String Search

Brute force
Rabin-Karp
Knuth-Morris-Pratt
Right-Left scan

String Searching

Text with N characters
Pattern with M characters

- match existence: any occurence of pattern in text?
- enumerate: how many occurences?
- match: return index of any occurence ← focus of this lecture
- all matches: return indices of all occurences

Sample problem: find avoctdfytvv in

```
kvjlixapejrbxeenpphkhthbkwyrwamnugzhppfxiyjyanhapfwbghx
mshrlyujfjhrsovkvveylnbxnawavgizyvmfohigeabgksfnbkmffxj
ffqbualeytqrphyrbjqdjqavctgxjifqgfgydhoiwhrvwqbxgrixydz
bpajnhopvlamhhf[redacted]ggikngkwzixgjtlxkozjlefilbrboi
gnbzsudssvqymnapbpqvlubdoyxkkwhcoudvtkmikansgsutdjythzl
apawlvliygjkmxorzeoafeoffbfxuhkzukeftnrfmocylculksedgrd
tvayjpgkrtedehwhrvvbbltdkctq
```

Assume that N >> M >> number of occurences

String searching context

Find M-char pattern in N-char text

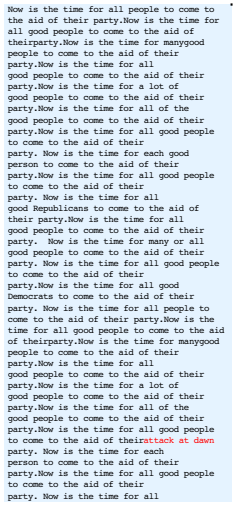
Applications

- word processors
- virus scanning
- text information retrieval (ex: Lexis/Nexis)
- digital libraries
- computational biology
- web search engines

Theoretical challenge: linear-time guarantee

- suffix-trie index costs ~NlgN

Practical challenge: avoid BACKUP



Modelling String Searching

Random pattern or text??

Sample problem 1: find unwillingly in

```
kvjlixapejrbxeenpphkhthbkwyrwamnugzhppfxiyjyanhapfwbghx
mshrlyujfjhrsovkvveylnbxnawavgizyvmfohigeabgksfnbkmffxj
ffqbualeytqrphyrbjqdjqavctgxjifqgfgydhoiwhrvwqbxgrixydz
bpajnhopvlamhhfavoctdfytvvggikngkwzixgjtlxkozjlefilbrboi
gnbzsudssvqymnapbpqvlubdoyxkkwhcoudvtkmikansgsutdjythzl
```

← random text

Sample problem 2: find avoctdfytvv in ← random pattern

```
all the world's a stage and all the men and women merely
players. They have their exits and their entrances, and
one man in his time plays many parts.  At first, the infant,
mewling and puking in the nurse's arms.  Then the whining
schoolboy, with his satchel and shining morning face, creeping
like snail unwillingly to school. And then the lover, sighing
```

Simple, effective algorithm: return "NOT FOUND"

- probability of match is less than N/(alphabet size)^M

.0000000000000084 for sample problems

Brute-force string searching

Check for pattern at every text position

using array to simplify alg descriptions
online apps need getchar()-based implementation

```
int brutesearch(char p[], char a[]) {
    int i, j;
    for (i = 0; i < strlen(a); i++) {
      for (j = 0; j < strlen(p); j++)          text loop
        if (a[i+j] != p[j]) break;             pattern loop
      if (j == strlen(p)) return i;
    }
    return strlen(a);
}
```

match

mismatch

- returns `i` if leftmost pattern occurence starts at `a[i]`

Problem with brute-force implementation

```
for (i = 0; i < strlen(a); i++)
```

In C, `strlen` takes time proportional to string length

- evaluated every time through loop
- running time is at least N^2
- same problem for simpler programs (ex: count the blanks)

PERFORMANCE BUG

Textbook example: Performance matters in ADT design

Exercise: implement string ADT with fast `strlen`

- need space to store length

Brute-force string searching (bug fixed)

