Elementary Sorts

- rules of the game
- selection sort
- insertion sort
- sorting challenges
- shellsort

Reference: Algorithms in Java, Chapter 6

Sorting problem

Ex. Student record in a University.

file →	Fox	1	A	243-456-9091	101 Brown
	Quilici	1	C	343-987-5642	32 McCosh
	Chen	2	A	884-232-5341	11 Dickinson
	Furia	3	A	766-093-9873	22 Brown
	Kanaga	3	B	898-122-9643	343 Forbes
record →	Andrews	3	A	874-088-1212	121 Whitman
	Rohde	3	A	232-343-5555	115 Holder
	Battle	4	C	991-878-4944	308 Blair
key →	Aaron	4	A	664-480-0023	097 Little
	Gazsi	4	B	665-303-0266	113 Walker

Sort. Rearrange array of N objects into ascending order.

Aaron	4	A	664-480-0023	097 Little
Andrews	3	A	874-088-1212	121 Whitman
Battle	4	C	991-878-4944	308 Blair
Chen	2	A	884-232-5341	11 Dickinson
Fox	1	A	243-456-9091	101 Brown
Furia	3	A	766-093-9873	22 Brown
Gazsi	4	B	665-303-0266	113 Walker
Kanaga	3	B	898-122-9643	343 Forbes
Rohde	3	A	232-343-5555	115 Holder
Quilici	1	C	343-987-5642	32 McCosh

Sample sort client

Goal. Sort any type of data.

Ex 1. Sort random numbers in ascending order.

```
public class Experiment
{
   public static void main(String[] args)
   {
      int N = Integer.parseInt(args[0]);
      Double[] a = new Double[N];
      for (int i = 0; i < N; i++)
         a[i] = StdRandom.uniform();
      Insertion.sort(a);
      for (int i = 0; i < N; i++)
         StdOut.println(a[i]);
   }
}
```

```
% java Experiment 10
0.08614716385210452
0.09054270895414829
0.10708746304898642
0.21166190071646818
0.363292849257276
0.460954145685913
0.5340026311350087
0.7216129793703496
0.9003500354411443
0.9293994908845686
```


Sample sort client

Goal. Sort any type of data.

Ex 2. Sort strings from standard input in alphabetical order.

```
public class StringSort
{
   public static void main(String[] args)
   {
      String[] a = StdIn.readAll().split("\\s+");
      Insertion.sort(a);
      for (int i = 0; i < N; i++)
         StdOut.println(a[i]);
   }
}
```

```
% more words3.txt
bed bug dad dot zoo ... all bad bin

% java StringSort < words.txt
all bad bed bug dad ... yes yet zoo
```

Sample sort client

Goal. Sort any type of data.

Ex 3. Sort the files in a given directory by filename.

```
import java.io.File;
public class Files
{
   public static void main(String[] args)
   {
      File directory = new File(args[0]);
      File[] files = directory.listFiles();
      Insertion.sort(files);
      for (int i = 0; i < files.length; i++)
         StdOut.println(files[i]);
   }
}
```

```
% java Files .
Insertion.class
Insertion.java
InsertionX.class
InsertionX.java
Selection.class
Selection.java
Shell.class
Shell.java
ShellX.class
ShellX.java
```


Callbacks

Goal. Sort any type of data.

Q. How can sort know to compare data of type `String`, `Double`, and `File` without any information about the type of an item?

Callbacks.

- Client passes array of objects to sorting routine.
- Sorting routine calls back object's compare function as needed.

Implementing callbacks.

- Java: interfaces.
- C: function pointers.
- C++: class-type functors.
- ML: first-class functions and functors.

