The Strategic Advantage of Negatively Interdependent Preferences*

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Abstract

We study certain classes of supermodular and submodular games which are symmetric with respect to material payoffs but in which not all players seek to maximize their material payoffs. Specifically, a subset of players have negatively interdependent preferences and care not only about their own material payoffs but also about their payoffs relative to others. We identify sufficient conditions under which members of the latter group have a strategic advantage in the following sense: at all intragroup symmetric equilibria of the game, they earn strictly higher material payoffs than do players who seek to maximize their material payoffs. These conditions are satisfied by a number of games of economic importance. We discuss the implications of these findings for the evolutionary theory of preference formation and the theory of strategic delegation.

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1 Introduction

A fundamental ingredient of most economic models is the hypothesis of independent preferences: agents choose their actions with the sole purpose of maximizing their own material payoffs regardless of how their actions affect the payoffs of other individuals. While this postulate is seldom given explicit justification, it appears to be based on the intuition that those individuals who are willing to make material sacrifices to affect the payoffs of others will lose wealth relative to those who are unwilling to do so, with the eventual consequence that the latter will come to dominate the economy. In this case, the maximization of one's own material payoffs would simply be a precondition for survival in an environment where a competitive selection process is at work. While this intuition may be persuasive in the context of perfectly competitive environments, it can be seriously misleading when applied to strategic settings, for it is *not* generally true in such environments that agents who pursue the maximization of their own material payoffs will obtain higher material payoffs in equilibrium than symmetrically placed individuals who maximize other objective functions.

This last point has been demonstrated in the literature mostly by means of particular specifications of Cournot oligopoly models. For instance, Vickers (1984) and Fershtman and Judd (1987) have shown in linear versions of such models that a firm whose objective function gives a positive weight to its relative profits or sales will outperform absolute profit maximizers in terms of *absolute* profits. Similar results obtain in some other strategic environments, such as common pool resource and public good games (Koçkesen et al., 1999), in which agents with *negatively interdependent* preferences (that is, those who care about *both* absolute *and* relative payoffs) may obtain greater absolute payoffs in equilibrium than do symmetrically placed absolute payoff maximizers. In such environments, interdependent preferences may be said to yield a *strategic advantage* to those who possess them.

The broader significance of such findings rests on the extent to which they are valid in a large set of economic environments. Accordingly, our aim in this paper is to provide a general analysis of the conditions under which negatively interdependent preferences yield an unambiguous strategic advantage over independent preferences. We consider classes of supermodular and submodular games in which only a subset of players have independent objective functions whereas the rest have negatively interdependent preferences. Informally stated, we identify several sets of sufficient conditions under which the members of the latter group have a strategic advantage in the following sense: at all intragroup symmetric equilibria, the interdependent individuals earn higher *material* payoffs than do players who seek to maximize their own material payoffs. It turns out that the class of games in which interdependent preferences have a strategic advantage in this sense is unexpectedly rich.

To motivate the present inquiry from a purely game-theoretic viewpoint, consider a two-person game in which player 1 wishes to maximize his monetary reward while player 2 cares also about 'beating' player 1, that is, making more money than player 1. The question we ask is: which of the two players will make more money in equilibrium? It is not difficult to see that the answer is not trivial, and depends on the structure of the game. Using the familiar notions of strategic complementarity and substitutability, we show in this paper that in many (but not all) games of interest, it is, in fact, player 2 who will outperform player 1 in equilibrium. Put differently, we identify subclasses of supermodular and submodular normal form games in which an envious concern with the payoffs of others leads one to have greater absolute payoffs in equilibrium than those obtained by (absolute) payoff maximizers. We contend that our results achieve a useful level of generality, for our sufficiency conditions are satisfied by a number of games which play central roles in various branches of economic theory, including strategic market games, search models, input and public good games, and arms races.

The analysis of strategic advantage also leads to interesting applications two of which we study here in some detail. Our first application concerns the analysis of oligopolistic industries, and stems from the fact that executive managers may in some circumstances either choose or be given incentives by owners to incorporate relative profit (or market share) concerns into their decision making. For instance, if managers' efforts are unobservable by owners and there is some common uncertainty affecting all firms in the industry, owners may benefit from making their managers' compensation contingent upon relative as well as absolute profits (Holmström, 1982). As a corollary of our results, we find here that such compensation schemes may yield an unplanned strategic advantage to a firm in terms of its absolute profits. This occurs in supermodular market games including the standard Bertrand model with differentiated products, and submodular games including the standard case of Cournot competition. In the submodular case, moreover, our results allow us to show that objective functions which value relative profits may arise as perfect equilibrium outcomes in two-stage delegation games in which owners select the objective functions of their managers in the first stage. This finding conforms with, and extends to a class of submodular games, the results obtained in the theory of strategic delegation by Vickers (1984) and Fershtman and Judd (1987). It is a useful illustration of the power of the strategic advantage approach that we develop here.

As a second application, we consider the theory of preference evolution. Evolutionary models of preference formation are typically based on the assumption that the selection dynamics are payoff monotonic: the population share of those endowed with preferences that are more highly rewarded materially increases relative to the population share of those who are less highly rewarded. In the presence of such selection dynamics, our results enable us to identify the evolutionary stability properties of absolute payoff maximizing behavior in a class of environments in which a finite population interacts strategically, with each player matched with each other player (the "playing the field" model.) Specifically, we identify environments in which the long run population composition cannot be a monomorphic one composed *only* of absolute payoff maximizers. These findings are then compared with the earlier evolutionary literature on the "spiteful effect," which reaches broadly similar conclusions. The idea behind the spiteful effect is conceptually close to the idea of strategic advantage, and it turns out that the presence of the spiteful effect is necessary but not sufficient for the strategic advantage of negatively interdependent preferences. Since our sufficient conditions are based on the primitives of a game (in contrast with the spiteful effect), they serve also as a means of identifying games in which the spiteful effect is present.

The paper is organized as follows. In Section 2 we introduce our general framework and formalize the nature of the present inquiry. Section 3 contains our main results which identify classes of supermodular and submodular games in which interdependent players have a strategic advantage over independent players at all intragroup symmetric equilibria. In Section 4, we elaborate on the implications of our main findings for the theories of oligopolistic competition and preference formation. Section 5 concludes.

2 The Framework

Since our ultimate aim is to compare the performance of different preference structures in terms of material outcomes, we shall concentrate on games in strategic form in which no player has an *a priori* advantage in terms of the primitives of the game. Consequently, our focus will be on *symmetric* games. Given any integer $n \geqslant 2$, we let Γ stand for a symmetric n-person game in normal form. That is,

$$\Gamma \equiv (X, \{\pi_r\}_{r=1,\dots,n})$$

where X and $\pi_r : X^n \to \mathbf{R}$ are the action space and the *absolute* payoff function of player r, and where $\pi_r(x) = \pi_q(x')$ for all $r, q = 1, ..., n$ and all $x, x' \in X^n$ such that x' is obtained from x by exchanging x_r and x_q. As is usually done in applied and experimental game theory, we interpret π_r as the material payoff function of player r. To be able to interpret the notion of "relative payoffs" in the usual manner, we assume that Γ satisfies the following nonnegativity conditions:

$$\pi_r(x) \geqslant 0 \quad \forall x \in X^n \text{ and } r = 1, ..., n. \tag{1}$$

Moreover, we endow X with a *linear order* $\succsim$ to obtain a *chain.*[1] The class of all Γ that satisfy these assumptions is denoted $\mathcal{G}$, and we let $N(\Gamma)$ stand for the set of all Nash equilibria of $\Gamma \in \mathcal{G}$.

[1] This is not an excessively demanding structural assumption insofar as applications are concerned. In many economic contexts one has $X \subseteq \mathbf{R}$ so that X is linearly ordered in a natural way.

For much of this paper we shall assume that the set of players consists of two different types, namely, independent and (negatively) interdependent types. The *independent* players are those who are absolute payoff maximizers in the usual sense; the objective function of an independent player i is precisely her own material payoff function π_i. On the other hand, (*negatively*) *interdependent* players are concerned not only with their absolute payoffs, but also with how their absolute payoffs compare with the average payoff in the game. Let $\bar{\pi} \equiv \frac{1}{n} \sum \pi_r$ denote the average (absolute) payoff function on X^n, and define the *relative payoff* of player j as follows:[2]

$$\rho_j = \begin{cases} \pi_j/\bar{\pi}, & \text{if } \pi_j > 0 \\ 0, & \text{if } \pi_j = 0. \end{cases} \tag{2}$$

The objective function of an interdependent player j is given by $x \mapsto F(\pi_j, \rho_j)$ where F is an arbitrary strictly increasing real function on $\mathbf{R}^2_+$. This particular way of representing negatively interdependent preferences has recently been proposed and axiomatically characterized by Ok and Koçkesen (1997). In particular, when Γ is played between individuals (as opposed to, say, firms), the preferences represented in this form can be interpreted as a compromise between the standard case in which individuals are assumed to care only about her monetary earnings π_j, and the other extreme in which they are concerned exclusively with their *relative payoff* ρ_j.[3] If, on the other hand, Γ is an oligopoly game, then an interdependent player with such an objective function can be thought of as a firm (or manager) which is not only concerned with the level of its profits, but cares also about its profit share in the industry.

