



# Scene Graphs & Modeling Transformations

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Princeton University  
COS 426, Spring 2008



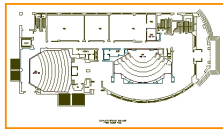
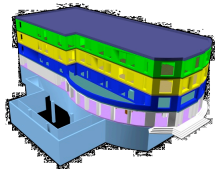
## 3D Object Representations

- Points
  - Range image
  - Point cloud
- Surfaces
  - Polygonal mesh
  - Subdivision
  - Parametric
  - Implicit
- Solids
  - Voxels
  - BSP tree
  - CSG
  - Sweep
- High-level structures
  - Scene graph
  - Application specific



## 3D Object Representations

- Desirable properties of an object representation
  - Easy to acquire
  - Accurate
  - Concise
  - Intuitive editing
  - Efficient editing
  - Efficient display
  - Efficient intersections
  - Guaranteed validity
  - Guaranteed smoothness
  - etc.

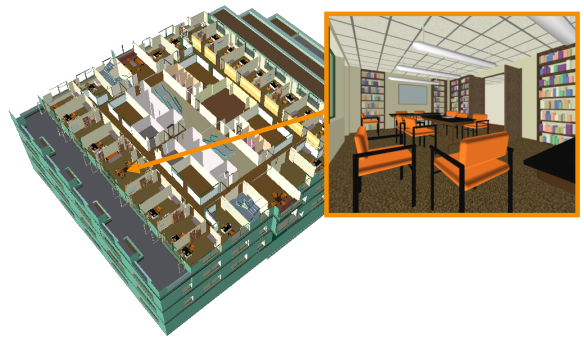


(CS Building, Princeton University)

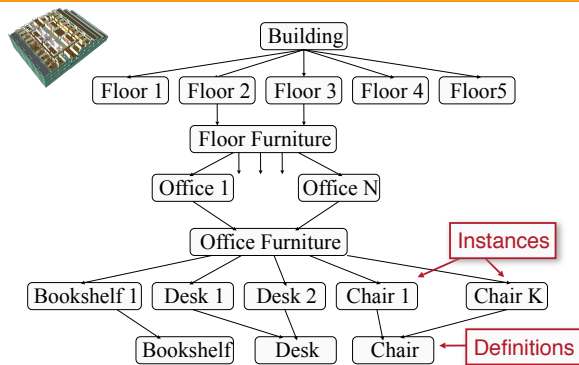


## 3D Object Representations

- What object representation is best for this?

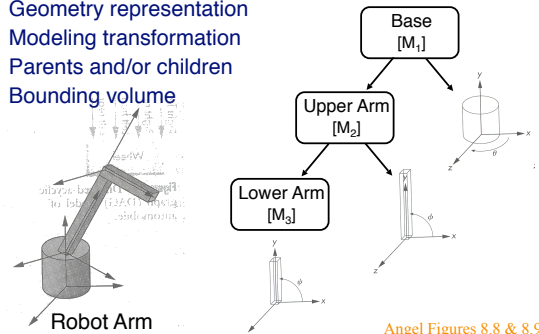


## Scene Graphs



## Scene Graphs

- Hierarchy of nodes, where each node may have:
  - Geometry representation
  - Modeling transformation
  - Parents and/or children
  - Bounding volume



Angel Figures 8.8 & 8.9

## Scene Graphs



- Advantages
  - Allows definitions of objects in own coordinate systems

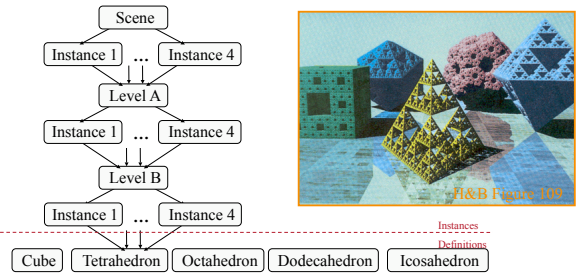


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## Scene Graphs



- Advantages
  - Allows definitions of objects in own coordinate systems
  - Allows use of object definition multiple times in a scene