Callbacks: roadmap

client

```
import java.io.File;
public class SortFiles
{
   public static void main(String[] args)
   {
      File directory = new File(args[0]);
      File[] files = directory.listFiles();
      Insertion.sort(files);
      for (int i = 0; i < files.length; i++)
         StdOut.println(files[i]);
   }
}
```

object implementation

```
public class File
implements Comparable<File>
{
   ...
   public int compareTo(File b)
   {
      ...
      return -1;
      ...
      return +1;
      ...
      return 0;
   }
}
```

interface

built in to Java

```
public interface Comparable<Item>
{
   public int compareTo(Item);
}
```

sort implementation

```
public static void sort(Comparable[] a)
{
   int N = a.length;
   for (int i = 0; i < N; i++)
      for (int j = i; j > 0; j--)
         if (a[j].compareTo(a[j-1]))
            exch(a, j, j-1);
         else break;
}
```

Key point: no reference to `File`

Comparable interface API

Comparable interface. Implement `compareTo()` so that `v.compareTo(w)`:

- Returns a negative integer if `v` is less than `w`.
- Returns a positive integer if `v` is greater than `w`.
- Returns zero if `v` is equal to `w`.

```
public interface Comparable<Item>
{
   public int compareTo(Item that);
}
```

Consistency. Implementation must ensure a total order.

- Transitivity: if (a < b) and (b < c), then (a < c).
- Trichotomy: either (a < b) or (b < a) or (a = b).

Built-in comparable types. `String`, `Double`, `Integer`, `Date`, `File`, ...

User-defined comparable types. Implement the `Comparable` interface.

Implementing the Comparable interface: example 1

Date data type. Simplified version of `java.util.Date`.

```
public class Date implements Comparable<Date>
{
   private final int month, day, year;

   public Date(int m, int d, int y)
   {
      month = m;
      day   = d;
      year  = y;
   }

   public int compareTo(Date that)
   {
      if (this.year  < that.year ) return -1;
      if (this.year  > that.year ) return +1;
      if (this.month < that.month) return -1;
      if (this.month > that.month) return +1;
      if (this.day   < that.day  ) return -1;
      if (this.day   > that.day  ) return +1;
      return 0;
   }
}
```

only compare dates to other dates

Implementing the Comparable interface: example 2

Domain names.

- Subdomain: `bolle.cs.princeton.edu`.
- Reverse subdomain: `edu.princeton.cs.bolle`.
- Sort by reverse subdomain to group by category.

```
public class Domain implements Comparable<Domain>
{
   private final String[] fields;
   private final int N;

   public Domain(String name)
   {
      fields = name.split("\\.");
      N = fields.length;
   }

   public int compareTo(Domain that)
   {
      for (int i = 0; i < Math.min(this.N, that.N); i++)
      {
         String s = fields[this.N - i - 1];
         String t = fields[that.N - i - 1];
         int cmp = s.compareTo(t);
         if      (cmp < 0) return -1;
         else if (cmp > 0) return +1;
      }
      return this.N - that.N;
   }
}
```

subdomains

```
ee.princeton.edu
cs.princeton.edu
princeton.edu
cnn.com
google.com
apple.com
www.cs.princeton.edu
bolle.cs.princeton.edu
```

reverse-sorted subdomains

```
com.apple
com.cnn
com.google
edu.princeton
edu.princeton.cs
edu.princeton.cs.bolle
edu.princeton.cs.www
edu.princeton.ee
```


Two useful sorting abstractions

Helper functions. Refer to data through compares and exchanges.

Less. Is object `v` less than `w` ?

```
private static boolean less(Comparable v, Comparable w)
{
   return v.compareTo(w) < 0;
}
```

Exchange. Swap object in array `a[]` at index `i` with the one at index `j`.

```
private static void exch(Comparable[] a, int i, int j)
{
   Comparable t = a[i];
   a[i] = a[j];
   a[j] = t;
}
```


Testing

Q. How to test if an array is sorted?

```
private static boolean isSorted(Comparable[] a)
{
   for (int i = 1; i < a.length; i++)
      if (less(a[i], a[i-1])) return false;
   return true;
}
```

Q. If the sorting algorithm passes the test, did it correctly sort its input?

A1. Not necessarily!

A2. Yes, if data accessed only through `exch()` and `less()`.

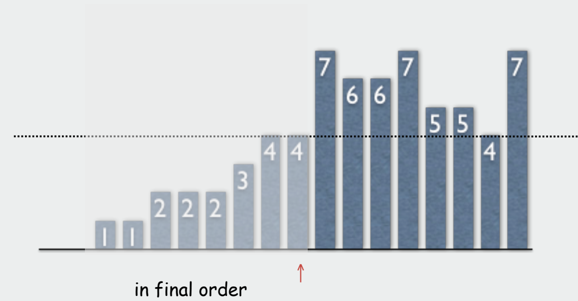
▸ rules of the game
▸ **selection sort**
▸ insertion sort
▸ sorting challenges
▸ shellsort

Selection sort

Algorithm. ↑ scans from left to right.

