COS 598A, Automated Theorem Proving
Spring 2003: Assignment 4

January 7, 2003

This assignment is to be done in the same logic (taken from "Hints on Proving Theorems in Twelf") as the last assignment.

State and prove the following theorems in Twelf. You may make use of any lemmas you find in the `core` directory. The axioms of our logic can be found in the `coreTCB` directory. Submit the file `as4.elf` electronically.

1. Let's start with something simple. Prove the $2+3=5$ and that for all $x \in \mathcal{R}$, $(x+2)+3 = x+5$. (Make your life eazier by using the first proof to help you complete the second - also remember that `congr` is your friend).
2. Prove that for all $x \in \mathcal{R}$[1], `succ (pred (pred (succ x))) = x`.
3. Prove that for $x_1, y_1, x_2, y_2 \in \mathcal{R}$, if $x_1 > y_1$, and $x_2 > y_2$ then $x_1 + x_2 > y_1 + y_2$.
4. Prove that for all $x \in \mathcal{R}$, $0 \cdot x = x \cdot 0 = 0$.
5. Prove that for all $x \in \mathcal{R}$, $-(-x) = x$, and that $x, y \in \mathcal{R}$, $(-x) \cdot y = x \cdot (-y)$.
6. Prove that for $x, y \in \mathcal{R}$, if $x \neq y$ then $x - y \neq 0$.
7. Prove that for $i, j \in \mathcal{Z}$, if $i > j$ then $i \geq j+1$. Note that this is not true for the reals in general (give a counter example in your solutions file as a comment before this proof).

[1] $\mathcal{R}$ = The real numbers, $\mathcal{Z}$ = The integers, $\mathcal{N}$ = The natural numbers (including zero).