Suppose that precisely $k \in \{1, ..., n-1\}$ players in $\Gamma \in \mathcal{G}$ are independent, and denote the set of all strictly increasing $F : \mathbf{R}^2_+ \to \mathbf{R}$ by $\mathbb{F}$. For any $F \in \mathbb{F}$, we define the n-person normal form game

$$\Gamma_F(k) \equiv (X, \{p_r\}_{r=1,...,n})$$

with

$$p_r \equiv \begin{cases} \pi_r, & \text{if } r \in I_k \\ F(\pi_r, \rho_r), & \text{if } r \in J_k \end{cases} \tag{3}$$

where $I_k \equiv \{1, ..., k\}$ and $J_k \equiv \{k+1, ..., n\}$. Clearly, in $\Gamma_F(k)$, the set of all *independent* players is I_k, and the set of all *interdependent* players is J_k. The crucial interpretation is that, while an uninformed outsider may only observe the payoffs associated with the game Γ, the players themselves are engaged in playing $\Gamma_F(k)$.

[2] We use the convention of setting $\rho_j(x) = 0$ whenever $\pi_j(x) = 0$ to avoid the difficulty of evaluating the indeterminate form $0/0$.

[3] Special cases of this representation of interdependent preferences are utilized in numerous economic contexts ranging from models of optimal income taxation to experimental bargaining games. We refer the reader to the references cited in Ok and Koçkesen (1997).

In this paper, we wish to analyze the nature of pure strategy Nash equilibria of an arbitrary $\Gamma_F(k)$. We note at the outset that there are two immediate difficulties which shall be assumed away in much of the analysis that follows. First, the existence of a Nash equilibrium of $\Gamma_F(k)$ is rather difficult to establish in general. Even if we take $X \subseteq \mathbf{R}^\ell$ and posit the standard requirement of quasiconcavity of π_r in x_r for all r (along with continuity of π_r, and compactness and convexity of X), the payoff function p_j, $j \in J_k$, need not inherit this property. Even the deeper existence theorems established in the literature (such as those of Topkis, 1979 and Dasgupta and Maskin, 1986) are not readily helpful in settling this existence problem. It appears that the best strategy at this stage is to ignore the existence problem, and search for some qualitative properties of the equilibria of $\Gamma_F(k)$, *when they exist*. In fact, in many examples of economic interest (such as the Cournot and Bertrand oligopolies, common pool resource and public good games, arms races, etc.) one can directly verify that the set of equilibria of $\Gamma_F(k)$ is nonempty, and hence our approach is fruitful.

The second difficulty is the analytical intractability of certain asymmetric equilibria of an arbitrary $\Gamma_F(k)$. The analysis is greatly simplified when we focus instead on the ***intragroup symmetric Nash equilibria*** of $\Gamma_F(k)$, denoted $N_{\text{sym}}(\Gamma_F(k))$, which is defined as

$$N_{\text{sym}}(\Gamma_F(k)) \equiv \{([a]_k, [b]_{n-k}) \in N(\Gamma_F(k)) : a, b \in X\}$$

where $[t]_l$ denotes the $l-$replication of the object t. One could, of course, advance a "focal point" argument to justify interest in $N_{\text{sym}}(\Gamma_F(k))$. Perhaps more importantly, we shall observe that in most of the economic examples considered below, we actually have $N(\Gamma_F(k)) = N_{\text{sym}}(\Gamma_F(k))$ so a focus on intragroup symmetric equilibria is unrestrictive. This is trivially the case in all two person games.

Finally, let us clarify what we mean by "studying the nature of $N_{\text{sym}}(\Gamma_F(k))$" given a $\Gamma \in \mathcal{G}$. Put precisely, we are interested in identifying some general subclasses of $\mathcal{G}$ where interdependent players have a *strategic advantage* over the independent players in terms of monetary payoffs, that is, where

$$\pi_j(\hat{x}) > \pi_i(\hat{x}) \quad \forall (i,j) \in I_k \times J_k \text{ and } \hat{x} \in N_{\text{sym}}(\Gamma_F(k)). \tag{4}$$

As noted in the introduction, there are at least two concrete economic considerations motivating this inquiry. Whether or not interdependent players (who do not directly maximize their absolute payoffs) obtain higher absolute payoffs than all independent players (who do target the maximization of their absolute payoffs) is a question of considerable interest in theories of preference evolution and strategic delegation. These applications are discussed in Section 4 below.

3 Main Results

3.1 Supermodular Games

An n-person normal form game $\Gamma \in \mathcal{G}$ is said to be ***supermodular*** whenever

$$\pi_r(x \vee y) + \pi_r(x \wedge y) \geqslant \pi_r(x) + \pi_r(y) \quad \forall x, y \in X^n \text{ and } r = 1, ..., n,$$

where $x \vee y$ is the lowest upper bound of $\{x, y\}$ in X^n (with respect to the product order induced by $\succsim$) and $x \wedge y$ is the greatest lower bound of $\{x, y\}$ in X^n.[4] We say that Γ is ***strictly supermodular*** if the above inequality holds strictly for all r and $x, y \in X^n$ such that $\{x \vee y, x \wedge y\} \neq \{x, y\}$. Supermodular games correspond to games in which the actions of two distinct players are *strategic complements* in the sense that the best response correspondences of the players are increasing (Bulow et al., 1985, Topkis, 1979, Vives, 1990). It is well known that if $X \subseteq \mathbf{R}^\ell$ is open and π_r is C^2, then Γ is supermodular if and only if $\partial^2 \pi_r / \partial x_r \partial x_q \geqslant 0$ for all $r \neq q$ (Topkis, 1978).

We next introduce the following subclass of $\mathcal{G}$.

Definition. *An n-person normal form game $\Gamma \in \mathcal{G}$ is said to be* ***positively (negatively) action monotonic*** *if, for all $x \in X^n$, $x_r \succ (\prec)\ x_q$ implies $\pi_r(x) > \pi_q(x)$.*

Action monotonicity is a property that requires a tight connection between payoffs and actions. While it is not a standard condition for normal form games, action monotonicity is nevertheless satisfied by a variety of symmetric games. In general, any $\Gamma \in \mathcal{G}$ with $\pi_r(x) = \Psi(x_r, \psi(x))$, where $\Psi : X \times \mathbf{R} \to \mathbf{R}_+$ is strictly increasing (decreasing) in the first component and $\psi : X^n \to \mathbf{R}$ is symmetric, is positively (negatively) action monotonic. Several widely studied symmetric games, including common pool resource extraction and public goods games, and Cournot oligopolies with constant average costs, are special cases of this general formulation. Thus all of these games are action monotonic.

Our first main result provides an answer to the question stated in the previous section within the class of all action monotonic strictly supermodular games:

Theorem 1. *Let $k \in \{1, ..., n-1\}$ and $F \in \mathbb{F}$. If $\Gamma \in \mathcal{G}$ is strictly supermodular and action monotonic, then for any $\hat{x} \in N_{\text{sym}}(\Gamma_F(k))$ with $\hat{x}_1 \neq \hat{x}_n$, we have $\pi_j(\hat{x}) > \pi_i(\hat{x})$ for all $(i, j) \in I_k \times J_k$.*

Proof. Take any $\hat{x} = ([a]_k, [b]_{n-k}) \in N_{\text{sym}}(\Gamma_F(k))$ such that $a \neq b$. Since $\pi_1 = p_1$ and $\hat{x}$ is an equilibrium, we have $\pi_1([a]_k, [b]_{n-k}) \geqslant \pi_1(b, [a]_{k-1}, [b]_{n-k})$. This, together with symmetry

[4] This definition is slightly more demanding than the usual one, which requires that π_r has increasing differences. As will become clear below, however, the present formulation results in little loss of generality and is very convenient for our purposes.

of Γ yields

$$\pi_n(b, [a]_{k-1}, [b]_{n-k-1}, a) \geqslant \pi_n(b, [a]_{k-1}, [b]_{n-k}). \tag{5}$$

By strict supermodularity, since $a \neq b$, we have

$$\pi_n([a]_k, [b]_{n-k}) + \pi_n(b, [a]_{k-1}, [b]_{n-k-1}, a) < \pi_n([a]_k, [b]_{n-k-1}, a) + \pi_n(b, [a]_{k-1}, [b]_{n-k})$$

which, together with (5), implies that

$$\pi_n(\hat{x}) = \pi_n([a]_k, [b]_{n-k}) < \pi_n([a]_k, [b]_{n-k-1}, a) = \pi_n(\hat{x}_{-n}, a). \tag{6}$$

Now assume that Γ is positively (negatively) action monotonic, and suppose, by way of contradiction, that $\rho_n(\hat{x}) \leq 1$. Then we must have either $\pi_n(\hat{x}) = 0$, or $\pi_n(\hat{x}) > 0$ and, by symmetry,

$$\frac{n\pi_n(\hat{x})}{k\pi_1(\hat{x}) + (n-k)\pi_n(\hat{x})} \leq 1.$$

In either case, we obtain $\pi_n(\hat{x}) \leq \pi_1(\hat{x})$. This is possible only if $a \succ (\prec)\ b$ by action monotonicity. But then applying action monotonicity again, we find $\pi_n(\hat{x}_{-n}, a) > \pi_j(\hat{x}_{-n}, a)$, $j = k+1, ..., n-1$, while (by symmetry) $\pi_n(\hat{x}_{-n}, a) = \pi_j(\hat{x}_{-n}, a)$, $j = 1, ..., k$. Hence we have $\rho_n(\hat{x}_{-n}, a) \geq 1$. Therefore, by (6) and the definition of p_r, we have $p_n(\hat{x}_{-n}, a) > p_n(\hat{x})$ which contradicts the hypothesis that $\hat{x}$ is an equilibrium. Therefore, we must have $\rho_n(\hat{x}) > 1$ so that, by symmetry, $\pi_j(\hat{x}) = \pi_n(\hat{x}) > \pi_i(\hat{x}) = \pi_1(\hat{x})$ for all $(i, j) \in I_k \times J_k$. *Q.E.D.*

Theorem 1 states that at any intragroup symmetric equilibrium of an action monotonic strictly supermodular game, the absolute payoffs to interdependent players are strictly greater than those to independent players, unless both groups take the same equilibrium action.[5] The significance of Theorem 1 is limited, however, by the fact that it deals only with intragroup symmetric equilibria and relies on the property of action monotonicity, which several important supermodular games do not satisfy.[6] We next show that both of these requirements can be relaxed for the case in which only one of the players has interdependent preferences.