Invariants.

- Elements to the left of ↑ (including ↑) fixed and in ascending order.
- No element to right of ↑ is smaller than any element to its left.



Selection sort inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```

- Identify index of minimum item on right.

```
int min = i;
for (int j = i+1; j < N; j++)
   if (less(a[j], a[min]))
      min = j;
```

- Exchange into position.

```
exch(a, i, min);
```

Selection sort: Java implementation

```
public class Selection {

   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
      {
         int min = i;
         for (int j = i+1; j < N; j++)
            if (less(a[j], a[min]))
               min = j;
         exch(a, i, min);
      }
   }

   private boolean less(Comparable v, Comparable w)
   {  /* as before */  }

   private boolean exch(Comparable[] a, int i, int j)
   {  /* as before */  }
}
```

Selection sort: mathematical analysis

Proposition A. Selection sort uses $(N-1) + (N-2) + \ldots + 1 + 0 \sim N^2/2$ compares and N exchanges.

i	min	0	1	2	3	4	5	6	7	8	9	10
		S	O	R	T	E	X	A	M	P	L	E
0	6	S	O	R	T	E	X	A	M	P	L	E
1	4	A	O	R	T	E	X	S	M	P	L	E
2	10	A	E	R	T	O	X	S	M	P	L	E
3	9	A	E	E	T	O	X	S	M	P	L	R
4	7	A	E	E	L	O	X	S	M	P	T	R
5	7	A	E	E	L	M	X	S	O	P	T	R
6	8	A	E	E	L	M	O	S	X	P	T	R
7	10	A	E	E	L	M	O	P	X	S	T	R
8	8	A	E	E	L	M	O	P	R	S	T	X
9	9	A	E	E	L	M	O	P	R	S	T	X
10	10	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in black are examined to find the minimum

entries in red are a[min]

entries in gray are in final position

Running time insensitive to input. Quadratic time, even if array is presorted.

Data movement is minimal. Linear number of exchanges.

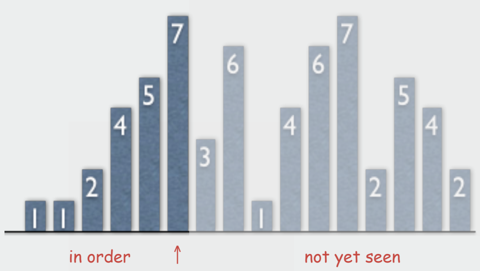
- rules of the game
- selection sort
- **insertion sort**
- sorting challenges
- shellsort

Insertion sort

Algorithm. ↑ scans from left to right.

Invariants.

- Elements to the left of ↑ (including ↑) are in ascending order.
- Elements to the right of ↑ have not yet been seen.



Insertion sort inner loop

To maintain algorithm invariants:

- Move the pointer to the right.

```
i++;
```

- Moving from right to left, exchange `a[i]` with each larger element to its left.

```
for (int j = i; j > 0; j--)
   if (less(a[j], a[j-1]))
        exch(a, j, j-1);
   else break;
```

Insertion sort: Java implementation

```
public class Insertion {

   public static void sort(Comparable[] a)
   {
      int N = a.length;
      for (int i = 0; i < N; i++)
         for (int j = i; j > 0; j--)
            if (less(a[j], a[j-1]))
               exch(a, j, j-1);
            else break;
   }

   private boolean less(Comparable v, Comparable w)
   {  /* as before */  }

   private boolean exch(Comparable[] a, int i, int j)
   {  /* as before */  }
}
```


Insertion sort: mathematical analysis

Proposition B. For randomly-ordered data with distinct keys, insertion sort uses ~ $N^2/4$ compares and $N^2/4$ exchanges on the average.

