COS 598A, Automated Theorem Proving
Spring 2003: Assignment 3

Get the tar file `proving.tar` from the web site and untar it in your working directory. Start the Twelf server as usual and proceed to complete the following exercises in this file.

State and prove the following theorems in Twelf. You may make use of any lemmas you find in the `core` directory. The axioms of our logic can be found in the `coreTCB` directory. To turn in this assignment simply email file `as3.elf` to your TA.

1. Prove the following sequents:
   (a) $\forall x \forall y P(x,y) \vdash \forall y \forall x P(x,y)$.
   (b) $\vdash \forall F \forall x \forall y F(x) \wedge \neg F(y) \rightarrow x \neq y$.
   (c) $\exists x (P(x) \wedge Q(x)), \forall y (P(y) \rightarrow R(y)) \vdash \exists x R(x) \wedge Q(x)$.

2. In file `coredefs.elf` which you can find in directory `coreTCB` we find the definition of $\leftrightarrow$ from assignment 2. Here it is called `equiv`. State and prove the following four sequents:
   (a) $A, B \vdash A \leftrightarrow B$
   (b) $\neg A, \neg B \vdash A \leftrightarrow B$
   (c) $\neg A, B \vdash \neg(A \leftrightarrow B)$
   (d) $A, \neg B \vdash \neg(A \leftrightarrow B)$
   (e) $A \leftrightarrow B \vdash \neg A \leftrightarrow \neg B$.

3. Again in file `coredefs.elf` we find the definition of `exists_uniq`. Prove the following introduction and ellimination rules for it:

```
exists_uniq_i:
  pf (A X) ->
    pf (forall [Y] A Y imp eq X Y) ->
       pf (exists_uniq A).

exists_uniq_i1:
  pf (A X) ->
   ({Y} pf (A Y) -> pf (eq X Y)) ->
       pf (exists_uniq A).

exists_uniq_e:
 pf (exists_uniq A) ->
  ({X:tm T}
    pf (A X) ->
     pf (forall [Y] A Y imp eq X Y) ->
      pf B) ->
   pf B.

exists_uniq_e1:
  pf (exists_uniq A) ->
   ({X:tm T} pf (A X) -> pf B) -> pf B.

exists_uniq_e2:
   pf (exists_uniq A) ->
     pf (A X) -> pf (A Y) -> pf (eq X Y).
```

4. Prove that $(c+b)+((a-1)\cdot b) = c+a\cdot b$. I've provided lots of hints in "lemma4" of the as3.elf file; there are 10 bigholes. If you can't fill in all the holes, leave some as bigholes for partial credit. (For a more interesting challenge, prove it without the hints.)