Theorem 2. *If $\Gamma \in \mathcal{G}$ is strictly supermodular, $F \in \mathbb{F}$ and $\hat{x} \in N(\Gamma_F(n-1))$ satisfies $\hat{x}_1 \neq \hat{x}_n$, then $\pi_n(\hat{x}) > \pi_i(\hat{x})$ for all $i \in I_{n-1}$.*

Proof. Take any $\hat{x} \in N(\Gamma_F(n-1))$. Suppose first that $\hat{x}$ is intragroup symmetric, so we may write $\hat{x} = ([a]_{n-1}, b)$. Since inequality (6) above was obtained without requiring action

[5] Theorem 1 remains valid if we replace strict supermodularity with the weaker notion of *strict quasisupermodularity*, which only requires that $\pi_r(x) \geqslant \pi_r(x \wedge y)$ imply $\pi_r(x \vee y) > \pi_r(y)$, for all r and $x, y \in X^n$ such that $\{x \vee y, x \wedge y\} \neq \{x, y\}$ (Milgrom and Shannon, 1994.)

[6] One can, however, show that the hypothesis of intragroup symmetry can be relaxed in Theorem 1 if we assume that Γ is positively (negatively) action monotonic *and* has negative (positive) spillovers property (see Section 3.2), in addition to being strictly supermodular.

monotonicity, we have $\pi_n(\hat{x}) < \pi_n(\hat{x}_{-n}, a)$. If $\hat{x}$ is an equilibrium, we must therefore have $\rho_n(\hat{x}) > \rho_n(\hat{x}_{-n}, a) = \rho_n([a]_k) = 1$. The theorem then follows from the symmetry of Γ.

To complete the proof, we show that $\hat{x} \in N_{\text{sym}}(\Gamma_F(n-1))$. This is trivial for the case $n = 2$, so suppose that $n \geq 3$. If $\hat{x}$ is not intragroup symmetric, then there exist i and $i' \in I_{n-1}$ such that $\hat{x}_i \neq \hat{x}_{i'}$. Without loss of generality, let $i = 1$, $i' = 2$, $\hat{x}_1 = a$, and $\hat{x}_2 = a'$ with $a \neq a'$. Since $\hat{x}$ is an equilibrium and Γ is symmetric, we have $\pi_1(a, a', \hat{x}_3, ..., \hat{x}_n) \geqslant \pi_1(a', a', \hat{x}_3, ..., \hat{x}_n)$ and

$$\pi_1(a', a, \hat{x}_3, ..., \hat{x}_n) = \pi_2(a, a', \hat{x}_3, ..., \hat{x}_n) \geqslant \pi_2(a, a, \hat{x}_3, ..., \hat{x}_n) = \pi_1(a, a, \hat{x}_3, ..., \hat{x}_n).$$

But, given that $a \neq a'$, these two inequalities contradict the strict supermodularity of Γ. Hence $a = a'$, and we conclude that $\hat{x}$ is intragroup symmetric. *Q.E.D.*

In particular, Theorem 2 shows that the interdependent player unambiguously holds the upper hand in any strictly supermodular *two-person* game. This simple case can be used to provide some intuition for the results of this section. Consider a two-person symmetric and strictly supermodular game Γ, and let $(a, b) \in X^2$ be any outcome in which the independent player 1 has a strictly higher payoff than the interdependent player 2. Clearly, the relative payoff of player 2 is strictly less than 1 at this action profile. But, as inequality (6) proves formally, symmetry and (strictly) supermodularity ensures the following: if $a \neq b$ and a is a best response to b, then a is a (strictly) better response to a than b is. Therefore, if (a, b) was an equilibrium, by switching to player 1's action a, player 2 could increase both her absolute and relative payoffs, which means that (a, b) cannot be an equilibrium.

This kind of argument underlies the proofs of Theorems 1 and 2. In particular, Theorem 1 states that this intuition generalizes for arbitrary n and k provided that Γ is also action monotonic and one restricts attention to intragroup symmetric equilibria; Theorem 2 states that neither action monotonicity nor intragroup symmetry are required when $k = n - 1$. As we argue below, the case $k = n - 1$ is important from an evolutionary perspective, since it helps us to identify environments in which the extinction of players with interdependent preferences cannot occur under any payoff monotonic evolutionary selection dynamics.

We conclude this section by demonstrating that action monotonicity alone is not sufficient to yield any of the above results. The aim of the following example is to illustrate the crucial role played by supermodularity in Theorems 1 and 2.

Example 1. Consider the two-person game $\Gamma \in \mathcal{G}$ represented by the bimatrix

$1, 1$	$1, 1/2$	$3, 2$
$1/2, 1$	$1, 1$	$2, 0$
$2, 3$	$0, 2$	$2, 2$

Here the strategy space of each agent is the chain $\{1, 2, 3\}$. This game is easily checked to be (negatively) action monotonic but not supermodular. It has three Nash equilibria, $N(\Gamma) = \{(1, 3), (3, 1), (2, 2)\}$. Taking $F(z_1, z_2) = z_1 z_2$ for all $z_1, z_2 \geqslant 0$, and adopting the convention of treating the column player as player 2, the game $\Gamma_F(1)$ is represented by the bimatrix

$1, 1$	$1, 1/3$	$3, 8/5$
$1/2, 4/3$	$1, 1$	$2, 0$
$2, 18/5$	$0, 4$	$2, 2$

Clearly, $N(\Gamma_F(1)) = \{(1, 3)\}$, and $\pi_1(1, 3) = 3 > 2 = \pi_2(1, 3)$. In this game, therefore, the player with interdependent preferences is subject to a strategic *disadvantage.* $\|$

Example 1 demonstrates that action monotonicity is consistent with the possibility of interdependent players having a strategic disadvantage against independent players. It should thus be formally clear that our main inquiry, that is, determining a general subclass of $\mathcal{G}$ the members of which satisfy (4) is not a trivial one. The following section deals with submodular games in this subclass.

3.2 Submodular Games with Spillovers

A variety of economically interesting games exhibit a *negative* (or *positive*) *spillover* effect. In such games, an increase in the level of action taken by a player decreases (or increases) the absolute payoffs of all other players. The strong form of this property is, however, too demanding for our purposes since it is not satisfied by games in which players have at least one potential action which would nullify the influence of other players. For instance, in the classical Cournot model of oligopoly, a firm may completely escape the effect of quantity choices of other firms on its profits simply by choosing to shut down. For this reason, we shall work here with a slightly weaker notion of the spillover effect (which is present in the Cournot game).

Definition. *Let* $\mathcal{A} \equiv \cup_{k=1}^{n-1}\{x \in N_{\text{sym}}(\Gamma_F(k)) : F \in \mathbb{F}\}$. *An n-person normal form game* $\Gamma \in \mathcal{G}$ *is said to have* ***negative spillovers****, if for any* $x \in \mathcal{A}$,

$$t^1 \succ x_r \succ t^2 \quad \textit{implies} \quad \pi_q(x_{-r}, t^1) < \pi_q(x) < \pi_q(x_{-r}, t^2)$$

for all r *and* $q \neq r$. *Games with* ***positive spillovers*** *are defined dually.*

It turns out that in games with negative spillovers, there is a tight connection between action monotonicity and the possibility of $\pi_j(\bar{x}) \geqslant \pi_i(\bar{x})$ holding for all $i \in I_k$ and all $j \in J_k$, as stated in the following result.

Lemma 1. *Let $k \in \{1, ..., n-1\}$ and $F \in \mathbb{F}$. For any $\Gamma \in \mathcal{G}$ with negative spillovers and any $\hat{x} \in N_{\text{sym}}(\Gamma_F(k))$,*

$$\pi_j(\hat{x}) \geqslant (>)\, \pi_i(\hat{x}) \quad \forall (i,j) \in I_k \times J_k$$

holds only if $\hat{x}_j \succsim (\succ)\, \hat{x}_i$ for all $(i,j) \in I_k \times J_k$. Moreover, if $k = n-1$, then, for any $i \in I_{n-1}$, we have $\pi_n(\hat{x}) \geqslant (>)\, \pi_i(\hat{x})$ if and only if $\hat{x}_n \succsim (\succ)\, \hat{x}_i$.

This lemma shows that positive action monotonicity at the equilibrium action profile is essentially a necessary condition for (4) to hold in the case of games with negative spillovers. It can be shown similarly that *negative* action monotonicity at the equilibrium action profile is a necessary condition for (4) to hold for games with *positive* spillovers.

