dream (Pixar Animation Studios)

Summary

- Ray casting (direct Illumination)
  - Usually use simple analytic approximations for light source emission and surface reflectance
- Recursive ray tracing (global illumination)
  - Incorporate shadows, mirror reflections, and pure refractions

All of this is an approximation so that it is practical to compute

More on global illumination next time!

Illumination Terminology

- Radiant power [flux] (Φ)
  - Rate at which light energy is transmitted (in Watts).
- Radiant Intensity (I)
  - Power radiated onto a unit solid angle in direction (in Watts/sr)
    - e.g.: energy distribution of a light source (inverse square law)
- Radiance (L)
  - Radiant intensity per unit projected surface area (in Watts/m^{2}sr)
    - e.g.: light carried by a single ray (no inverse square law)
- Irradiance (E)
  - Incident flux density on a locally planar area (in Watts/m^2)
    - e.g.: light hitting a surface at a point
- Radiosity (B)
  - Exitant flux density from a locally planar area (in Watts/m^2)