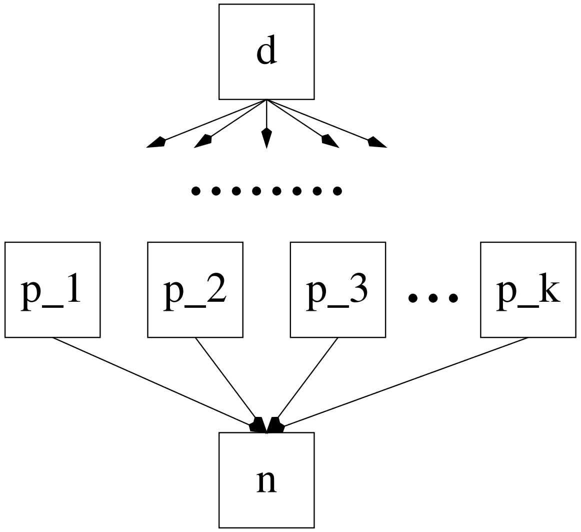
# The Back-End

- Chapter 17: Data Flow Analysis
  - 17.2: Data Flow Analysis (reaching definitions, available expressions, etc.)
  - 17.3: Using data flow analysis (constant propagation, common subexpression elimination, etc.)
  - 17.4: Speeding up data flow analysis (not responsible)
  - 17.5: Alias analysis
- Chapter 11 (11.1-11.4): Register Allocation (graph coloring, move coalescing, implementation: your last assignment)
- Chapter 18: Loop optimizations
  - 18.1: Finding loops (Ctrl-flow analysis)
  - 18.2: Loop-invariant computations
  - 18.3: Induction variables and strength reduction

# Dominators: Reminder

- Assume every Control Flow Graph (CFG) has *start* node $s_0$ with no predecessors.
- Node $d$ *dominates* node $n$ if every path of directed edges from $s_0$ to $n$ must go through $d$.
- Every node dominates itself.
- Consider:




- If $d$ dominates each of the $p_i$, then $d$ dominates $n$.
- If $d$ dominates $n$, then $d$ dominates each of the $p_i$.

# Loop Preheaders

**Recall:**

- A *loop* is a set of CFG nodes $S$ such that:
  1. there exists a *header* node $h$ in $S$ that dominates all nodes in $S$.
     - there exists a path of directed edges from $h$ to any node in $S$.
     - $h$ is the only node in $S$ with predecessors not in $S$.
  2. from any node in $S$, there exists a path of directed edges to $h$.
- A loop is a single entry, multiple exit region.

**Loop Preheaders:**

- Some loop optimizations (loop invariant code removal) need to insert statements immediately before loop header.
- Create a loop *preheader* - a basic block before the loop header block.

# Loop Preheader Example

# Loop Invariant Computations

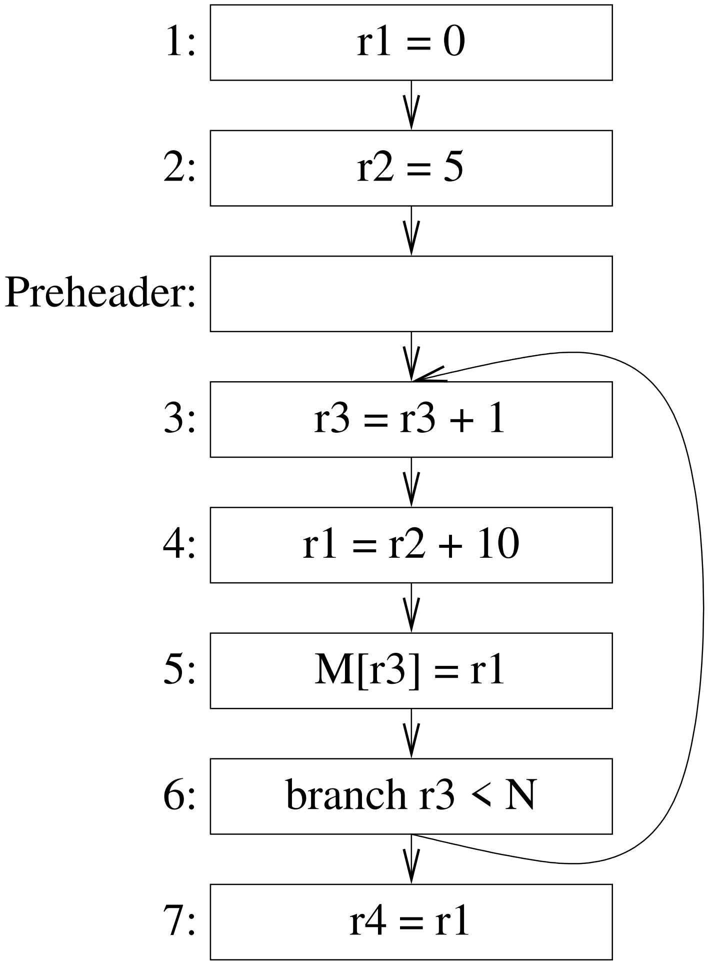
- Given statements in loop $s$: `t = ` $a_1$ ` op ` $a_2$:
  - $s$ is loop-invariant if $a_1$, $a_2$ have same value each loop iteration.
  - may sometimes be possible to hoist $s$ outside loop.
- Cannot always tell whether $a$ will have same value each iteration $\rightarrow$ conservative approximation.
- $d$: `t = ` $a_1$ ` op ` $a_2$ is loop-invariant within loop $L$ if for each $a_i$:
  1. $a_i$ is constant, or
  2. all definitions of $a_i$ that reach $d$ are outside $L$, or
  3. only one definition of $a_i$ reaches $d$, and is loop-invariant.