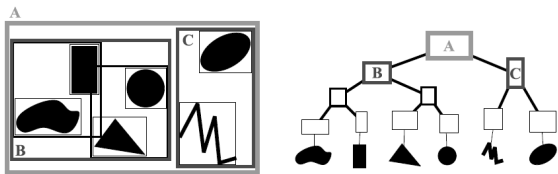


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## Scene Graphs



- Advantages
  - Allows definitions of objects in own coordinate systems
  - Allows use of object definition multiple times in a scene
  - Allows hierarchical processing (e.g., intersections)



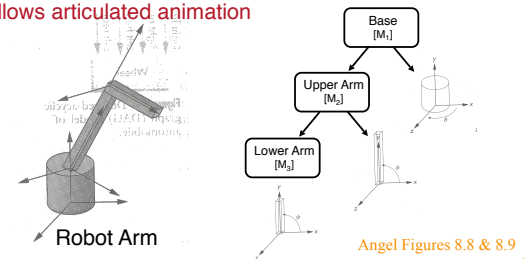
Haverkort

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## Scene Graphs



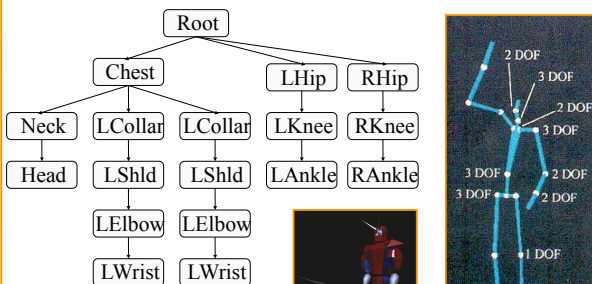
- Advantages
  - Allows definitions of objects in own coordinate systems
  - Allows use of object definition multiple times in a scene
  - Allows hierarchical processing (e.g., intersections)
  - Allows articulated animation



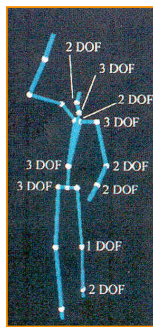
Angel Figures 8.8 & 8.9

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## Scene Graph Example



Mike Marr, COS 426



Rose et al. '96

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## Overview



- Scene graphs
  - Geometry & attributes
  - Transformations
  - Bounding volumes
- Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
  - 3D transformations

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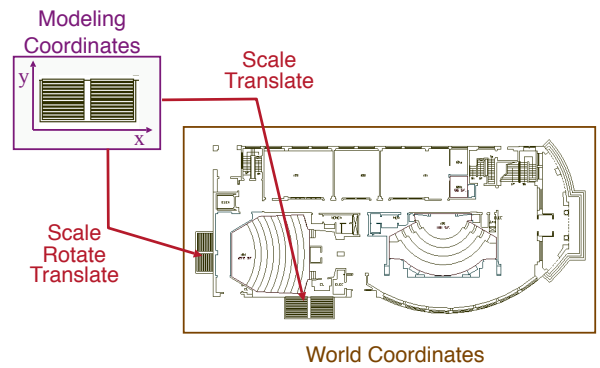
## Overview



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- Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
  - 3D transformations

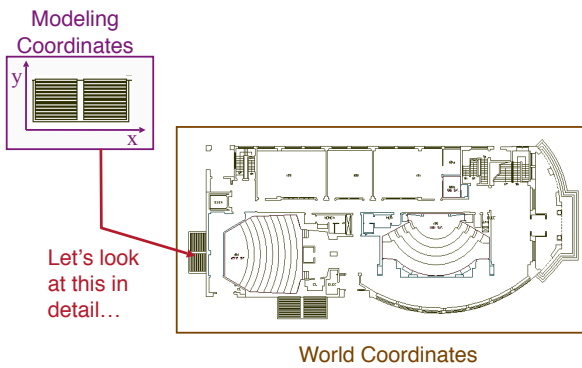
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## 2D Modeling Transformations



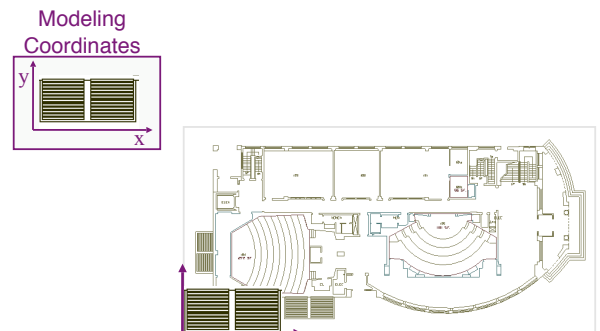
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## 2D Modeling Transformations