Pf. For randomly data, we expect each element to move halfway back.

i	j	a[0]	a[1]	a[2]	a[3]	a[4]	a[5]	a[6]	a[7]	a[8]	a[9]	a[10]
		S	O	R	T	E	X	A	M	P	L	E
1	0	O	S	R	T	E	X	A	M	P	L	E
2	1	O	R	S	T	E	X	A	M	P	L	E
3	3	O	R	S	T	E	X	A	M	P	L	E
4	0	E	O	R	S	T	X	A	M	P	L	E
5	5	E	O	R	S	T	X	A	M	P	L	E
6	0	A	E	O	R	S	T	X	M	P	L	E
7	2	A	E	M	O	R	S	T	X	P	L	E
8	4	A	E	M	O	P	R	S	T	X	L	E
9	2	A	E	L	M	O	P	R	S	T	X	E
10	2	A	E	E	L	M	O	P	R	S	T	X
		A	E	E	L	M	O	P	R	S	T	X

entries in gray do not move

entry in red is a[j]

entries in black moved one position right for insertion

Trace of insertion sort (array contents just after each insertion)

Insertion sort: best and worst case

Best case. If the input is in ascending order, insertion sort makes N-1 compares and 0 exchanges.

A E E L M O P R S T X

Worst case. If the input is in descending order (and no duplicates), insertion sort makes ~ $N^2/2$ compares and ~ $N^2/2$ exchanges.

X T S R P O M L E E A

Insertion sort: partially sorted inputs

Def. An inversion is a pair of keys that are out of order.

A E E L M O T R X P S

T-R T-P T-S X-P X-S

(5 inversions)

Def. An array is partially sorted if the number of inversions is O(N).

- Ex 1. A small array appended to a large sorted array.
- Ex 2. An array with only a few elements out of place.

Proposition C. For partially-sorted arrays, insertion sort runs in linear time.

Pf. Number of compares equals the number of inversions.

- rules of the game
- selection sort
- insertion sort
- sorting challenges
- shellsort

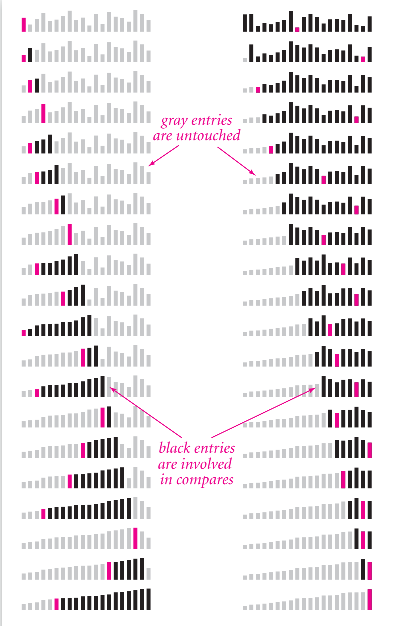
Sorting challenge 0

Input. Array of doubles.

Plot. Data proportional to length.

Name the sorting method.

- Insertion sort.
- Selection sort.



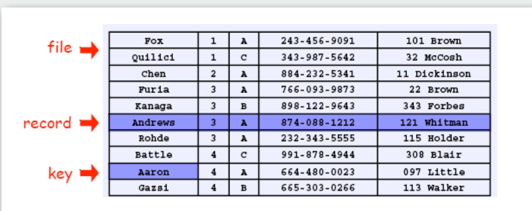
Sorting challenge 1

Problem. Sort a file of huge records with tiny keys.

Ex. Reorganize your MP3 files.

Which sorting method to use?

- System sort.
- Insertion sort.
- Selection sort.



Fox	1	A	243-456-9091	101 Brown
Quilici	1	C	343-987-5642	32 McCosh
Chen	2	A	884-232-5341	11 Dickinson
Furia	3	A	766-093-9873	22 Brown
Kanaga	3	B	898-122-9643	343 Forbes
Andrews	3	A	874-088-1212	121 Whitman
Rohde	3	A	232-343-5555	115 Holder
Battle	4	C	991-878-4944	308 Blair
Aaron	4	A	664-480-0023	097 Little
Gazsi	4	B	665-303-0266	113 Walker

Sorting challenge 1

Problem. Sort a file of huge records with tiny keys.

Ex. Reorganize your MP3 files.

Which sorting method to use?