# Loop Invariant Computation: Algorithm

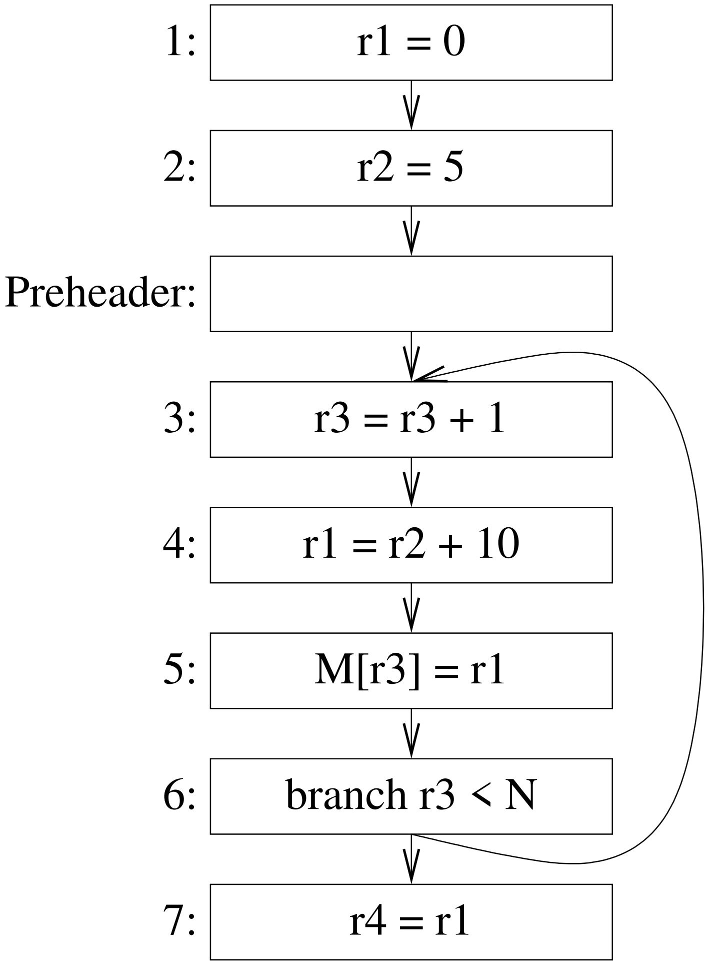
Iterative algorithm for determining loop-invariant computations:

- mark ”invariant” all definitions whose operands
  - are constant, or
  - whose reaching definitions are outside loop.
- WHILE (changes have occurred) mark ”invariant” all definitions whose operands
  - have a single reaching definition in loop marked invariant.