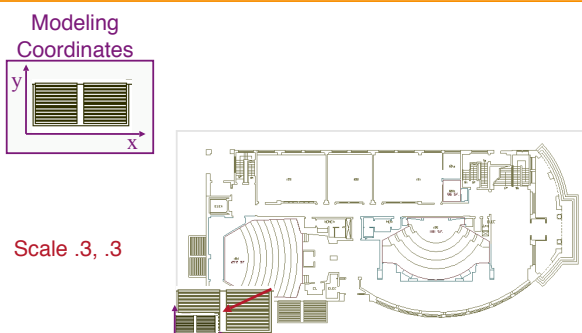
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## 2D Modeling Transformations



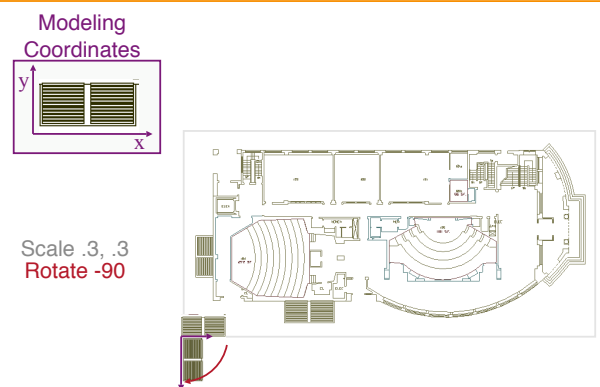
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## 2D Modeling Transformations



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## 2D Modeling Transformations



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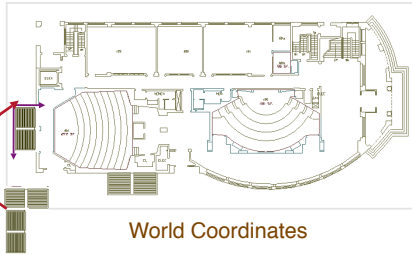
## 2D Modeling Transformations



Modeling Coordinates



Scale .3, .3  
Rotate -90  
Translate 5, 3



World Coordinates

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## Basic 2D Transformations

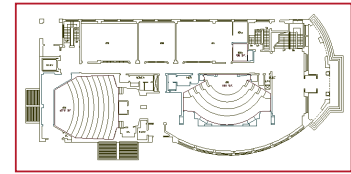


- Translation:
  - $x' = x + tx$
  - $y' = y + ty$

- Scale:
  - $x' = x * sx$
  - $y' = y * sy$

- Shear:
  - $x' = x + hx*y$
  - $y' = y + hy*x$

- Rotation:
  - $x' = x*cos\theta - y*sin\theta$
  - $y' = x*sin\theta + y*cos\theta$



Transformations can be combined (with simple algebra)

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## Basic 2D Transformations

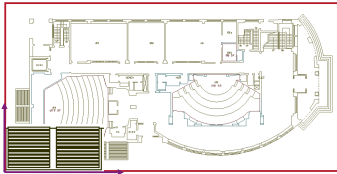


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## Basic 2D Transformations

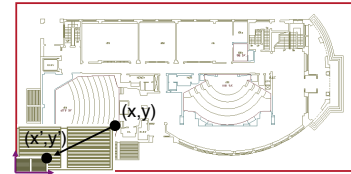


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$$\begin{matrix} x' = x*sx \\ y' = y*sy \end{matrix}$$

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## Basic 2D Transformations

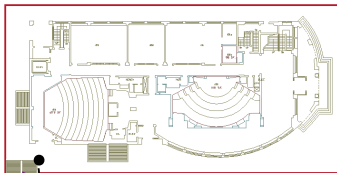


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- Rotation:
  - $x' = x*cos\theta - y*sin\theta$
  - $y' = x*sin\theta + y*cos\theta$



$$\begin{matrix} x' = (x*sx)*cos\theta - (y*sy)*sin\theta \\ y' = (x*sx)*sin\theta + (y*sy)*cos\theta \end{matrix}$$

