

COS 116
The Computational Universe
Homework 3

Against logic there is no armor like ignorance.

Laurence J. Peter (1919 - 1988)

Logic merely enables one to be wrong with authority.

Doctor Who

Due Thursday, March 30, in lecture.

Q1) Describe the effect of running the following Turing-Post program when *00010101* is placed all the way to the left of the tape, and the machine is started on top of the first *0* on the left. Your answer should mimic Figure 2 on page 248 of the Martin Davis article. Also, describe in words what this Turing-Post program does.

1. GO TO STEP 3 IF 0 SCANNED
2. GO TO STEP 5 IF 1 SCANNED
3. GO RIGHT
4. GOTO STEP 1 IF 0 SCANNED
5. STOP

Q2) Modify the Turing-Post program in Figure 1 of the Davis article so that it *triples* the number of 1's rather than doubling it. (Keep in mind that there is a typo in that Figure; the correct program was given in the lecture slides.) Hand in both the program and its binary code (see p. 251 of Davis article).

Q3) Consider the following facts:

*Matt will go to the party if John and Brian go. Brian will go if
Karen goes or Sue doesn't go. Sue will go if John doesn't. Karen will go if
Sue does.*

Represent whether or not Matt will go to the party as a truth table, boolean circuit, and boolean expression. Simplify your boolean expression as much as you can.

Q4) What problem can arise when combinational circuits are allowed to have cycles?

Q5) In class we saw that every Boolean function can be computed using a Boolean circuit that consists only of AND, OR and NOT gates. Briefly argue that in fact a single type of gate suffices to build all Boolean circuits: the NAND gate. This is a gate with two inputs and has the following truth table:

<i>A</i>	<i>B</i>	<i>A NAND B</i>
0	0	1
0	1	1
1	0	1
1	1	0

Q6 (From Ray Smullyan) On the last of the one thousand and one nights, the Sultan of Arabia was going to finally send Scheherazade to her death. As her final wish, she begged him to answer one question with a Yes or No. The Sultan agreed, saying "I always give a straight Yes/No answer, and always stand by what I say." The wily Scheherazade — well-versed in Boolean logic — then asked a tricky question. The Sultan pondered his possible answers, and realized that no matter how he answered, he would be forced to spare Scheherazade's life.

Your task is to figure out Scheherazade's question. Hint: It consists of asking whether or not a certain boolean expression is true. It has two variables: *K* (which is 1 if and only if the King answers "yes" to the entire question) and *S* (which is 1 if and only if the King decides to spare Scheherazade).