# Loop Invariant Code Dectection



# Loop Invariant Code Dectection

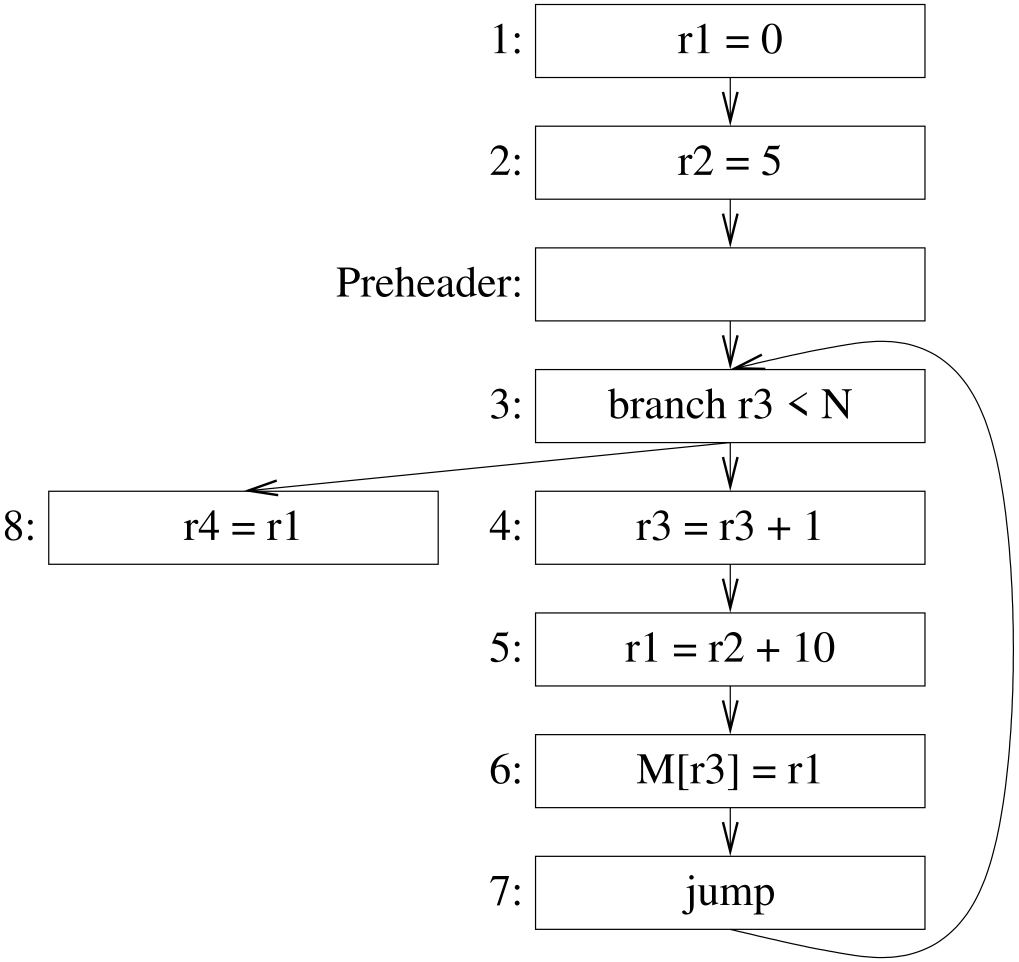



Hoist loop-invariant code to pre-header — **when possible**.

# Loop Invariant Code Motion: Constraint 1

*d*: `t = a op b`

*d* must dominate all loop exit nodes where `t` is live out.



# Loop Invariant Code Motion: Constraint 1

$d$: `t = a op b`

$d$ must dominate all loop exit nodes where `t` is live out.

Problem with while loops: Loop exit dominates all nodes in the loop. eg:

```
while x > 0 {x = x - 1; ...}
```

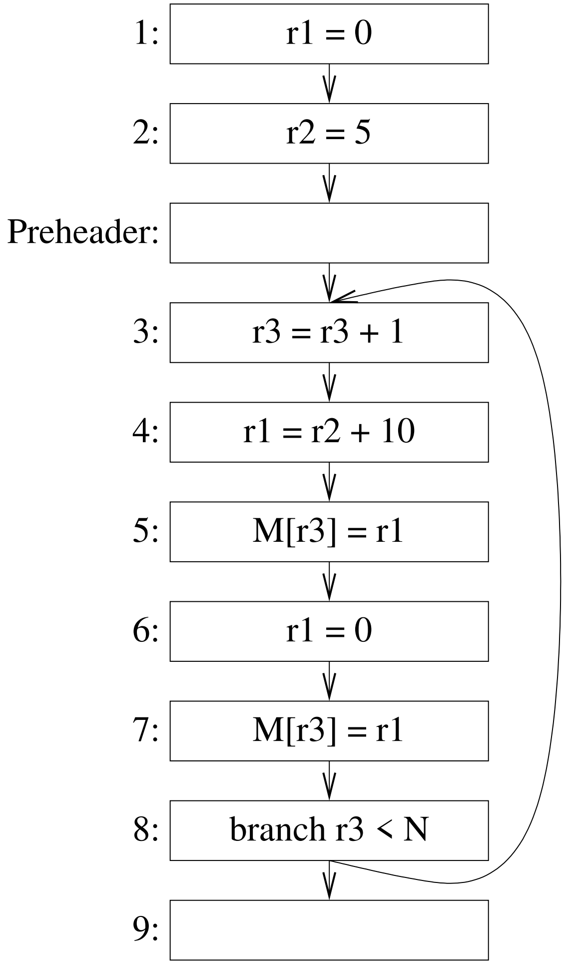
Solution: Transform into repeat:

```
if (x > 0) {
  x = x - 1; ...;
  repeat {x = x - 1; ...} until (x <= 0)
}
```