Check for pattern at every text position

```
int brutesearch(char p[], char a[]) {
   int M = strlen(p), N = strlen(a);
   int i, j;
   for (i = 0; i < N; i++) {
      for (j = 0; j < M; j++)                  text loop
        if (a[i+j] != p[j]) break;             pattern loop
      if (j == M) return i;
   }
   return N;
}
```

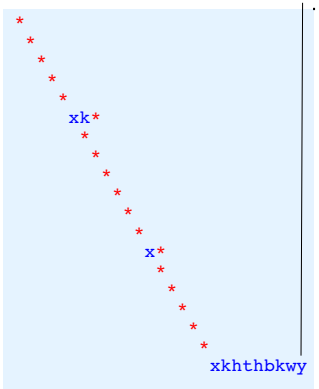
lengths won't change; precompute them

- returns `i` if leftmost pattern occurence starts at `a[i]`

Brute-force typical case

pattern: `xkhthbkwy`

text: `kvjlixkpejrbxeenppxkhthbkwy`



Brute-force worst case

pattern: 000000001

text: 00000000000000000000000001

```
00000000*
 00000000*
  00000000*
   00000000*
    00000000*
     00000000*
      00000000*
       00000000*
        00000000*
         00000000*
          00000000*
           00000000*
            00000000*
             00000000*
              00000000*
               00000000*
                00000001
```

Backs up in text:
need to retain M-char buffer

Rabin-Karp algorithm

Idea 1: Use hashing

- compute hash function for each text position
- NO TABLE needed: just compare with pattern hash

Example: search for 59265 in 3141592653589793238462643 3

pattern hash: 59265 = 95 (mod 97)

text hashes: 3141592653589793238462643 3

```
31415 = 84 (mod 97)
 14159 = 94 (mod 97)
  41592 = 76 (mod 97)
   15926 = 18 (mod 97)
    59265 = 95 (mod 97)
```

Rabin-Karp algorithm (continued)

Idea 2: Use hash for previous position to compute hash

```
14159     = (   31415     -  30000      )*10 + 9

14159 mod 97 = (31415 mod 97 - 30000 mod 97)*10 + 9 (mod 97)
                  ↓ known from previous position
             = (    84       -    3*9      )*10 + 9 (mod 97)
                                  ↑ precompute 9 = 10000 (mod 97)

             = 579 mod 97 = 94
```

Key point: all ops involve small numbers
No restriction on N and M

Example: search for 59265 in 3141592653589793238462643 3

pattern hash: 59265 = 95 (mod 97)

text hashes for 3141592653589793238462643 3:

```
31415 mod 97 = 84
14159 mod 97 = (84 - 3*9)*10 + 9 (mod 97) = 94
41592 mod 97 = (94 - 1*9)*10 + 2 (mod 97) = 76
15926 mod 97 = (76 - 4*9)*10 + 6 (mod 97) = 18
59265 mod 97 = (18 - 1*9)*10 + 5 (mod 97) = 95
```

Implementation of Rabin-Karp algorithm

```
#define q 3355439
#define d 256
```

random q much larger than N
q much smaller than (alphabet size)^M
d*q smaller than max integer

```
int rksearch(char *p, char *a)
  { int M = strlen(p), N = strlen(a);
    int i, j, dM = 1, h1 = 0, h2 = 0;
    for (j = 1; j < M; j++)                    radix hash
      dM = (d*dM) % q;
    for (j = 0; j < M; j++)                    pattern hash
      h1 = (h1*d + p[j]) % q;
    for (i = 0; i < M; i++)                    initial text hash
      h2 = (h2*d + a[i]) % q;
    for (i = M; i < N; i++)                    main search loop
      {
        if (h1 == h2) return i-M;
        h2 = (h2 + d*q - a[i-M]*dM) % q;
```