- System sort. probably no, selection sort simpler and faster
- Insertion sort. no, too many exchanges
- Selection sort. yes, linear time under reasonable assumptions

Ex: 5,000 records, each 2 million bytes with 100-byte keys.

- Cost of comparisons: $100 \times 5000^2 / 2 = 1.25$ billion.
- Cost of exchanges: $2{,}000{,}000 \times 5{,}000 = 10$ trillion.
- System sort might be a factor of log (5000) slower.

Sorting challenge 2

Problem. Sort a huge randomly-ordered file of small records.
Ex. Process transaction records for a phone company.

Which sorting method to use?

- System sort.
- Insertion sort.
- Selection sort.

file →	Fox	1	A	243-456-9091	101 Brown
	Quilici	1	C	343-987-5642	32 McCosh
	Chen	2	A	884-232-5341	11 Dickinson
	Furia	3	A	766-093-9873	22 Brown
	Kanaga	3	B	898-122-9643	343 Forbes
record →	Andrews	3	A	874-088-1212	121 Whitman
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	Battle	4	C	991-878-4944	308 Blair
key →	Aaron	4	A	664-480-0023	097 Little
	Gazsi	4	B	665-303-0266	113 Walker

Sorting challenge 2

Problem. Sort a huge randomly-ordered file of small records.
Ex. Process transaction records for a phone company.

Which sorting method to use?

- System sort. yes, it's designed for this problem
- Insertion sort. no, quadratic time for randomly ordered files
- Selection sort. no, always quadratic time

Sorting challenge 3

Problem. Sort a huge number of tiny files (each file is independent)
Ex. Daily customer transaction records.

Which sorting method to use?

- System sort.
- Insertion sort.
- Selection sort.

file →	Fox	1	A	243-456-9091	101 Brown
	Quilici	1	C	343-987-5642	32 McCosh
	Chen	2	A	884-232-5341	11 Dickinson
	Furia	3	A	766-093-9873	22 Brown
	Kanaga	3	B	898-122-9643	343 Forbes
record →	Andrews	3	A	874-088-1212	121 Whitman
	Rohde	3	A	232-343-5555	115 Holder
	Battle	4	C	991-878-4944	308 Blair
key →	Aaron	4	A	664-480-0023	097 Little
	Gazsi	4	B	665-303-0266	113 Walker

Sorting challenge 3

Problem. Sort a huge number of tiny files (each file is independent)
Ex. Daily customer transaction records.

Which sorting method to use?

- System sort. no, too much overhead
- Insertion sort. yes, less overhead than system sort
- Selection sort. yes, less overhead than system sort

Ex: 4 record file.

- $4 N \log N + 35 = 70$
- $2N^2 = 32$

Sorting challenge 4

Problem. Sort a huge file that is already almost in order.
Ex. Resort a huge database after a few changes.

Which sorting method to use?
- System sort.
- Insertion sort.
- Selection sort.

file → record → key →

Fox	1	A	243-456-9091	101 Brown
Quilici	1	C	343-987-5642	32 McCosh
Chen	2	A	884-232-5341	11 Dickinson
Furia	3	A	766-093-9873	22 Brown
Kanaga	3	B	898-122-9643	343 Forbes
Andrews	3	A	874-088-1212	121 Whitman
Rohde	3	A	232-343-5555	115 Holder
Battle	4	C	991-878-4944	308 Blair
Aaron	4	A	664-480-0023	097 Little
Gazsi	4	B	665-303-0266	113 Walker

Sorting challenge 4

Problem. Sort a huge file that is already almost in order.
Ex. Resort a huge database after a few changes.