An n-person normal form game $\Gamma \in \mathcal{G}$ is said to be ***submodular*** whenever

$$\pi_r(x \vee y) + \pi_r(x \wedge y) \leqslant \pi_r(x) + \pi_r(y) \quad \forall x, y \in X^n \text{ and } r = 1, ..., n.$$

We say that Γ is ***strictly submodular*** if the above inequality holds strictly for all r and $x, y \in X^n$ such that $\{x \vee y, x \wedge y\} \neq \{x, y\}$. In contrast with supermodular games, submodular games are those in which actions of any two players are strategic substitutes in the sense that the best response maps of all players are decreasing (Bulow et al., 1985).

Finally, we shall need the following concept for the analysis of this subsection.

Definition. *An n-person normal form game $\Gamma \in \mathcal{G}$ is called* ***symmetric in equilibrium*** *if it does not possess an asymmetric Nash equilibrium.*

While symmetry in equilibrium is admittedly a demanding property, it is satisfied by a variety of commonly studied symmetric games such as the stag hunt game, prisoner's dilemma, the common pool resource game, many symmetric Cournot and Bertrand oligopoly models, and public good games. In fact, for a strictly submodular game Γ, this property is nothing other than the requirement of uniqueness of equilibrium, for, if $([a]_n), ([b]_n) \in N(\Gamma)$, then

$$\begin{aligned} \pi_1((a, [b]_{n-1}) \wedge (b, [a]_{n-1})) + \pi_1((a, [b]_{n-1}) \vee (b, [a]_{n-1})) &= \pi_1([a]_n) + \pi_1([b]_n) \\ &\geqslant \pi_1(b, [a]_{n-1}) + \pi_1(a, [b]_{n-1}) \end{aligned}$$

so that unless $a = b$, Γ cannot be strictly submodular. When combined with symmetry, this observation leads us to the following

Lemma 2. *Let $\Gamma \in \mathcal{G}$ be a strictly submodular game such that $N(\Gamma) \neq \emptyset$. Then, Γ is symmetric in equilibrium if, and only if, it has a unique equilibrium.*

Our main result takes as primitives those games in $\mathcal{G}$ where the common strategy set of the players is a convex and compact subchain of $\mathbf{R}^\ell$, and the payoff function of the rth player is continuous and quasiconcave in x_r, for all r. Denoting the class of all such games by $\mathcal{G}_*$, we are now ready to state the following strategic advantage result the interpretation of which is similar to that of Theorem 1.

Theorem 3. *Let $\Gamma \in \mathcal{G}_*$, $k \in \{1, ..., n-1\}$ and $F \in \mathbb{F}$. If Γ is a positively (negatively) action monotonic and strictly submodular game with negative (positive) spillovers, and is symmetric in equilibrium, then, for any $\hat{x} \in N_{\text{sym}}(\Gamma_F(k))$ such that $\hat{x}_1 \neq \hat{x}_n$ and $\pi_r(\hat{x}) > 0$ for all r, we have $\pi_j(\hat{x}) > \pi_i(\hat{x})$ for all $(i, j) \in I_k \times J_k$.*

Remark 2. Lemma 1 demonstrates the necessity of action monotonicity for the conclusion of Theorem 3 to hold. Since we think of submodular games with spillovers as primitives in the above analysis, the only question about the tightness of this result concerns the relaxation of the symmetry in equilibrium condition. To see that this condition too cannot be relaxed in Theorem 3, consider the following "hawk-dove" game represented by the bimatrix

$10, 10$	$5, 15$
$15, 5$	$1, 1$

and define Γ as its mixed strategy extension. One can easily verify that Γ satisfies all the hypotheses of Theorem 3 except for symmetry in equilibrium, and that $((1,0),(0,1)) \in N_{\text{sym}}(\Gamma_F(1))$ where $F(z_1, z_2) = (z_1 + 1)(z_2 + 1)$ for all $z_1, z_2 \geqslant 0$. Hence there exists an equilibrium in which the player with interdependent preferences obtains a strictly lower absolute payoff. ||

To provide intuition for Theorem 3, let us consider a two-person symmetric and strictly submodular game Γ which satisfies positive action monotonicity, negative spillovers, and symmetry in equilibrium, and let $(a, b) \in X^2$ be an equilibrium of $\Gamma_F(1)$ in which a exceeds b and hence the independent player 1 has a strictly higher payoff than the interdependent player 2. By raising her action, player 2 can lower player 1's material payoff (due to negative spillovers) so if (a, b) is to be an equilibrium of $\Gamma_F(1)$, any such change must also lower player 2's material payoff. In fact, since (a, b) cannot be an equilibrium of Γ (due to symmetry in equilibrium), player 2's material payoff maximizing response to a must lie strictly *below* b. If one constructs a sequence in which, starting from (a, b), each player chooses, in turn, a material payoff maximizing response to the other player's previous choice, it can be shown that (due to the strict submodularity of Γ) this sequence leads to increasing choices for player 1, decreasing choices for player 2, and converges to an asymmetric equilibrium of Γ. Hence, if Γ is symmetric in equilibrium, the premise that a exceeds b must be false, and hence the independent player 1 cannot have a higher payoff than the interdependent player 2 at an equilibrium of $\Gamma_F(1)$. While this particular reasoning applies only to the two person case, it generalizes to yield Theorem 3.

In closing, we note that one can again relax the requirement of action monotonicity when there is only one interdependent player in the game. The following is then a counterpart to Theorem 2.

Theorem 4. *Let $F \in \mathbb{F}$. If $\Gamma \in \mathcal{G}_*$ is a strictly submodular game with negative or positive spillovers, and is symmetric in equilibrium, then, for any $\hat{x} \in N_{\text{sym}}(\Gamma_F(n-1))$ such that $\hat{x}_1 \neq \hat{x}_n$ and $\pi_r(\hat{x}) > 0$ for all r, we have $\pi_n(\hat{x}) > \pi_i(\hat{x})$ for all $i \in I_{n-1}$.*

Theorem 4 provides sufficient conditions for the single player with interdependent preferences to have a strategic advantage with respect to the remaining players in submodular games. In particular, it implies that if $\Gamma \in \mathcal{G}_*$ is a two-person strictly submodular game with spillovers and a unique equilibrium, then we have $\pi_2(\hat{x}) > \pi_1(\hat{x})$ for any Nash equilibrium $\hat{x}$ of $\Gamma_F(1)$ with $\hat{x}_1 \neq \hat{x}_2$ and $\min\{\pi_1(\hat{x}), \pi_2(\hat{x})\} > 0$. This is analogous to our earlier observation about two-person supermodular games.

4 Applications

The usefulness of the theorems established above hinges on the degree to which they may be applied to games of economic interest. There are a variety of environments to which Theorems 1-4 apply, among which are input and public good games, search models and arms races. In each of these examples, it is possible to verify that, under standard conditions, interdependent preferences yield a strategic advantage over independent preferences.[7] To illustrate, we shall discuss here the implications of the strategic advantage results of the previous section for certain oligopoly games, and in particular, provide an application to the theory of strategic delegation. We shall then briefly discuss the implications of our results for the theory of preference evolution.

4.1 Strategic Advantage and Delegation in Market Games

4.1.1 Strategic Advantage. Objective functions which incorporate relative payoff concerns are particularly easy to justify in the case of firms that separate management from ownership. For instance, owners may benefit from writing contracts with managers in which the compensation of the latter is based, in part, on the performance of their firm relative to that of other firms, or relative to some industry average (Holmström, 1982). This, in turn, would provide an incentive for managers to pursue the maximization of interdependent objective functions. The results of Section 3 can thus be used to show that such contracts may have the unplanned effect of yielding a strategic advantage to a firm, enabling it to achieve a higher level of profitability than its profit-maximizing competitors.

[7] For a detailed analysis of these examples we refer the reader to Koçkesen et al. (1997).

To illustrate, consider the case of Cournot competition. For expositional simplicity, we consider a duopolistic industry composed of two firms with identical cost structures producing a homogenous product.[8] Firm r chooses an output level $x_r \in X \equiv [0, \bar{Q}]$, $0 < \bar{Q} < \infty$, where $\bar{Q}$ is interpreted as the capacity limit on a firm's output level. The profit function of firm r is given by $\pi_r(x) = x_r P(x_1 + x_2) - C(x_r)$, $x \in X^2$, where the inverse demand function P is a strictly positive and twice differentiable function on $[0, 2\bar{Q}]$ and the cost function C is a twice differentiable function on $[0, \bar{Q}]$. We make the standard assumptions that demand is downward sloping and average cost is non-decreasing: $P' < 0,\ C(0) = 0,\ C' > 0,\ C'' \geqslant 0$. We also assume that the game is strictly submodular.[9] These assumptions imply that Γ^C has a unique equilibrium (Corchón, 1996, Proposition 1.3), which must therefore be symmetric. Let us denote the resulting Cournot game by $\Gamma^C \in \mathcal{G}$, and make the additional assumptions that $P(2\bar{Q}) > C(\bar{Q})/\bar{Q}$, to ensure positive profits for each firm at any output profile, and $P(2\bar{Q}) + \bar{Q}P'(2\bar{Q}) < C'(\bar{Q})$ and $P(0) > C'(0)$ to ensure that the equilibrium occurs in the interior. The last condition also guarantees that Γ^C has the negative spillovers property. It is a straightforward matter to show that under the stated conditions, any equilibrium $\hat{x} \in N(\Gamma^C_F(1))$ satisfies $\hat{x}_1 \neq \hat{x}_2$.[10] Since Γ^C is strictly submodular with negative spillovers, and is symmetric in equilibrium, the following then follows from Theorem 4.

