# Clustering and the $k$-means Algorithm 

David M. Blei<br>COS424<br>Princeton University

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- MySpace users according to interests
- A museum catalog according to image similarity


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- Define a distance function between data, $d\left(\mathbf{x}_{n}, \mathbf{x}_{m}\right)$.
- Goal: segment the data into $k$ groups

$$
\left\{z_{1}, \ldots, z_{N}\right\} \quad \text { where } \quad z_{i} \in\{1, \ldots, K\} .
$$

## Example data



500 2-dimensional data points: $\mathbf{x}_{n}=\left\langle x_{n, 1}, x_{n, 2}\right\rangle$

## Example data



- What is a good distance function here?


## Example data


-What is a good distance function here?

- Squared Euclidean distance is reasonable

$$
d\left(\mathbf{x}_{n}, \mathbf{x}_{m}\right)=\sum_{i=1}^{p}\left(x_{n, i}-x_{m, i}\right)^{2}=\left\|x_{n}-x_{m}\right\|^{2}
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- Goal: segment this data into $k$ groups.
- What should $k$ be?
- Automatically choosing $k$ is complicated; for now, 4.


## $k$-means



- Different clustering algorithms use the data and distance measurements in different ways


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- The goal of $k$-means is to assign data to clusters and deine these clusters with their means.


## $k$-means algorithm

(1) Initialization

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(3) Until assignments $\mathbf{z}_{1: N}$ do not change

## $k$-means example



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## Objective function

- How can we measure how well our algorithm is doing?


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- The $k$-means objective function is the sum of the squared distances of each point to each assigned mean

$$
F\left(z_{1: N}, \mathbf{m}_{1: k}\right)=\frac{1}{2} \sum_{n=1}^{N}\left\|\mathbf{x}_{n}-\mathbf{m}_{z_{n}}\right\|^{2}
$$

## $k$-means example (look at the objective)



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- Holding the assignments fixed, computing the centroids of each cluster minimizes $F$ with respect to $\mathbf{m}_{1: k}$.
- Thus, $k$-means is a coordinate descent algorithm.
- It finds a local minimum. (Multiple restarts are often necessary.)


## Objective for the example data



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- Each pixel is associated with a red, green, and blue value
- A $1024 \times 1024$ image is a collection of 1048576 values $\left\langle x_{1}, x_{2}, x_{3}\right\rangle$, which requires 3 M of storage
- How can we use $k$-means to compress this image?


## Vector quantization


$\square$

- Replace each pixel $\mathbf{x}_{n}$ with its assignment $\mathbf{m}_{z_{n}}$ ("paint by numbers").


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- The $k$ means are called the codebook.
- With $k=100$, we need 7 bits per pixel plus $100 \times 3$ bits $\approx 897 \mathrm{~K}$.


## Charlie Brown and Linus VQ



2 means

## Charlie Brown and Linus VQ



4 means

## Charlie Brown and Linus VQ



8 means

## Charlie Brown and Linus VQ



16 means

## Charlie Brown and Linus VQ



32 means

## Charlie Brown and Linus VQ



64 means

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128 means

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256 means

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- For more clusters, the picture is less distorted.


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- No need to define the mean.
- Each of the clusters is associated with its most typical example


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- Clustering customers for $k$ salespeople in a business
- Usually, we seek the "natural" clustering, but what does this mean?
- It is not well-defined.


## What happens as $k$ increases?



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- This suggests that 4 is the right number of clusters.
- Tibshirani (2001) presents a method for finding this kink.


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- Choose $k$ very carefully, with a complicated computational technique.



## Computational Biology

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- Clustered genes based on their response in different tissues
- (No mention of how $k=23$ was chosen.)