# Loop Invariant Code Motion: Constraint 2

$d$: `t = a op b`

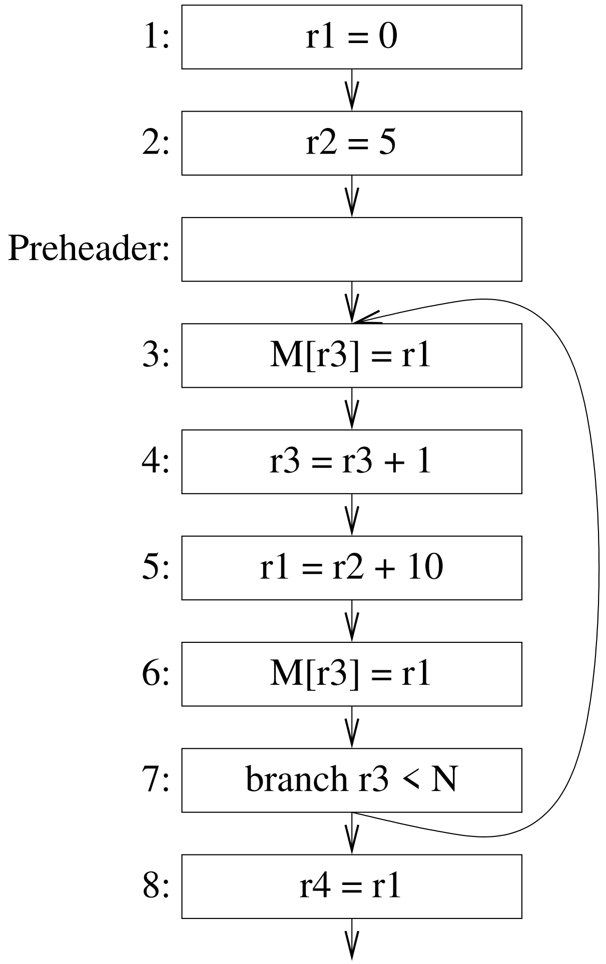
there must be only one definition of `t` inside loop.



# Loop Invariant Code Motion: Constraint 3

$d$: `t = a op b`

$t$ must not be live-out of loop preheader node (live-in to loop)



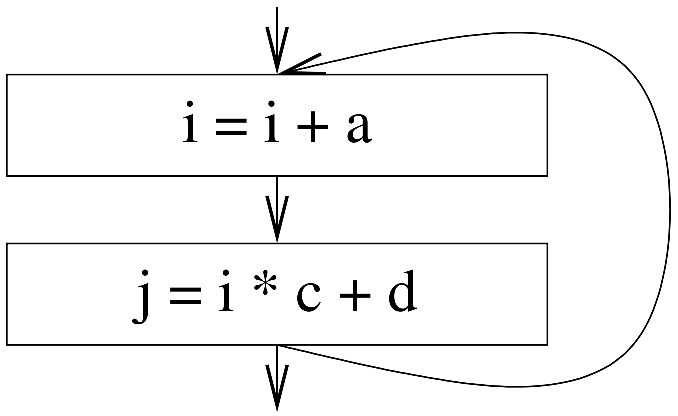
# Loop Invariant Code Motion

Algorithm for code motion:

- Examine invariant statements of $L$ in same order in which they were marked.
- If invariant statement $s$ satisfies three criteria for code motion, remove $s$ from $L$, and insert into preheader node of $L$.

# Induction Variables

Variable `i` in loop $L$ is called an **induction variable** of $L$ if each time `i` changes value in $L$, it is incremented/decremented by loop-invariant value.




Assume `a, c` loop-invariant.

- `i` is an induction variable
- `j` is an induction variable
  - `j = i * c` is equivalent to `j = j + a * c`
  - compute `e = a * c` outside loop: `j = j + e` ⇒ **strength reduction**
  - may not need to use `i` in loop ⇒ **induction variable elimination**

# Induction Variable Detection

Scan loop $L$ for two classes of induction variables:

- **basic** induction variables - variables (`i`) whose only definitions within $L$ are of the form `i = i + c` or `i = i - c`, `c` is loop invariant.
- **derived** induction variables - variables (`j`) defined only once within $L$, whose value is linear function of some basic induction variable $L$.

Associate triple `(i, a, b)` with each induction variable `j`

- `i` is basic induction variable; `a` and `b` are loop invariant.
- value of `j` at point of definition is `a + b * i`
- `j` belongs to the family of `i`

# Induction Variable Detection: Algorithm

Algorithm for induction variable detection:

- Scan statements of $L$ for basic induction variables `i`
  - for each `i`, associate triple `(i, 0, 1)`
  - `i` belongs to its own family.
- Scan statements of $L$ for derived induction variables `k`:
  1. there must be single assignment to `k` within $L$ of the form `k = j * c` or `k = j + d`, `j` is an induction variable; `c,d` loop-invariant, and
  2. if `j` is a derived induction variable belonging to the family of i, then:
     - the only definition of `j` that reaches `k` must be one in $L$, and
     - no definition of `i` must occur on any path between definition of `j` and definition of `k`