Proposition 1. *Take any Cournot duopoly Γ^C and $F \in \mathbb{F}$. Then, at any $\hat{x} \in N(\Gamma^C_F(1))$, we have $\pi_1(\hat{x}) < \pi_2(\hat{x})$, i.e., the firm with interdependent preferences obtains a strictly higher profit than does the independent firm at any equilibrium.*

While the analysis above pertains to the classical Cournot duopoly, similar results may be obtained for other market games. For instance, in the case of a Bertrand duopoly with differentiated products and constant marginal costs, we may apply Theorem 2 (under the standard conditions that ensure the supermodularity of the game) to conclude that the interdependent firms will be more profitable than profit-maximizing firms. In such environments, therefore, managers who include relative profit considerations in their decision making process (say, due to incentive contracts) will obtain higher profits in equilibrium than those who do not. This finding has an immediate implication for industries in which the entry and exit of firms occurs on the basis of profitability: behavior that corresponds to an interdependent objective function will thrive at the expense of profit maximizing behavior in the long run.

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[8] All of our results extend with minor modifications to n-firm industries; we focus on the duopoly case only to avoid uninteresting technical details.

[9] That is, $P'(x_1 + x_2) + x_r P''(x_1 + x_2) < 0$ for all $x \in [0, \bar{Q}]^2$.

[10] Suppose $\hat{x} = (a, a)$. The boundary conditions posited above imply $a \in (0, \bar{Q})$ so that $\partial\pi_1(a, a)/\partial x_1 = 0$ and $\partial p_2(a, a)/\partial x_2 = 0$. Since, by symmetry, the former equation implies that $\partial\pi_2(a, a)/\partial x_2 = 0$, the latter equation holds only if $(\pi_2 F_2/\bar{\pi}^2)\partial\pi_1/\partial x_2 = 0$ at the action profile (a, a). Yet this is a contradiction, for we have $\pi_i(a, a) > 0$, $i = 1, 2$, $F_2 > 0$ and $\partial\pi_1(a, a)/\partial x_2 < 0$ in this model.

4.1.2 Strategic Delegation. An interesting application of Proposition 1 concerns the theory of strategic delegation as developed by Vickers (1984) and Fershtman and Judd (1987). Consider an extension of the Cournot duopoly game Γ^C in which one of the firms has the option of hiring a manager and delegating the output choice to her. In the first stage of the game, the owner of firm 2 (henceforth, owner 2) gives a compensation contract to her manager, the nature of which is common knowledge. In the second stage, the manager and firm 1 engage in a standard Cournot competition in which the manager's objective is to maximize her compensation, which is determined by the following objective function:

$$\Pi\left(x,\theta\right) \equiv \left(1-\theta\right)\pi_2\left(x\right) + \theta\rho_2\left(x\right) \qquad \forall (x,\theta) \in [0,\bar{Q}]^2 \times [0,1], \tag{7}$$

where ρ_2 is the relative profit of firm 2 as defined in (2). While owner 2 chooses θ to maximize π_2, we assume for simplicity that the owner of the firm 1 is a standard profit-maximizer.

We denote the resulting 3-person extensive form game by Γ^D. The subgame of Γ^D that is reached when owner 2 chooses $\theta \in [0,1]$ is denoted by Γ^D_θ. Clearly, Γ^D_θ is identical to the family of 2-person normal-form games considered in Proposition 1. More formally, we have $\Gamma^D_\theta = \Gamma^C_{F_\theta}(1)$ where F_θ is defined on $\mathbf{R}^2_+$ by $F_\theta(z_1,z_2) = (1-\theta)z_1 + \theta z_2$. In particular, we have $\Gamma^D_0 = \Gamma^C$. In what follows, we refer Γ^D as a *delegation game*, and assume that Γ^D_0 carries the structure of the Cournot duopoly model described in the beginning of this section. The main issue in the theory of strategic delegation is the question of whether owner 2 will choose to delegate in equilibria of Γ^D, that is, whether she will set $\theta > 0$. Given earlier findings in the literature, our answer will not come as a surprise: delegation occurs in any perfect equilibrium of Γ^D.

Proposition 2. *At any subgame perfect equilibrium of Γ^D, the owner of firm 2 chooses to delegate.*

Provided that the contract space is limited to that assumed above, there is reason to expect managerial compensation schemes to embody relative profitability concerns at least to some extent.[11] Since we exogenously limit ourselves to a particular contract space, Proposition 2 cannot be used to predict the form of the equilibrium contract in general. It tells us, however, that no-delegation cannot be an equilibrium so long as the contract space is rich enough to include those contracts given by (7). In contrast to Vickers (1984) and Fershtman and Judd (1987), Proposition 2 shows that this conclusion obtains without assuming the linearity of the demand and cost functions.[12]

[11] We are, of course, neglecting objections that could be raised on the grounds of contract unobservability (see Katz, 1991 and Koçkesen and Ok, 1999 for more on this issue).

[12] Formally speaking, however, Proposition 2 does not directly generalize the corresponding result of, say, Vickers (1984) since Vickers identifies a compensation contract with the mapping $x \mapsto \pi_2(x) - \theta\bar{\pi}(x)$, $\theta \geq 0$, as opposed to (7). However, the proof of Proposition 2 may be easily adapted (by setting $F_\theta(z_1,z_2) \equiv z_1(1-\theta/z_2)$ so that Proposition 1 may be applied) to account for this case.

Remark 3. The findings of Vickers (1984) have also been generalized by Salas Fumas (1992), whose Proposition 1(a) is similar to our Proposition 2. Neither result, however, implies the other. In particular, our hypotheses are posited only on the primitive Cournot game Γ_0^D (as opposed to all the subgames Γ_θ^D). Perhaps more importantly, our proof (given in the appendix) is quite elementary and hence readily generalizes to the usual n-firm scenario. In fact, it is easy to see that the same proof (with only minor modifications) enables one to replace the Cournot game considered in Proposition 2 with essentially *any* strictly submodular game that satisfies the hypotheses of Theorem 4. ‖

4.2 Preference Evolution and the Spiteful Effect

While it is conventional in economic models to posit that individuals are material payoff maximizers, there is now mounting experimental evidence that contradicts this strong "independence" hypothesis. The theoretical plausibility of this assumption has accordingly been questioned recently by several economists on evolutionary grounds.[13] The theory of preference evolution is based on the premise that individual preferences come to being as a result of an unplanned process in which children inherit the preferences of their parents either by genetic transmission or imitation. The population composition is typically assumed to evolve according to a *payoff monotonic* selection dynamic: those behaviors which yield the highest material rewards are replicated with greatest frequency from one generation to the next. The evolutionary scenario we consider here is one in which each individual interacts with each other member of the population in each period; the so-called "playing the field" model. This is an environment in which strategic advantage has particularly transparent evolutionary implications. Dispensing with formalities for brevity, we briefly discuss these implications next.

Consider a finite population of size n, such that the constituent individuals are engaged in playing an n-person game $\Gamma \in \mathcal{G}$. Not all players need be material payoff maximizers; we allow for the possibility that some players are negatively interdependent. With the initial preference distribution in the population given exogenously, suppose that this distribution evolves over time in accordance with the hypothesis of payoff monotonicity. It is apparent from Theorem 2 that if Γ is strictly supermodular, then the long-run population composition cannot consist exclusively of material payoff maximizers unless the initial state consists exclusively of such preferences. If, due to a 'mutation', a single individual in the population happens to be negatively interdependent (denote the resulting game by Γ_F), then this individual would obtain at least as great a material payoff as any independent individual, and consequently, independent preferences could not expel such a mutant.[14] On the basis of

[13]See, for instance, Bester and Güth, 1998, Fershtman and Weiss, 1998, and Koçkesen et al., 1999.

[14]While the basic idea here is quite transparent, it is nevertheless informal. To examine the issue at

Theorem 4, the same conclusion holds if Γ is strictly submodular, symmetric in equilibrium, and has negative or positive spillovers, provided that all independent agents take the same action at any equilibrium of Γ_F. We therefore conclude that there are evolutionary reasons to expect that the population will *not* be composed only of absolute payoff maximizers in the long run.[15]

Broadly similar conclusions have been reached earlier in the evolutionary literature on the 'spiteful effect'. The spiteful effect occurs when it is possible for a player to deviate from a Nash equilibrium action profile in such a manner as to reduce the payoffs of other players more severely than the payoffs of the deviating player are reduced (Rhode and Stegeman, 1996, Vega-Redondo, 1997). In this case payoff monotonic selection dynamics will lead to the spread of a 'spiteful' mutant who adopts such a deviation. It turns out that the presence of the spiteful effect is necessary but not sufficient for the strategic advantage of negatively interdependent preferences. To see that it is necessary, consider for simplicity a two-person game in which the spiteful effect is *not* present at any equilibrium of Γ. In particular, it is not present at any equilibrium $\hat{x}$ at which $\pi_1(\hat{x}) \geq \pi_2(\hat{x})$. Since Γ is symmetric, there must exist at least one such equilibrium. In the absence of the spiteful effect, any deviation from this equilibrium by Player 2 will lower her payoffs at least as much as it lowers the payoff of Player 1. Such a deviation can raise neither the absolute payoff, nor relative payoff of the deviating player. Hence $\hat{x}$ is also an equilibrium of Γ_F when Player 1 is independent and Player 2 negatively interdependent. As a result, interdependent players cannot have a strategic advantage in the absence of the spiteful effect.