Example for q = 97, d = 10

10000 (mod 97) = 9
10*10*10*10
mod after each op

59265 (mod 97) = 95
Horner's method
mod after each op

31415 (mod 97) = 84
Horner's method
mod after each op

31415 (mod 97) = 84

95 != 84

84 - 3*9 (mod 97) = 57

Randomized algorithms

A randomized algorithm uses random numbers to gain efficiency

- quicksort with random partitioning element
- randomized BSTs
- Rabin-Karp

Las Vegas algorithm

- expected to be fast
- guaranteed to be correct

Examples: quicksort, randomized BSTs, Rabin-Karp with match check

Monte Carlo algorithm

- guaranteed to be fast

String search implementations cost summary

Search for an M-character pattern in an N-character text

	typical	worst
brute-force	$N^{=}$	N*M
Rabin-Karp	$N^{=}$	$N^{=}$

= assumes appropriate model

= assumes system can produce "random" numbers

Knuth-Morris-Pratt algorithm

Observation: On mismatch at pattern char `j` we know the previous `j-1` chars in the text (they are also in the pattern)

Idea: precompute what to do on mismatch

Example 1: mismatch `00000*` when searching for `000001` in binary text

- text had `000000`
- compare next text char with last pattern char

Example 2: mismatch `000*` when searching for `000001` in binary text

- text had `0001`
- compare next text char with first pattern char

char to check is completely deduced from pattern

KMP algorithm

- precompute table of pattern char to check on mismatch, indexed by pattern position

KMP examples

pattern: `10100110`
text: `10011101001010101010011000011 1`

```
10*
   1*
    1*
     101001*
          1010*
            1010*
              1010*
                10100110
0120111234562343434345678
```

mismatch table

0	1	2	3	4	5	6	7
0	1	0	1	3	0	2	1

pattern: `00000001`
text: `00100000100000000000001`

```
00*
   00000*
         0000000*
          0000000*
           0000000*
            0000000*
             0000000*
              0000000*
```

mismatch table

0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	7

KMP implementation

Check for pattern at every text position

- char match: increment both `i` and `j`
- char mismatch: set `j` to `mismatch[j]`

```
int kmpsearch(char p[], char a[], int mismatch[])
  { int M = strlen(p), N = strlen(a);
    int i, j = 0;
    for (i = 0; i < N; i++) {
      if (a[i] == p[j]) j++;
      else j = mismatch[j];
      if (j == M) return i-M+1;
    }
   return N;
  }
```

no pointer backup in this version

Differs from brute-force in two very significant ways

KMP mismatch table construction

Table builds itself (!!)

mismatch table

0	1	2	3	4	5	6	7
0	1	0	1	3	0	1	0

Idea 1: Simulate restart ala brute-force

Ex: `mismatch[6]` for `10100110`

- if mismatch at `101001*` then text was `1010010`
- for [redacted]`010010x`, `x` compares to `p[0]`
- note also: for [redacted]`010011x`, `x` compares to `p[1]`

```
100111010010[redacted]
   101001*
    1010010110
```

pattern: 10100110
text: 010010x
*
10*
10
0012012

Idea 2: Remember simulation for previous entry

Ex: `mismatch[7]` for `10100110`

- if mismatch at `1010011*` then text was `10100111`
- just noted: for [redacted]`010011x`, `x` compares to `p[1]`

pattern: 10100110
text: 010011x
*
10*
1*
0012011

KMP mismatch table construction implementation

`mis[j]`: index of pattern char to compare against next text char on mismatch on jth pattern character

`t`: index of pattern char that brute-force algorithm would compare against next text char on iteration after mismatch

To compute `mis[j]`, compare `p[j]` with `p[t]`

match:

- `mismatch[j] = mismatch[t]` since mismatch action same as for `t`
- `t = t+1` since we know that brute-force algorithm will find match

mismatch: opposite assignment

```
t = 0; mismatch[0] = 0;
for (int j = 1; j < M; j++)
  if (p[j] == p[t])
    { mismatch[j] = mismatch[t]; t = t+1; }
  else
```