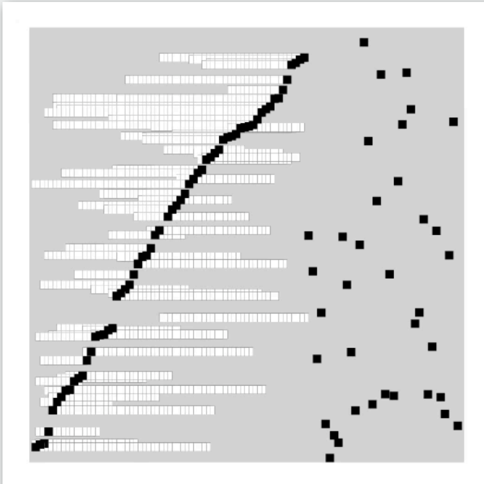
Which sorting method to use?
- System sort. no, insertion sort simpler and faster
- Insertion sort. yes, linear time for most definitions of "in order"
- Selection sort. no, always takes quadratic time

Ex.
- A B C D E F H I J G P K L M N O Q R S T U V W X Y Z
- Z A B C D E F G H I J K L M N O P Q R S T U V W X Y

- rules of the game
- selection sort
- insertion sort
- animations
- **shellsort**

Insertion sort animation

Insertion sort animation

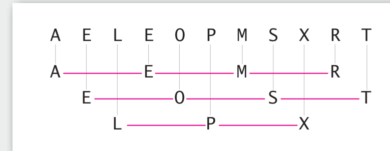


Reason it is slow: excessive data movement.

Shellsort overview

Idea. Move elements more than one position at a time by h-sorting the file.

a 3-sorted file is 3 interleaved sorted files



Shellsort. h-sort the file for a decreasing sequence of values of h.

input	S	O	R	T	E	X	A	M	P	L	E
7-sort	M	O	L	E	E	X	A	S	P	R	T
3-sort	A	E	L	E	O	P	M	S	X	R	T
1-sort	A	E	E	L	M	O	P	R	S	T	X

Shellsort trace (array contents after each pass)

h-sorting

How to h-sort a file? Insertion sort, with stride length h.

3-sorting a file

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

Why insertion sort?

- Big increments ⇒ small subfiles.
- Small increments ⇒ nearly in order. [stay tuned]

Shellsort example

input

S	O	R	T	E	X	A	M	P	L	E

7-sort

S	O	R	T	E	X	A	M	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	L	T	E	X	A	S	P	R	E
M	O	L	E	E	X	A	S	P	R	T

3-sort

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

1-sort

A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	E	L	O	P	M	S	X	R	T
A	E	E	L	O	P	M	S	X	R	T
A	E	E	L	O	P	M	S	X	R	T
A	E	E	L	M	O	P	S	X	R	T
A	E	E	L	M	O	P	S	X	R	T
A	E	E	L	M	O	P	S	X	R	T
A	E	E	L	M	O	P	R	S	X	T
A	E	E	L	M	O	P	R	S	T	X

result

A	E	E	L	M	O	P	R	S	T	X

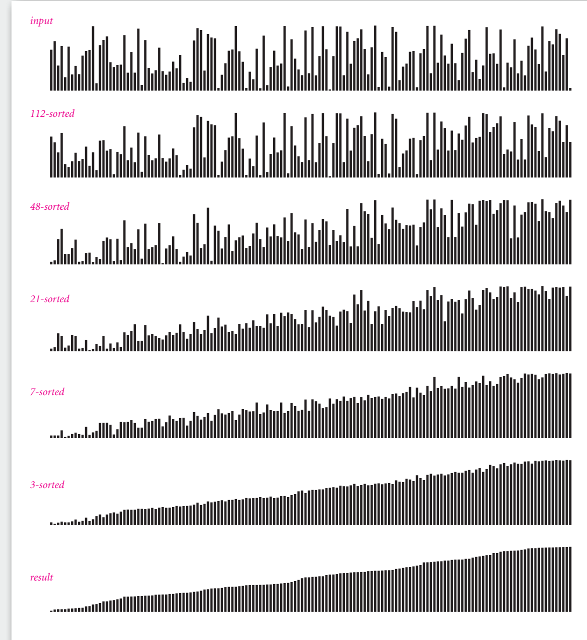
Shellsort: Java implementation

```
public class Shell
{
   public static void sort(Comparable[] a)
   {
      int N = a.length;
      int[] incs = { 1391376, 463792, 198768, 86961,
                     33936, 13776, 4592, 1968, 861,
                     336, 112, 48, 21, 7, 3, 1
                   };
      for (int k = 0; k < incs.length; k++)
      {
         int h = incs[k];
         for (int i = h; i < N; i++)
            for (int j = i; j >= h; j-= h)
               if (less(a[j], a[j-h]))
                  exch(a, j, j-h);
               else break;
      }
   }

   private boolean less(Comparable v, Comparable w)
   {  /* as before */  }

   private boolean exch(Comparable[] a, int i, int j)
   {  /* as before */  }
}
```