To see that the spiteful effect is not sufficient for strategic advantage, one need only examine Example 1 above. The game Γ has three Nash equilibria when both players have independent preferences, and the spiteful effect is present at each one of them. However, if exactly one of the players is negatively interdependent, this player may end up with strictly lower material payoffs than her opponent at the unique equilibrium of the resulting game Γ_F. Hence the presence of the spiteful effect does not guarantee the existence of strategic advantage for negatively interdependent preferences. The reason why the two criteria yield different predictions is because the spiteful effect is based on the hypothesis that

hand formally, we need a *set-valued* extension of the concept of an evolutionarily stable state for a finite population (Riley, 1979, Shaffer, 1988) since, if the preference space is sufficiently rich, there will typically exist preferences that are behaviorally indistinguishable in particular game forms. (For instance, in the multi-person prisoners' dilemma, defection will be a dominant strategy for a very wide range of objective functions including all negatively interdependent and independent preferences.) Such a stability concept is introduced in Koçkesen et al. (1997) where the evolutionary ideas discussed above are formalized.

[15] Other interesting environments in which this issue can be investigated include random matching models as well as models of local interaction. Results obtained by Fershtman and Weiss (1998) and Bester and Güth (1998) for pairwise random matching, and by Eshel et al. (1998) for local interaction on a circle suggest that even altruistic preferences can survive in such cases.

the incumbent population does not respond optimally to the appearance of the mutant (and continues, instead, to adopt the same strategies that were previously optimal.) In contrast, we postulate rational behavior by all players and study the properties of the equilibrium set conditional on the underlying preference distribution. The introduction of an individual with interdependent preferences into an incumbent population of absolute payoff maximizers causes the latter to adjust their actions in such a manner as to locate a new equilibrium. Since the notion of strategic advantage is *global* (having been defined conditional on the equilibria of Γ_F), it cannot be deduced by examining the consequences of *local* deviations from equilibria of Γ, which is what defines the spiteful effect. Furthermore, verification of the presence of the spiteful effect requires one to examine the properties of equilibria, which are not primitives of a game. In contrast, the strategic advantage results of Section 3 are obtained by positing conditions on the payoff (fitness) functions, and are therefore easily verified in arbitrary games.[16] Given that the spiteful effect is necessary for the strategic advantage of negatively interdependent preferences, the main results of this paper provide sufficient conditions for the presence of the spiteful effect in a given normal form game.

5 Conclusion

We have aimed in this paper to uncover the generality of the statement that negatively interdependent preferences provide individuals with a strategic advantage over others who are motivated exclusively by a concern with their own material payoffs. While the plausibility of this phenomenon has been noted earlier in a variety of contexts, the conditions under which it arises have not previously been systematically explored. It turns out that there is a broad class of strategic environments in which such an advantage exists, although this class is not exhaustive. Our results identify the properties of strategic complementarity and substitutability as particularly relevant to the possibility that interdependent preferences earn greater absolute payoffs than do (absolute) payoff maximizers. While we find this observation interesting from a purely game-theoretic viewpoint, it also has important implications for the theories of strategic delegation and preference evolution. We note, however, that our results fall short of characterizing the class of games in which interdependent players have a strategic advantage over independent players (Koçkesen, et al., 1999). Determining precisely the class of all normal-form games for which this phenomenon occurs remains as

[16] The only non-primitive property that we used above is symmetry of equilibrium. But this property too can be replaced by any set of primitive assumptions that ensure uniqueness of equilibrium in our context (recall Lemma 2). For instance, in Theorems 3 and 4, the symmetry in equilibrium property can be replaced with the requirements that (i) payoff functions are strictly quasiconcave in own actions, and (ii) the associated best response functions are contraction maps.

open problem.

Appendix: Proofs

Proof of Lemma 1. Assume that $\hat{x} = ([a]_k, [b]_{n-k})$ for some $a, b \in X$ with $a \succ b$. Then, by hypothesis, negative spillovers effect, and symmetry of Γ,

$$\pi_1([a]_k, [b]_{n-k}) \leqslant \pi_n([a]_k, [b]_{n-k}) < \pi_n(b, [a]_{k-1}, [b]_{n-k}) = \pi_1(b, [a]_{k-1}, [b]_{n-k})$$

and this contradicts that playing a is a best response for player 1 against $([a]_{k-1}, [b]_{n-k})$. The first assertion follows by the completeness and antisymmetry of $\succsim$.

To prove the second assertion, let $k = n - 1$ and notice that all we have to show is that $\pi_n(\hat{x}) > \pi_1(\hat{x})$ whenever $b \succ a$. Assume then for contradiction that

$$b \succ a \quad \text{and} \quad \pi_n([a]_{n-1}, b) \leqslant \pi_1([a]_{n-1}, b). \tag{8}$$

Since $\alpha \mapsto \alpha/(\tau + \alpha)$ is a strictly increasing mapping in $\alpha \geqslant 0$ for any $\tau > 0$, (8) implies that

$$\frac{\rho_n(\hat{x})}{n} = \frac{\pi_n(\hat{x})}{(n-1)\pi_1(\hat{x}) + \pi_n(\hat{x})} \leqslant \frac{\pi_1(\hat{x})}{(n-1)\pi_1(\hat{x}) + \pi_1(\hat{x})} = \frac{1}{n}.$$

Hence $\rho_n(\hat{x}) \leq 1$ and since b is a best response of player n against $[a]_{n-1}$ in $\Gamma_F(n-1)$, we must have $\pi_n([a]_{n-1}, b) \geqslant \pi_n([a]_n)$. Therefore, by the negative spillovers effect and symmetry of Γ, we have $\pi_1([a]_{n-1}, b) < \pi_1([a]_n) = \pi_n([a]_n) \leqslant \pi_n([a]_{n-1}, b)$ which contradicts (8). *Q.E.D.*

Proof of Theorem 3. This theorem is an immediate consequence of the following

Lemma A. *Let $\Gamma \in \mathcal{G}_*$, $k \in \{1, ..., n-1\}$ and take any strictly increasing $F : \mathbf{R}^2_+ \to \mathbf{R}$. If Γ is a strictly submodular game with negative spillovers, and is symmetric in equilibrium, then, for any $\hat{x} \in N_{\text{sym}}(\Gamma_F(k))$ with $\pi_r(\hat{x}) > 0$ for all r, we have*

$$\hat{x}_j \succsim \hat{x}_i \quad \forall (i, j) \in I_k \times J_k.$$

Proof. Let $\hat{x} = ([a]_k, [b]_{n-k}) \in N_{\text{sym}}(\Gamma_F(k))$ for some $a, b \in X$. Clearly, since a is a best response of player 1 against $([a]_{k-1}, [b]_{n-k})$ in $\Gamma_F(k)$, we have

$$\pi_1([a]_k, [b]_{n-k}) \geqslant \pi_1(t, [a]_{k-1}, [b]_{n-k}) \quad \forall t \in X. \tag{9}$$

We claim that

$$\pi_n([a]_k, [b]_{n-k}) > \pi_n([a]_k, [b]_{n-k-1}, t) \quad \forall t \in \{t' \in X : t' \succ b\}. \tag{10}$$

To see this, let us assume for contradiction that

$$\pi_n(\hat{x}_{-n}, t) = \pi_n([a]_k, [b]_{n-k-1}, t) \geqslant \pi_n([a]_k, [b]_{n-k}) = \pi_n(\hat{x}) > 0 \tag{11}$$

holds for some $t \in X$ with $t \succ b$. Since $\alpha \mapsto \alpha/(\tau+\alpha)$ is a strictly increasing mapping in $\alpha \geqslant 0$ for any $\tau > 0$, we must then have

$$\frac{\pi_n(\hat{x})}{\sum_{r=1}^{n-1}\pi_r(\hat{x}) + \pi_n(\hat{x})} \leqslant \frac{\pi_n(\hat{x}_{-n}, t)}{\sum_{r=1}^{n-1}\pi_r(\hat{x}) + \pi_n(\hat{x}_{-n}, t)} \tag{12}$$

But since $t \succ b$, the negative spillovers effect yields $\pi_r(\hat{x}) > \pi_r(\hat{x}_{-n}, t)$ for all $r \neq n$ so that

$$\frac{\pi_n(\hat{x}_{-n}, t)}{\sum_{r=1}^{n-1}\pi_r(\hat{x}) + \pi_n(\hat{x}_{-n}, t)} < \frac{\pi_n(\hat{x}_{-n}, t)}{\sum_{r=1}^{n-1}\pi_r(\hat{x}_{-n}, t) + \pi_n(\hat{x}_{-n}, t)}.$$

But this inequality, (12) and (11) yield $p_n(\hat{x}_{-n}, t) > p_n(\hat{x})$ which contradicts that $\hat{x}$ is a Nash equilibrium of $\Gamma_F(k)$. Therefore, we must conclude that (10) holds.

Now, for any $\alpha, \beta \in X$, define the correspondences $K_\beta : X \rightrightarrows X$ and $L_\alpha : X \rightrightarrows X$ as

$$K_\beta(\alpha) \equiv \arg\max_{t \in X} \pi_1(t, [\alpha]_{k-1}, [\beta]_{n-k}) \quad \text{and} \quad L_\alpha(\beta) \equiv \arg\max_{t \in X} \pi_n([\alpha]_k, [\beta]_{n-k-1}, t).$$

We define next the double sequence $(a_m, b_m) \in X^2$ recursively as follows:

$$a_0 = a, \quad b_0 = b, \quad a_m \in K_{b_m}(a_m) \quad \text{and} \quad b_m \in L_{a_{m-1}}(b_m), \; m = 1, 2, ...$$

Claim 1. (a_m, b_m) is well-defined.