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## Basic 2D Transformations

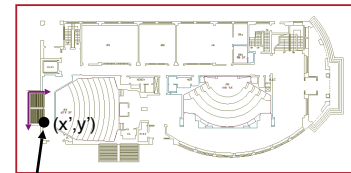


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- Rotation:
  - $x' = x*cos\theta - y*sin\theta$
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$$\begin{matrix} x' = ((x*sx)*cos\theta - (y*sy)*sin\theta) + tx \\ y' = ((x*sx)*sin\theta + (y*sy)*cos\theta) + ty \end{matrix}$$

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## Basic 2D Transformations

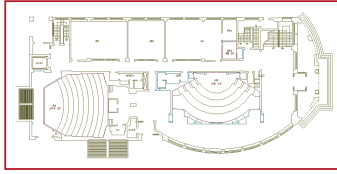


- Translation:

- $x' = x + tx$
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- Scale:

- $x' = x * sx$
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- Shear:

- $x' = x + hx*y$
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$$\begin{aligned} x' &= ((x*sx)*\cos\theta - (y*sy)*\sin\theta) + tx \\ y' &= ((x*sx)*\sin\theta + (y*sy)*\cos\theta) + ty \end{aligned}$$

- Rotation:

- $x' = x*\cos\theta - y*\sin\theta$
- $y' = x*\sin\theta + y*\cos\theta$

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## Overview



- Scene graphs
  - Geometry & attributes
  - Transformations
  - Bounding volumes
- Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
  - 3D transformations

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## Matrix Representation



- Represent 2D transformation by a matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- Multiply matrix by column vector

⇔ apply transformation to point

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad \begin{aligned} x' &= ax + by \\ y' &= cx + dy \end{aligned}$$

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## Matrix Representation



- Transformations combined by multiplication

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} i & j \\ k & l \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Matrices are a convenient and efficient way to represent a sequence of transformations!

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## 2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

### 2D Identity?

$$\begin{aligned} x' &= x \\ y' &= y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

### 2D Scale around (0,0)?

$$\begin{aligned} x' &= sx * x \\ y' &= sy * y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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## 2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

### 2D Rotate around (0,0)?

$$\begin{aligned} x' &= \cos\theta * x - \sin\theta * y \\ y' &= \sin\theta * x + \cos\theta * y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

### 2D Shear?

$$\begin{aligned} x' &= x + shx * y \\ y' &= shy * x + y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & shx \\ shy & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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## 2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

### 2D Mirror over Y axis?

$$\begin{aligned} x' &= -x \\ y' &= y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

### 2D Mirror over (0,0)?

$$\begin{aligned} x' &= -x \\ y' &= -y \end{aligned} \quad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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## 2x2 Matrices



- What types of transformations can be represented with a 2x2 matrix?

### 2D Translation?

$$\begin{aligned} x' &= x + tx \\ y' &= y + ty \end{aligned} \quad \text{NO!}$$

Only linear 2D transformations can be represented with a 2x2 matrix

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## Linear Transformations



- Linear transformations are combinations of ...

- Scale,
- Rotation,
- Shear, and
- Mirror

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Properties of linear transformations:

- Satisfies:  $T(s_1\mathbf{p}_1 + s_2\mathbf{p}_2) = s_1T(\mathbf{p}_1) + s_2T(\mathbf{p}_2)$
- Origin maps to origin
- Lines map to lines
- Parallel lines remain parallel
- Ratios are preserved
- Closed under composition

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## 2D Translation



- 2D translation represented by a 3x3 matrix

- Point represented with *homogeneous coordinates*

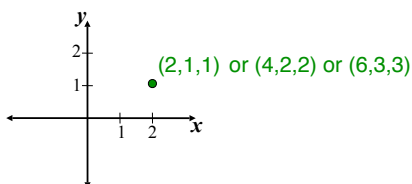
$$\begin{aligned} x' &= x + tx \\ y' &= y + ty \end{aligned} \quad \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

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## Homogeneous Coordinates



- Add a 3rd coordinate to every 2D point
  - $(x, y, w)$  represents a point at location  $(x/w, y/w)$
  - $(x, y, 0)$  represents a point at infinity
  - $(0, 0, 0)$  is not allowed