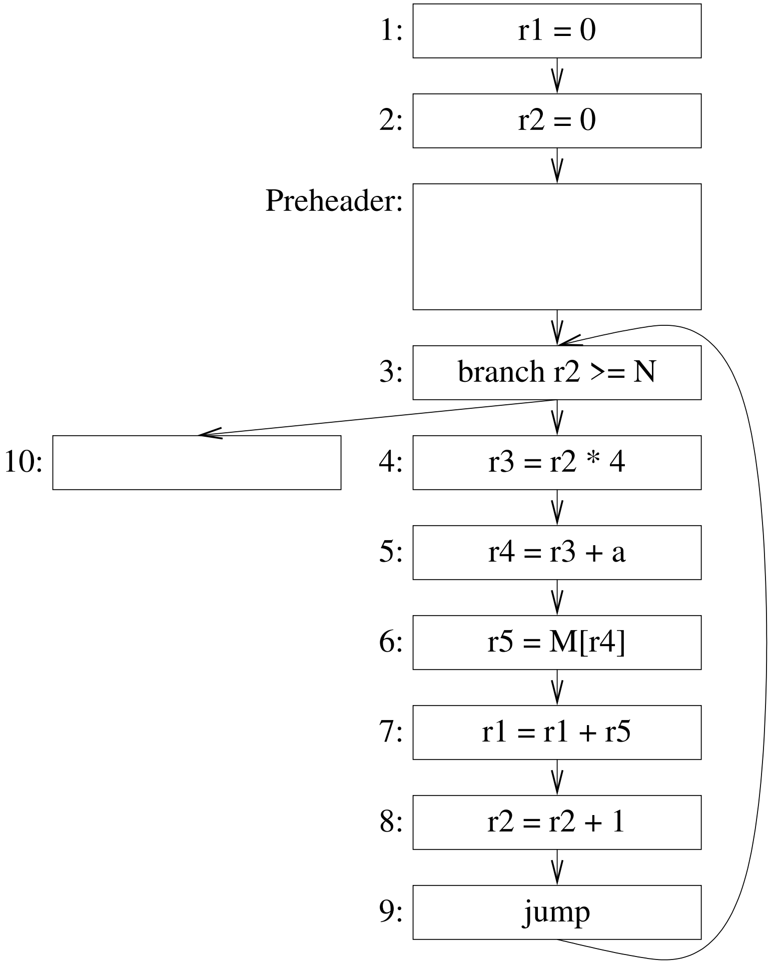
# Induction Variable Detection: Algorithm

Algorithm for induction variable detection:

- Assume `j` associated with triple `(i, a, b)`: `j = a + b * i` at point of definition.
- Can determine triple for `k` based on triple for `j` and instruction defining `k`:
  - `k = j * c` $\rightarrow$ `(i, a*c, b*c)`
  - `k = j + d` $\rightarrow$ `(i, a + d, b)`