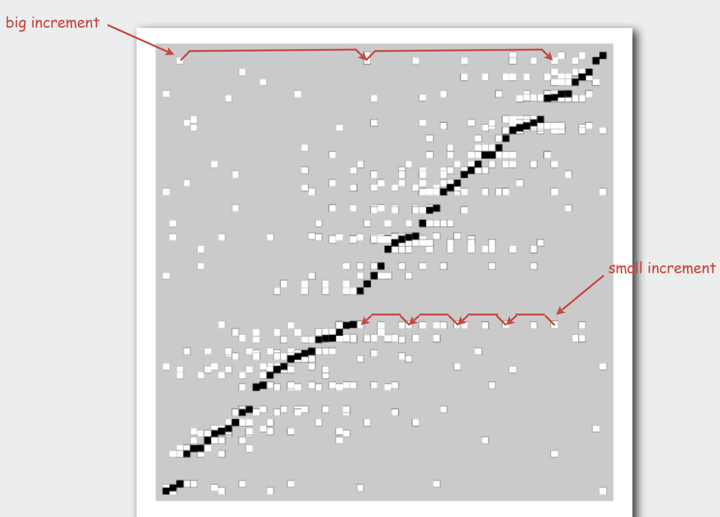
magic increment sequence

insertion sort

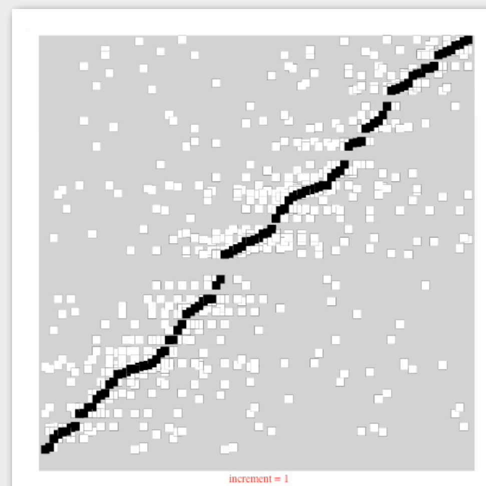
Visual trace of shellsort



Shellsort animation



Shellsort animation



Bottom line: substantially faster than insertion sort!

Empirical analysis of shellsort

Property. The number of compares used by shellsort with the increments 1, 4, 13, 40, ... is at most by a small multiple of N times the # of increments used.

N	comparisons	$N^{1.289}$	2.5 N lg N
5,000	93	58	106
10,000	209	143	230
20,000	467	349	495
40,000	1022	855	1059
80,000	2266	2089	2257

measured in thousands

Remark. Accurate model has not yet been discovered (!)

Shellsort: mathematical analysis

Proposition. A g-sorted array remains g-sorted after h-sorting it.

Pf. Harder than you'd think!

7-sort

M	O	R	T	E	X	A	S	P	L	E
M	O	R	T	E	X	A	S	P	L	E
M	O	L	T	E	X	A	S	P	R	E
M	O	L	E	E	X	A	S	P	R	T
M	O	L	E	E	X	A	S	P	R	T

3-sort

M	O	L	E	E	X	A	S	P	R	T
E	O	L	M	E	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
E	E	L	M	O	X	A	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	X	M	S	P	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T
A	E	L	E	O	P	M	S	X	R	T

still 7-sorted

Proposition. The worst-case number of compares for shellsort using the 3x+1 increment sequence 1, 4, 13, 40, 121, 364, … is $O(N^{3/2})$.

Why are we interested in shellsort?

Example of simple idea leading to substantial performance gains.

Useful in practice.

- Fast unless file size is huge.
- Tiny, fixed footprint for code (used in embedded systems).
- Hardware sort prototype.

Simple algorithm, nontrivial performance, interesting questions

- Asymptotic growth rate?
- Best sequence of increments?
- Average case performance?

open problem: find a better increment sequence

Lesson. Some good algorithms are still waiting discovery.