Proof of Claim 1. Fix any $\alpha, \beta \in X$. Since X is a convex compact set, and π_1 and π_n are continuous, K_β and L_α must be nonempty (by Weierstrass' theorem) and must have closed graphs (by Berge's maximum theorem). Moreover, quasiconcavity of π_1 and π_n entail that K_β and L_α are convex-valued. Therefore, by Kakutani's fixed point theorem, there exist fixed points of K_β and L_α. Since α and β were arbitrary in this reasoning, we may conclude that (a_m) and (b_m) are well-defined sequences. $\|$

Let $B^r : X^{n-1} \rightrightarrows X$ be the best response correspondence of player r in Γ. We note that, for any $\alpha, \beta \in X$,

$$B^i([\alpha]_{k-1}, [\beta]_{n-k}) = K_\beta(\alpha) \quad \forall i \in I_k \tag{13}$$

and

$$B^j([\alpha]_k, [\beta]_{n-k-1}) = L_\alpha(\beta) \quad \forall j \in J_k \tag{14}$$

hold by symmetry of Γ.

Claim 2. If $a \succ b$, then $a_0 \prec a_1 \prec a_2 \prec \cdots$ and $\cdots \prec b_2 \prec b_1 \prec b_0$.

Proof of Claim 2. Let $a \succ b$. We shall first establish that $b_1 \neq b_0$. If $b_1 = b$, then $b_1 \in L_a(b_1)$ implies by (14) that $b \in B^j([a]_k, [b]_{n-k-1})$ for all $j \in J_k$. But then since $a \in B^i([a]_{k-1}, [b]_{n-k})$ for all $i \in I_k$, it follows that $([a]_k, [b]_{n-k}) \in N(\Gamma)$, contradicting that Γ is symmetric in equilibrium. If, on the other hand, $b_1 \succ b$, then (10) yields that $\pi_n(\hat{x}) >$

$\pi_n([a]_k, [b]_{n-k-1}, b_1)$. But then by submodularity of π_n, $\pi_n([a]_k, [b_1]_{n-k}) < \pi_n([a]_k, [b_1]_{n-k-1}, b)$ which, in turn, contradicts that $b_1 \in L_a(b_1)$. We thus conclude that $b \succ b_1$.

Next, we claim that $a_1 \succsim a_0$. But by (9) and the fact that $a_1 \in K_{b_1}(a_1)$, we have $\pi_1([a]_k, [b]_{n-k}) \geqslant \pi_1(a_1, [a]_{k-1}, [b]_{n-k})$ and $\pi_1([a_1]_k, [b_1]_{n-k}) \geqslant \pi_1(a, [a_1]_{k-1}, [b_1]_{n-k})$. Clearly, given that $b \succ b_1$, if $a \succ a_1$, the last two inequalities would contradict the strict submodularity of π_1. Therefore, $a_1 \succsim a$ must hold. In fact, $a_1 \neq a$, for otherwise, $a_1 \in K_{b_1}(a_1)$ and $b_1 \in L_a(b_1)$ would yield that $([a]_k, [b_1]_{n-k}) \in N(\Gamma)$, and this would contradict Γ's symmetry in equilibrium since then $a \succ b \succ b_1$ would have to hold. By linearity of $\succsim$, therefore, we have $a_1 \succ a_0$.

Finally, we claim that $b_1 \succ b_2$. (Since we used (10) in establishing that $b_0 \succ b_1$, this step is necessary to be able to complete the proof by induction.) This claim follows from the fact that $b_1 \in L_a(b_1)$ and $b_2 \in L_{a_1}(b_2)$ imply that $\pi_n([a]_k, [b_1]_{n-k}) \geqslant \pi_n([a]_k, [b_1]_{n-k-1}, b_2)$ and $\pi_n([a_1]_k, [b_2]_{n-k}) \geqslant \pi_n([a_1]_k, [b_2]_{n-k-1}, b_1)$, respectively. If $b_2 \succ b_1$ held, given that $a_1 \succ a$, these inequalities would contradict the strict submodularity of π_n. Moreover, if $b_1 = b_2$, then $b_1 \in L_{a_1}(b_1)$ holds, and since $a_1 \in K_{b_1}(a_1)$, we obtain $([a_1]_k, [b_1]_{n-k}) \in N(\Gamma)$ contradicting that Γ is symmetric in equilibrium (because $a_1 \succ a \succ b \succ b_1$). We conclude that $b_1 \succ b_2$.

Proof is completed by a straightforward induction argument. $\|$

Since X is compact, there exist convergent subsequences (a_{v_m}) and (b_{v_m}) such that $(a_{v_m}, b_{v_m}) \to (a^*, b^*) \in X^2$ as $m \to \infty$. We now claim that $(a^*, b^*) \in N(\Gamma)$. To see this, notice that $a_{v_m} \in K_{b_{v_m}}(a_{v_m})$ implies that

$$a_{v_m} \in B^1([a_{v_m}]_{k-1}, [b_{v_m}]_{n-k}) \quad m = 1, 2, ...$$

But since X is compact and π_1 is continuous, B^1 must have a closed graph, and therefore,

$$a^* = \lim_{m\to\infty} a_{v_m} \in B^1\left(\lim_{m\to\infty}([a_{v_m}]_{k-1}, [b_{v_m}]_{n-k})\right) = B^1([a^*]_{k-1}, [b^*]_{n-k}).$$

Moreover, by symmetry of Γ, $a^* \in B^i([a^*]_{k-1}, [b^*]_{n-k})$ for all $i \in I_k$. Similarly, we can show that $b^* \in B^i([a^*]_k, [b^*]_{n-k-1})$ for all $i \in J_k$. We thus conclude that $(a^*, b^*) \in N(\Gamma)$ as is sought. Therefore, if $a \succ b$ held, by Claim 2 there would exist an $(a^*, b^*) \in N(\Gamma)$ with $a^* \succ b^*$, contradicting that Γ is symmetric in equilibrium. *Q.E.D.*

Remark A. (a) If $n = 2$, we may drop the hypotheses of convexity of X and quasiconcavity of π_rs from the statement of Lemma A, for then one does not need Kakutani's fixed point theorem in proving that (a_m, b_m) is well-defined.

(b) Lemma A remains valid if the chain X is any compact convex subset of a locally convex topological vector space, and π_rs are continuous with respect to the subspace topology. The proof of this claim is essentially identical to that of Lemma A, the only major modification being the use of Tychonoff-Fan fixed point theorem (Berge, 1963, p.251) instead of Kakutani's theorem in proving Claim 1.

(c) Continuity of π_r can be weakened in Lemma A to upper semicontinuity of π_r and lower semicontinuity of $V_r(x_{-r}) \equiv \max_{a\in X} \pi_r(x_{-r}, a)$ for all $x_{-r} \in X^{n-1}$, which is well-defined when π_r is upper semicontinuous. We only need to check that these conditions guarantee the nonemptiness of K_β (and L_α) and the closed graph property of B^r. The former is immediate from upper semicontinuity of π_r. To see the latter, take any sequence x^m in X^n such that $x^m \to x^*$ as $m \to \infty$, and let $x_r^m \in B^r(x_{-r}^m)$ for all m. If $x_r^* \notin B^r(x_{-r}^*)$, then it must be the case that $V_r(x_{-r}^*) > \pi_r(x_{-r}^*, x_r^*)$. But then using the upper semicontinuity of π_r, the fact that $x_r^m \in B^r(x_{-r}^m)$ for all m, and the lower semicontinuity of V_r, we reach to the following contradiction:[17]

$$\begin{aligned} \pi_r(x_{-r}^*, x_r^*) &\geqslant \limsup_{m\to\infty} \pi_r(x_{-r}^m, x_r^m) = \limsup_{m\to\infty} V_r(x_{-r}^m) \\ &\geqslant \liminf_{m\to\infty} \pi_r(x_{-r}^m, x_r^m) \geqslant V_r(x_{-r}^*) > \pi_r(x_{-r}^*, x_r^*). \end{aligned}$$

Proof of Theorem 4. Immediate from Lemma 1 and Lemma A. *Q.E.D.*

Proof of Proposition 2. Let $(a, a) \in N(\Gamma^C)$ and notice that we must have $a \in (0, \bar{Q})$. Let b_1 denote the restriction of the best response correspondence of player 1 to $(0, \bar{Q})$ in the game Γ^C. By strict concavity of π_2, b_1 can be considered as a function. Define the function $\varphi : (0, \bar{Q}) \to \mathbf{R}$ by $\varphi(q) \equiv \pi_2(b_1(q), q)$. By the implicit function theorem, b_1 and φ are C^1 on $(0, \bar{Q})$, and since $a = b_1(a)$ and $\partial \pi_2(a, a)/\partial x_2 = 0$, we have $\varphi'(a) = aP'(2a)b_1'(a) > 0$, where the inequality follows from $P' < 0$ and strict submodularity of P. In view of continuity of φ, we then have

Claim 1: There exists an $\bar{\varepsilon} > 0$ such that $\pi_2(b_1(a+\varepsilon), a+\varepsilon) > \pi_2(b_1(a), a)$ for all $\varepsilon \in (0, \bar{\varepsilon}]$.