# Induction Variable Detection: Example

```
s = 0;
for(i = 0; i < N; i++)
  s += a[i];
```



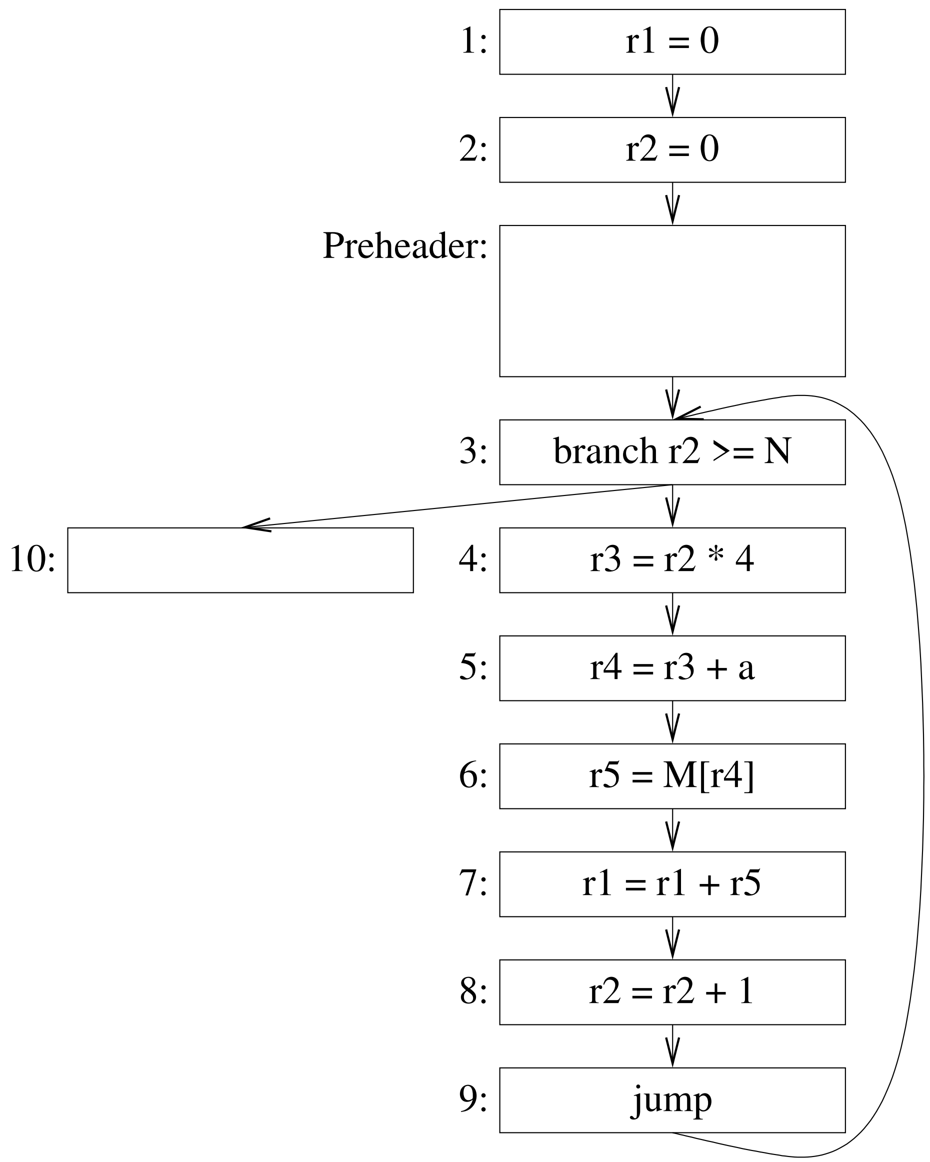
# Strength Reduction

1. For each derived induction variable `j` with triple `(i, a, b)`, create new `j'`.
   - all derived induction variables with same triple `(i, a, b)` may share `j'`
2. After each definition of `i` in $L$, `i = i + c`, insert statement:
   `j' = j' + b * c`
   - `b * c` is loop-invariant and may be computed in preheader or during compile time.
3. Replace unique assignment to `j` with `j = j'`.
4. Initialize `j'` at end of preheader node:

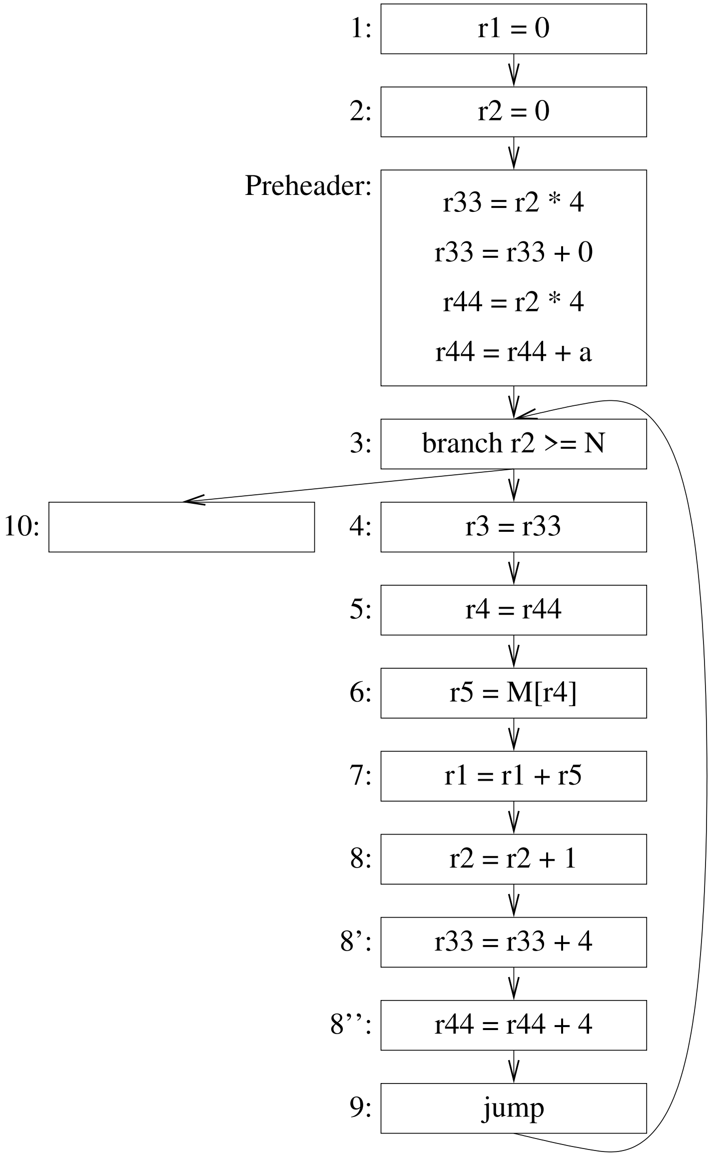
```
j' = b * i
j' = j' + a
```

- Strength reduction still requires multiplication, but multiplication now performed outside loop.
- `j'` also has triple `(i, a, b)`