		0	1	2	3	4	5	6	7
		1	1	1	0	0	1	1	0
j	t								
	0	0							
1	0	0	1						
2	1	0	1	0					

Optimized KMP implementation

Easy to create specialized program for given pattern (build in mismatch table)

assumes pattern in text (can use sentinel)

pattern 10100110

```
int kmpsearch(char a[])
  { int i = 0;
    s0: if (a[i] != '1') { i++; goto s0; }
    s1: if (a[i] != '0') { i++; goto s1; }
    s2: if (a[i] != '1') { i++; goto s0; }
    s3: if (a[i] != '0') { i++; goto s1; }
    s4: if (a[i] != '0') { i++; goto s3; }
    s5: if (a[i] != '1') { i++; goto s0; }
    s6: if (a[i] != '1') { i++; goto s2; }
    s7: if (a[i] != '0') { i++; goto s1; }
    return i-8;
  }
```

mismatch table

String search implementations cost summary

Search for an M-character pattern in an N-character text

	typical	worst	
brute-force	$N^{=}$	N*M	
Rabin-Karp	$N^{=}$	$N^{=}$	← inner loop with several arithmetic instructions
Knuth-Morris-Pratt	N	N	← tiny inner loop

= assumes appropriate model

= assumes system can produce "random" numbers

Right-left pattern scan

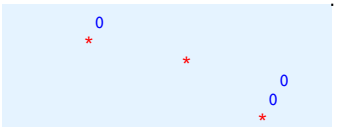
Sublinear algorithms

- move right to left in pattern
- move left to right in text

Q: Does binary string have 9 consecutive 0s?

pattern: 000000000

text: 100111010010100010100111000111



A: No. (Needed to look at only 6 of 30 chars.)

Idea effective for general patterns, larger alphabet

Right-left scan example

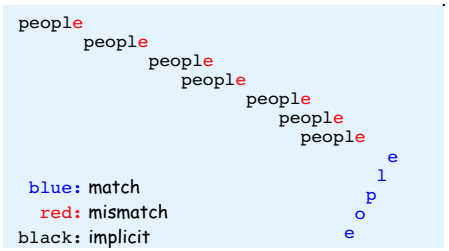
Text char not in pattern: skip forward M chars

Text char in pattern: skip to end of pattern

char	skip
a	6
b	6
...	6
e	0
...	6
l	1
...	6
o	3
p	2
...	6
z	6

pattern: people

text: now is the time for all good people to



Implementation of right-left pattern scan

build skip table

```
void initskip(char *p)
  { int j, M = strlen(p);
    for (j = 0; j < 256; j++) skip[j] = M;
    for (j = 0; j < M; j++) skip[p[j]] = M-j-1;
  }

int mischarsearch(char *p, char *a)
  { int M = strlen(p), N = strlen(a);
    int i, j;
    initskip(p);
    for (i = M-1, j = M-1; j >= 0; i--, j--)
      while (a[i] != p[j])
        {
          i += max(M-j, skip[a[i]]);
          if (i >= N) return N;
          j = M-1;
```

right-to-left scan

String search implementations cost summary

Search for an M-character pattern in an N-character text

	typical	worst
brute-force	$N^{=}$	N*M
Rabin-Karp	$N^{=}$	$N^{=}$
Knuth-Morris-Pratt	N	N
Boyer-Moore	$N/M^{=}$	N

beats optimal by a factor of 100 for M = 100

= assumes appropriate model

= assumes system can produce "random" numbers

String search summary

Ingenious algorithms for a fundamental problem

Rabin-Karp

- easy to implement
- extends to more general settings (ex: 2D search)

Knuth-Morris-Pratt

- quintessential solution to theoretical problem
- works well in practice, too (no backup, tight inner loop)

Right-left scan

- simple idea leads to dramatic speedup for long patterns

Tip of the iceberg (stay tuned)