PRINCETON UNIVERSITY COS 522: COMPUTATIONAL COMPLEXITY

# Homework 1

Out: *February 14* Due: *February 26*

*You can collaborate with your classmates, but be sure to list your collaborators with your answer. Also, limit your answers to one page or less —you just need to give enough detail to convince me. If you suspect a problem is open, just say so and give reasons for your suspicion.*

§1 A *branching program with inputs* $x_1, x_2, \ldots, x_n$ is a directed graph with outdegree 2 where every node is labelled with one of the $x_i$'s and the two outgoing edges are labelled 0 and 1. There is a special "start" node and a node labelled "accept" that has no outgoing edges. The branching program computes some boolean function on $n$-bit inputs (i.e., assignments to $x_1, x_2, \ldots, x_n$) as follows: starting in the "start" node we follow a path, taking at every node —labelled say with $x_i$— the edge that is labelled with the value of $x_i$. See if the path ends in the accept node.

(a) Show that if $L \in \mathbf{SPACE}(s(n))$ then for every $n$ the decision problem for $L$ can be computed using a branching program of size $2^{O(S(n))}$ size.

(b) What is the complexity of the following decision problem: Given two $n$-input branching programs of size $\text{poly}(n)$, identify whether they compute the same function.

§2 Show that $\Sigma_2^p = \mathbf{NP}^{\text{SAT}}$.

§3 Show that $\mathbf{SPACE}(n) \neq \mathbf{NP}$. (Hint: Use padding, mentioned in the notes for Lecture 1.)

§4 Can you give a definition of **NEXPTIME** in terms of certificates as we did for **NP**? If not, report your best attempt.

§5 Say that a class $C_1$ is *superior to* a class $C_2$ if there is a machine $M_1$ in class $C_1$ such that for every machine $M_2$ in class $C_2$ and every large enough $n$, there is an input of size between $n$ and $n^2$ on which $M_1$ and $M_2$ answer differently.

(a) Is $\mathbf{DTIME}(n^{1.1})$ superior to $\mathbf{DTIME}(n)$?

(b) Is $\mathbf{NTIME}(n^{1.1})$ superior to $\mathbf{NTIME}(n)$?

§6 Suppose we define the *logspace hierarchy* in analogy with the polynomial hierarchy using logspace machines that can use alternation. Does this hierarchy collapse by Immerman's theorem ($\mathbf{NL} = \mathbf{coNL}$)?