# Strength Reduction: Example



# Strength Reduction: Example

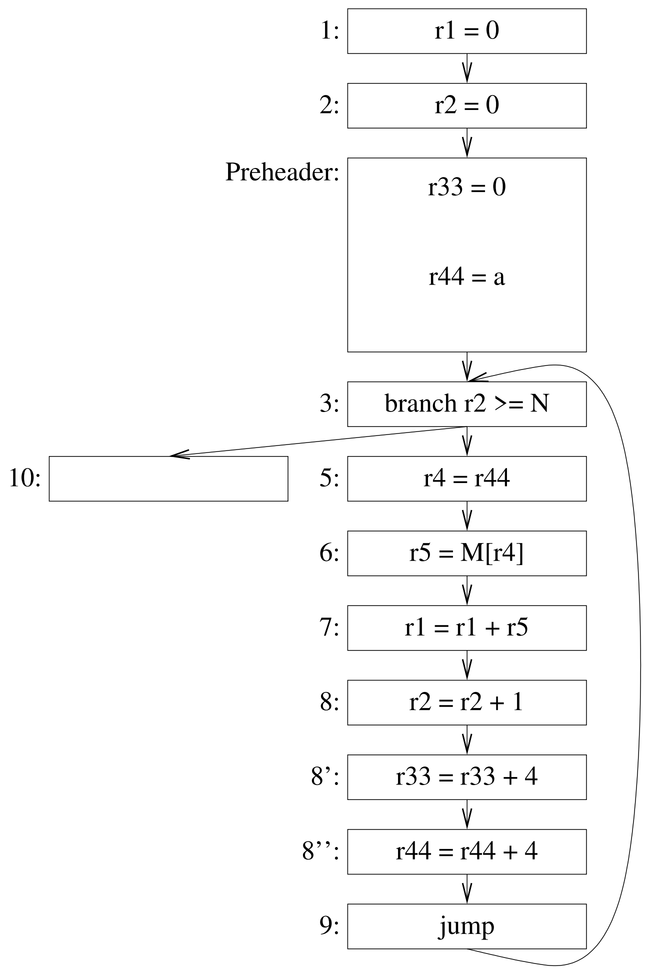


# Induction Variable Elimination

After strength reduction has been performed:

- some induction variables are only used in comparisons with loop-invariant values.
- some induction variables are *useless*
  - dead on all loop exits, used only in definition of itself.
  - dead code elimination will not remove useless induction variables.

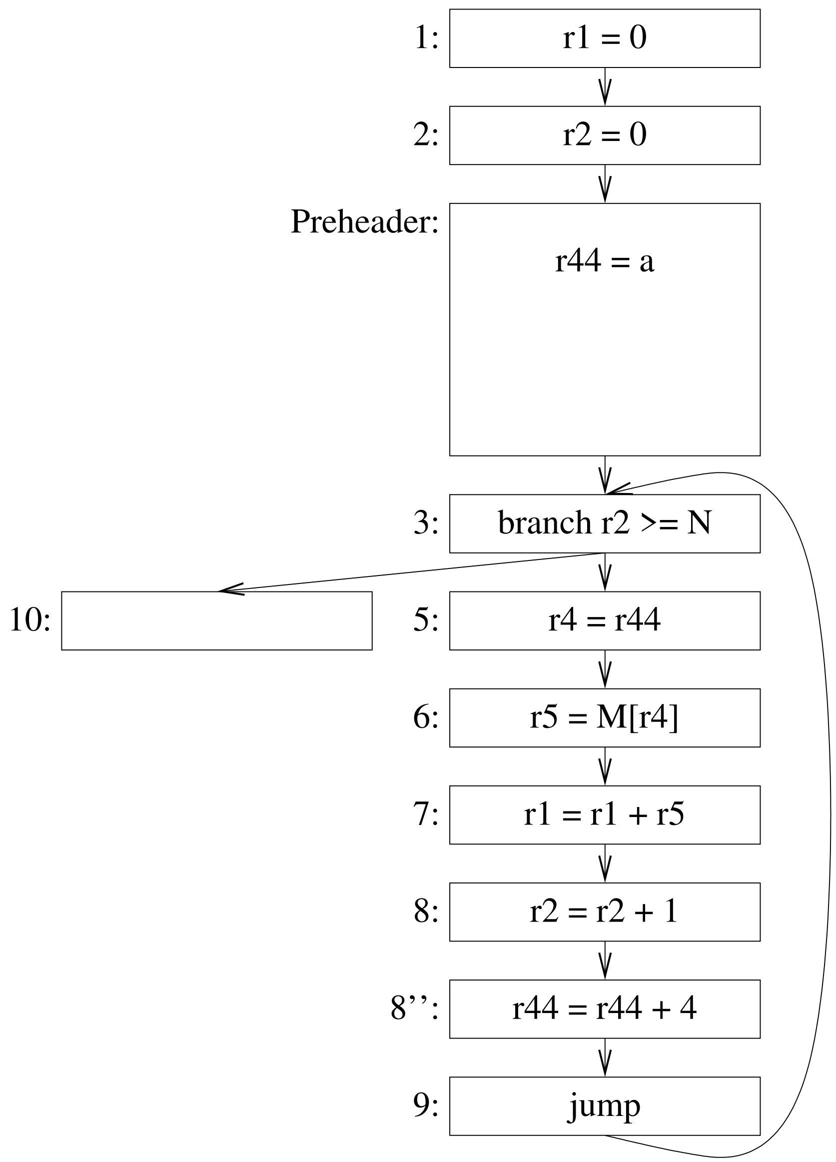
# Induction Variable Elimination: Example