Convenient coordinate system to represent many useful transformations

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## Basic 2D Transformations



- Basic 2D transformations as 3x3 matrices

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{Translate}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{Scale}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{Rotate}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & shx & 0 \\ shy & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \quad \text{Shear}$$

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## Affine Transformations



- Affine transformations are combinations of ...
  - Linear transformations, and
  - Translations

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of affine transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines remain parallel
  - Ratios are preserved
  - Closed under composition

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## Projective Transformations



- Projective transformations ...
  - Affine transformations, and
  - Projective warps

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

- Properties of projective transformations:
  - Origin does not necessarily map to origin
  - Lines map to lines
  - Parallel lines do not necessarily remain parallel
  - Ratios are not preserved (but "cross-ratios" are)
  - Closed under composition

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## Overview



- Scene graphs
  - Geometry & attributes
  - Transformations
  - Bounding volumes
- Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
  - 3D transformations

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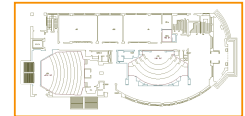
## Matrix Composition



- Transformations can be combined by matrix multiplication

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 \\ \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\mathbf{p}' = \mathbf{T}(tx,ty) \quad \mathbf{R}(\Theta) \quad \mathbf{S}(sx,sy) \quad \mathbf{p}$$



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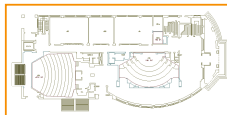
## Matrix Composition



- Matrices are a convenient and efficient way to represent a sequence of transformations
  - General purpose representation
  - Hardware matrix multiply
  - Efficiency with premultiplication
    - Matrix multiplication is associative

$$\mathbf{p}' = (\mathbf{T} * (\mathbf{R} * (\mathbf{S} * \mathbf{p})))$$

$$\mathbf{p}' = (\mathbf{T} * \mathbf{R} * \mathbf{S}) * \mathbf{p}$$



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## Matrix Composition

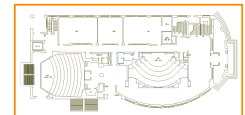


- Be aware: order of transformations matters
  - Matrix multiplication is not commutative

$$\mathbf{p}' = \mathbf{T} * \mathbf{R} * \mathbf{S} * \mathbf{p}$$

←
→

“Global”
“Local”



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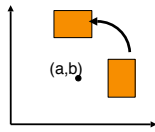
## Matrix Composition



- Rotate by  $\Theta$  around arbitrary point (a,b)
  - $M=T(a,b) * R(\Theta) * T(-a,-b)$

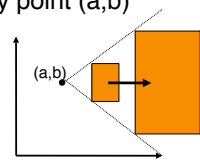
The trick:

First, translate (a,b) to the origin.  
Next, do the rotation about origin.  
Finally, translate back.



- Scale by  $s_x, s_y$  around arbitrary point (a,b)
  - $M=T(a,b) * S(s_x, s_y) * T(-a, -b)$

(Use the same trick.)



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## Overview



- Scene graphs
  - Geometry & attributes
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  - Bounding volumes
- Transformations
  - Basic 2D transformations
  - Matrix representation
  - Matrix composition
  - 3D transformations

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## 3D Transformations



- Same idea as 2D transformations
  - Homogeneous coordinates: (x,y,z,w)
  - 4x4 transformation matrices

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

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## Basic 3D Transformations



$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Identity

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Scale

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Translation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Mirror on Y/Z plane

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## Basic 3D Transformations



Rotate around Z axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \Theta & -\sin \Theta & 0 & 0 \\ \sin \Theta & \cos \Theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around Y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} \cos \Theta & 0 & -\sin \Theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin \Theta & 0 & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

Rotate around X axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \Theta & -\sin \Theta & 0 \\ 0 & \sin \Theta & \cos \Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

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## Summary



- Scene graphs
  - Hierarchical
  - Modeling transformations
  - Bounding volumes
- Coordinate systems
  - World coordinates
  - Modeling coordinates
- 3D modeling transformations
  - Represent most transformations by 4x4 matrices
  - Composite with matrix multiplication (order matters)

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