D. Blei


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- I.e., the levels of encouragement are corrected for
- Chose the number of clusters to get nice results

Table 3. Five-Cluster Solution: $Z$ scores on Each Clustering Variable

|  | Cluster 1 | Cluster 2 | Cluster 3 | Cluster 4 | Cluster 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Teacher caring | -.5 | -.5 to .5 | -.5 to .5 | -.5 | 1.0 |
| Peers' academic support | 1.0 | -.5 | 1.0 | -.5 | -.5 to .5 |
| Parents' academic support | .5 | -1.0 | -.5 to .5 | -.5 to .5 | 1.0 |

Table 4. Means and Standard Deviations for Each Cluster on Grade 8 Motivational Variables

| Cluster | Academic Self-Efficacy |  | Intrinsic <br> Valuing of Education |  | Teacher-Rated Effort |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | M | SD | M | SD | M | SD |
| 1. All positive | 3.59 | . $48{ }^{\text {a }}$ | 2.99 | . $55^{\text {a }}$ | 3.74 | . $6^{\text {a }}$ |
| 2. Peer negative, parents very negative | 2.44 | . $66^{\text {b }}$ | 2.16 | . $51{ }^{\text {b }}$ | 3.05 | . $61{ }^{\text {b }}$ |
| 3. Peer positive | 3.01 | .73 ${ }^{\text {c }}$ | 2.43 | . $66^{\text {b }}$ | 3.26 | . $66^{\text {b }}$ |
| 4. Negative teacher and peer | 2.47 | . $63{ }^{\text {b }}$ | 2.24 | . $51{ }^{\text {b }}$ | 3.17 | . $59{ }^{\text {b }}$ |
| 5. Positive teacher and parents | 3.19 | . $65{ }^{\text {c }}$ | 2.89 | . $62^{\text {a }}$ | 3.54 | . $47^{\text {a }}$ |

D. Blei Clustering 01

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- Draw the conclusion that patterns exist. What's wrong with this?


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- Draw the conclusion that patterns exist. What's wrong with this?
- $k$-means will find patterns everywhere!


## TABLE 2. Percentage distribution of participants, by cluster, and behavioral patterns defining each cluster

Cluster type and behavioral patterns \%

Light substance dabblers-infrequent or no current use of substancest
None have had sex
Abstainers-none have ever used substancest or had sex
Sex dabblers-all have had sex
Median no. of partners=1
60\% used a condom at last sex
Infrequent use of substances $\dagger$
Drinkers-all consumed alcohol in past 12 mos.
49\% report binge drinking
Infrequent or no illicit drug use
None have had sex
Smokers-all smoke cigarettes daily
Infrequent use of alcohol/illicit drugs
$62 \%$ have had sex
Alcohol-and-sex dabblers-all drink occasionally; all have had sex Infrequent tobaccorillicit drug use

Binge drinkers-all binge frequently Infrequent cigarette, marijuana and other drug use $60 \%$ binge $\geq 1$ time/wk
45\% have had sex
Heavy dabblers-all smoke, drink and binge drink with moderate frequency

## Combination sex and drug use-all have had sex; all used alcohol/ilicit drug at last sex

74\% have had sex

## Multiple partners-all report $\geq 14$ sexual partners

Sex for drugs or money-all have had sex for drugs or money
$50 \%$ report low or moderate use of substancest
Median no. of partners=3
High marijuana use and sex-all use marijuana frequently; all have had sex
All used alcohol/other drug at last se
$82 \%$ have had $>1$ partner (median $=6$ )
Marijuana and other drug users-95\% report heavy marijuana use; all use other illicit drugs $68 \%$ have had sex
$28 \%$ used alcohol/other drug at last sex
Injection-drug users-all have injected drugs
$82 \%$ have had sex
Median no. of partners=4
Males who have sex with males-all are males who have had sex with another male 0.3
$78 \%$ have had multiple partners (median=5)
$40 \%$ used marijuana in past 30 days
$50 \%$ use alcohol $\geq 1$ time/mo.
$17 \%$ have had sex for drugs or money

## Summary