Claim 2. There exists a $\bar{\theta} \in (0, 1]$ such that $N(\Gamma_\theta^D) \neq \emptyset$ for all $\theta \in [0, \bar{\theta}]$.

Proof of Claim 2. Take any $\varepsilon \in (0, \bar{Q})$ and $\theta \in [0, 1]$, and consider the 2-person game $\Gamma_{\theta,\varepsilon}^D \equiv (X_\varepsilon^2, \{\pi_1, \Pi(\cdot, \theta)\})$ where $X_\varepsilon \equiv [\varepsilon, \bar{Q}]$. First, note that $\Pi(\cdot, \theta)$ is continuous on X_ε^2.[18] Second, since

$$\frac{\partial^2 \Pi(x, \theta)}{\partial x_2^2} = (1-\theta)\frac{\partial^2 \pi_2(x)}{\partial x_2^2} + \theta \frac{\partial^2 \rho_2(x)}{\partial x_2^2} \quad \text{and} \quad \frac{\partial^2 \pi_2(x)}{\partial x_2^2} < 0,$$

there exists a $\theta(\varepsilon) \in (0, 1]$ such that $\Pi(x_1, \cdot, \theta)$ is concave on X_ε for all $\theta \in [0, \theta(\varepsilon)]$. We can therefore use Nash's existence theorem to conclude that $N(\Gamma_{\theta,\varepsilon}^D) \neq \emptyset$ for all $\theta \in [0, \theta(\varepsilon)]$. Now, choose any $x(\varepsilon) \in N(\Gamma_{\theta,\varepsilon}^D)$ with $\theta \in [0, \theta(\varepsilon)]$. Since

$$\lim_{\varepsilon \downarrow 0} \sup\nolimits_{x_2 \in [0,\varepsilon)} \Pi((x_1(\varepsilon), x_2), \theta) = 0 < \Pi(x(\varepsilon), \theta)$$

[17] Dasgupta and Maskin (1986) use an analogous reasoning in proving their Theorem 2.

[18] Notice, however, that $\Pi(\cdot, \theta)$ is *not* continuous at $(0, 0)$.

there exists an $\varepsilon > 0$ such that $\Pi((x_1(\varepsilon), x_2), \theta) \leqslant \Pi(x(\varepsilon), \theta)$ for all $x_2 \in [0, \varepsilon)$. Choosing such an $\varepsilon > 0$, therefore, $x_2(\varepsilon)$ is a best response of player 2 to $x_1(\varepsilon)$ in the game Γ_θ^D for all $\theta \in [0, \theta(\varepsilon)]$. That $x_1(\varepsilon) \in \arg\max_{x_1 \in X} \pi_1(x_1, x_2(\varepsilon))$ is, on the other hand, easily verified. Hence, by choosing $\bar{\theta} = \theta(\varepsilon)$, we have $N\left(\Gamma_\theta^D\right) \neq \emptyset$ for all $\theta \in [0, \bar{\theta}]$. $\|$

Claim 3. Let $\hat{x}(\theta) \in N\left(\Gamma_\theta^D\right)$ for any $\theta \in (0, \bar{\theta}]$. There exists a $\theta_0 \in (0, \bar{\theta}]$ such that $\pi_2(\hat{x}(\theta)) > \pi_2(a, a)$ for all $\theta \in (0, \theta_0]$.

Proof of Claim 3. Take any sequence $\theta_n \in [0, \bar{\theta}]$ that converges to 0, and note that, by Claim 2, $N(\Gamma_{\theta_n}^D) \neq \emptyset$ for all n. Let $\hat{x}(\theta_n) \in N(\Gamma_{\theta_n}^D)$, $n \geqslant 1$. Since b_1 is strictly decreasing (due to strict submodularity of Γ^C), Proposition 1 implies that $\hat{x}_2(\theta_n) > a$ for all $n \geqslant 1$. There must then exist a subsequence $\hat{x}_2(\theta_{n_\ell})$ such that $\lim \hat{x}_2(\theta_{n_\ell}) \geqslant a$. Let $y_2 \equiv \lim \hat{x}_2(\theta_{n_\ell})$, and notice that $\hat{x}(\theta_{n_\ell}) \to (b_1(y_2), y_2)$ (as $\ell \to \infty$) by continuity of b_1. But then we must have $(b_1(y_2), y_2) \in N\left(\Gamma^C\right)$ since Nash equilibrium correspondence has a closed graph. Given the uniqueness of the equilibrium of Γ^C, it follows that $(b_1(y_2), y_2) = (a, a)$. Thus, since $\hat{x}_2(\theta_n) > a$ for all $n \geqslant 1$, there exists a positive integer ℓ^* such that $\hat{x}_2(\theta_{n_\ell}) \in (a, a+\bar{\varepsilon}]$ for all $\ell \geqslant \ell^*$, where $\bar{\varepsilon}$ is defined as in Claim 1. Applying Claim 1, we find $\pi_2(\hat{x}_2(\theta_{n_\ell})) > \pi_2(a, a)$ for all $\ell \geqslant \ell^*$. Choose $\theta_0 = \theta_{n_{\ell^*}}$ and the claim follows. $\|$

The proof of Proposition 2 is now easily completed. The Nash equilibrium of the subgame induced by the choice of $\theta = 0$, i.e. $N\left(\Gamma^C\right)$, is given by (a, a) where the payoff of owner 2 is $\pi_2(a, a)$. However, Claim 3 shows that there exists a $\theta_0 \in (0, 1]$ such that $\pi_2(\hat{x}(\theta)) > \pi_2(a, a)$ for all $\theta \in (0, \theta_0]$ which implies that $\theta = 0$ cannot obtain in any subgame perfect equilibrium of Γ^D. *Q.E.D.*

References

[1] Berge, C. (1963): *Topological Spaces*, Macmillan, New York.

[2] Bester, H. and W. Güth (1998): "Is Altruism Evolutionary Stable?" *Journal of Economic Behavior and Organization,* 34, 193-209.

[3] Bulow, J. I., J. D. Geanakopolos and P. D. Klemperer (1985): "Multimarket Oligopoly: Strategic Substitutes and Complements," *Journal of Political Economy*, 93, 488-511.

[4] Corchón, L. (1996). *Theories of Imperfectly Competitive Markets.* Berlin: Springer-Verlag.

[5] Dasgupta, P. and E. Maskin (1986): "The Existence of Equilibrium in Discontinuous Economic Games I: Theory," *Review of Economic Studies*, 53, 1-26.

[6] Eshel, I., L. Samuelson and A. Shaked (1998): "Altruists, Egoists and Hooligans in a Local Interaction Model," *American Economic Review*, 88, 157-79.

[7] Fershtman, C. and K.L. Judd (1987): "Incentive Equilibrium in Oligopoly," *American Economic Review,* 77, 927-40.

[8] Fershtman, C. and Y. Weiss (1998): "Social Rewards, Externalities and Stable Preferences," *Journal of Public Economics*, 70, 53-73.

[9] Holmström, B. (1982): "Moral Hazard in Teams," *Bell Journal of Economics*, 13, 324-40.

[10] Katz, M. (1991): "Game-Playing Agents: Unobservable Contracts as Precommitments," *RAND Journal of Economics*, 22, 307-28.

[11] Koçkesen, L., E.A. Ok and R. Sethi (1997). "The Strategic Advantage of Negatively Interdependent Preferences," C.V. Starr Center Working Paper 97-34, New York University.

[12] Koçkesen, L., E.A. Ok and R. Sethi (1999). "Evolution of Interdependent Preferences in Aggregative Games," *Games and Economic Behavior*, forthcoming.

[13] Koçkesen, L. and E.A. Ok (1999). "Strategic Delegation by Unobservable Incentive Contracts," mimeo, New York University .

[14] Milgrom, P. and C. Shannon (1994): "Monotone Comparative Statics," *Econometrica*, 62, 157-180.

[15] Ok, E. A. and L. Koçkesen (1997): "Negatively Interdependent Preferences," C.V. Starr Center Working Paper 97-02, New York University.

[16] Rhode, P. and M. Stegeman (1996): "Learning, Mutation, and Long-Run Equilibria in Games: A Comment," *Econometrica,* 64, 443-49.

[17] Riley, J. (1979): "Evolutionarily Equilibrium Strategies," *Journal of Theoretical Biology*, 76, 109-123.

[18] Salas Fumas, V. (1992): "Relative Performance Evaluation of Management," *International Journal of Industrial Organization*, 10, 473-89.

[19] Schaffer, M. (1988): "Evolutionary Stable Strategies for a Finite Population and a variable contest size," *Journal of Theoretical Biology*, 132, 469-78.

[20] Topkis, D.M. (1978): "Minimizing a Submodular Function on a Lattice," *Operations Research*, 26, 305-321.

[21] Topkis, D.M. (1979): "Equilibrium Points in Nonzero-Sum n-Person submodular games," *SIAM Journal on Control and Optimization*, 17, 773-787.

[22] Vega-Redondo, F. (1997): "The Evolution of Walrasian Behavior," *Econometrica*, 65, 375-84.

[23] Vickers, J. (1984): "Delegation and the Theory of the Firm," *Economic Journal*, Supplement, 95, 138-147.

[24] Vives, X. (1990): "Nash Equilibrium with Strategic Complementarities," *Journal of Mathematical Economics*, 19, 305-321.