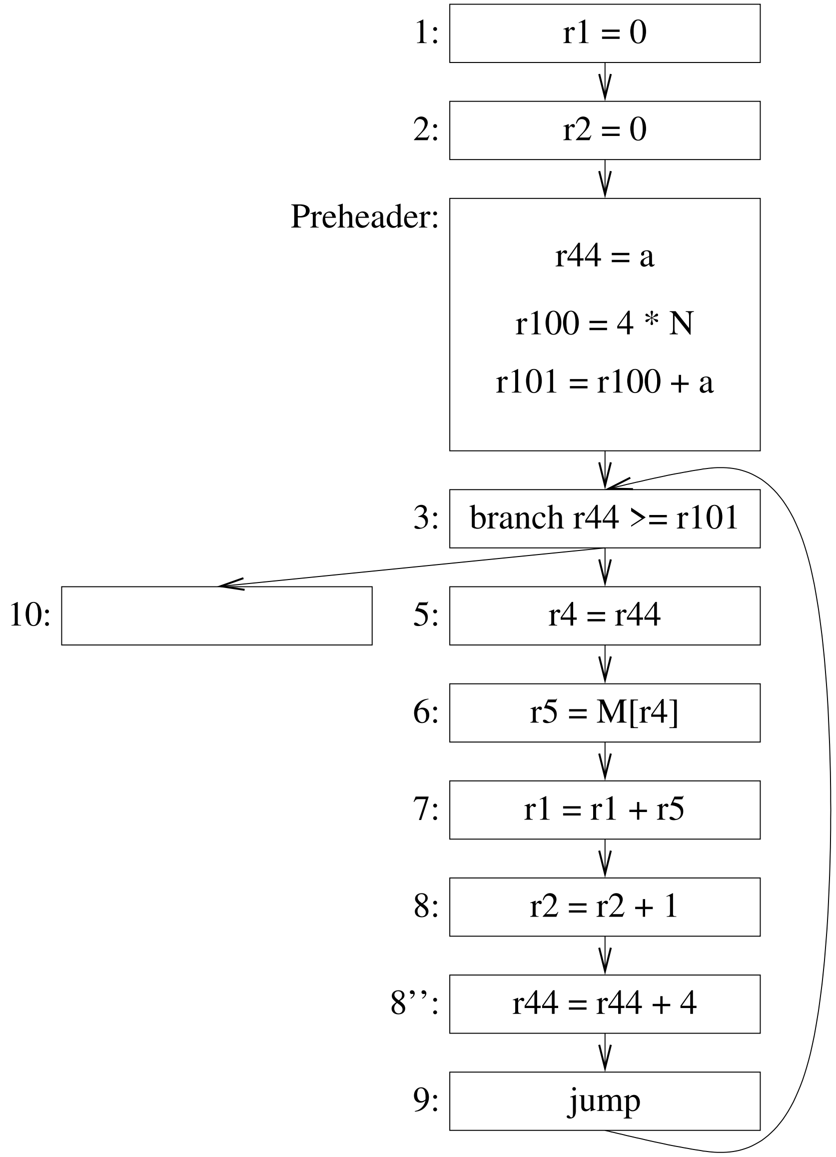
# Induction Variable Elimination

- Variable `k` is *almost useless* if it is only used in comparisons with loop-invariant values, and there exists another induction variable `t` in the same family as `k` that is not useless.
- Replace `k` in comparison with `t`
  $\rightarrow$ `k` is useless

# Induction Variable Elimination: Example



# Induction Variable Elimination: Example



# Loop Unrolling

Read Section 18.5 on loop unrolling by next lecture.