COS 226 Lecture 12: String searching

TEXT: N characters
PATTERN: M characters

Existence: Any occurrence of pattern in text?
Enumerate: How many occurrences of pattern in text?
Search: Find an occurence of pattern in text
Search all: Find all occurences of pattern in

Three parameters N, M, C (number of occurences)

- start with N >> M >> C
- Ex: N = 100,000, M = 100, C = 5

Other factors

- multiple patterns (preprocessing)
- binary vs. ascii text
- avoid backup in text

String searching examples

Text string: find gjkmxorzeoa in

```
kvjlixapejrbxeenpphkhthbkwyrwamnugzhppfx
iyjyanhapfwbghxmshwrlyujfjhrsovkvveylnbx
nawavgizyvmfohigeabgksfnbkmffxjffqbualey
tqrphyrbjqdjqavctgxjifqgfgydhoiwhrvwqbxg
rixydzbpajnhopvlamhhfavoctdfytvvggikngkw
zixgjtlxkozjlefilbrboignbzsudssvqymnapbp
qvlubdoyxkkwhcoudvtkmikansgsutdjythzlapa
wlvliygjkmxorzeoafeoffbfxuhkzukeftnrfmoc
ylculksedgrdivayjpgkrtedehwhrvvbbltdkctq
```

Binary: find 101001111010111011 in

```
1000100101011010001001101011010110110110
0111111110101001110101000000010111101001
1011100000011000110000100110000111101101
1100000011011110101111101111000111100001
1011111001110011001111011001000000011101
1111100001011001110011010010110101001001
0001111111111011100111101100101010011110
1111001110110010010010011000100010011100
0010000001110001010111010100110110010001 1
```


String search analysis

Idea to test algorithms:

- use random pattern or random text

Ex: binary text

- ~N possible different patterns
- Probability of search success < N/2^M
- < .00000000000000000000001 for N = 100,000, M = 100

Probabilities are much lower for bigger alphabets

NOT FOUND is a simple algorithm that works

Better idea to test algorithms:

- search for fixed patterns in fixed text
- use random position for successful search
- use random perturbations for unsuccessful search

Brute-force string searching

Check for pattern at every text position

```
int brutesearch(char *p, char *a)
  {
    int i, j, cnt = 0;
    for (i = 0; i < strlen(a); i++)
      for (j = 0; j < strlen(p); j++)
        {
          if (a[i+j] != p[j]) break;
          if (j == strlen(p)-1) return i;
        }
    return strlen(a)+1;
  }
```

DON'T USE THIS PROGRAM!

Problem with brute-force implementation

```
for (i = 0; i < strlen(a); i++)
```

In C, strlen takes time proportional to string length

- running time at least N^2
- even for simpler programs (count the a's)

PERFORMANCE BUG

Performance matters in ADT design!

Exercise: implement string ADT with fast strlen

- need space to store length
- need to update length when changing string
- ...

Brute-force algorithm (bug fixed)

```
int brutesearch(char *p, char *a)
  { int i, j;
    int M = strlen(p), N = strlen(a), cnt = 0;
    for (i = 0; i < N; i++)
      for (j = 0; j < M; j++)
        {
          if (a[i+j] != p[j]) break;
          if (j == M-1) return i;
        }
    return N+1;
  }
```


Brute-force algorithm (alternative)

Different implementation (same algorithm)

- char match: increment both i and j
- char mismatch: set j to 0, reset i

```
int brutesearch(char *p, char *a)
  {
    int i, j, M = strlen(p), N = strlen(a);
    for (i = 0, j = 0; j < M && i < N; i++, j++)
      while (a[i] != p[j]) { i -= j-1; j = 0; }
    if (j == M) return i-M; else return i;
  }
```


Analysis of brute-force algorithm

Running time depends on text and pattern

Worst case: search for 000001

```
.    000000000000000000000000000001
.    00000*
.     00000*
.      00000*
.       00000*
.        00000*
```

M*N bit compares

[Too slow when N and M are both large]

Average-case analysis

Fixed pattern, random text

```
.    10011010010100010100111000111
.    *
.     00*
.      0*
.       *
.        *
.         0*
.          *
.           001
```

long pattern: 2*N compares

short pattern:

- precise cost depends on pattern
- (first 001 appears before first 000, on average)

Ref: Flajolet and Sedgewick

Rabin-Karp algorithm

Idea: Use hashing

- compute hash function for each text position
- NO TABLE! (just compare with pattern hash)

Ex: "table" size 97, M = 5

Search for 15926 = 18 (mod 97)

```
.     31415926535897932384626433
.     31415 = 84 (mod 97)
.      14159 = 94 (mod 97)
.       41592 = 76 (mod 97)
.        15926 = 18 (mod 97)
.         59265 = 95 (mod 97)
```

Problem: hash function depends on M characters

- (running time N*M)

Rabin-Karp algorithm (continued)

Solution: use previous hash to compute next hash

```
31415 = 84  (mod 97)
14159 = (31415 - 30000)*10 + 9
        = (84 - 3*9)*10 + 9 (mod 97)
        = 579 = 94 (mod 97)
41592 = (94 - 1*9)*10 + 2 = 76 (mod 97)
15926 = (76 - 4*9)*10 + 6 = 18 (mod 97)
59265 = (18 - 1*9)*10 + 5 = 95 (mod 97)
```

Improves running time from N*M to N+M

Slight problem: need full compare on collision

```
92653 = (95 - 5*9)*10 + 3 = 18 (mod 97)
```

Solution: use giant (virtual) table!

- limit on table size: overflow on arithmetic ops

RK algorithm implementation

```
#define q 3355439
#define d 256
int rksearch(char *p, char *a)
  {
    int i, j, dM = 1, h1 = 0, h2 = 0;
    int M = strlen(p), N = strlen(a);
    for (j = 1; j < M; j++) dM = (d*dM) % q;
    for (j = 0; j < M; j++)
      {
        h1 = (h1*d + p[j]) % q;
        h2 = (h2*d + a[j]) % q;
      }
    for (i = M; i < N; i++)
      {
        if (h1 == h2) return i-M;
        h2 = (h2 + d*q - a[i-M]*dM) % q;
        h2 = (h2*d + a[i]) % q;
      }
    return N;
  }
```

Randomized algorithm: take random (huge) table size

Knuth-Morris-Pratt algorithm

Text characters that match are in the pattern!
PRECOMPUTE what to do on mismatch

Ex: when searching for 000001, mismatch 00000* implies
- text had 000000
- if next text char is 1, full match found
- if next text char is 0, back in same state

Ex: when searching for 000001, mismatch 000* implies
- text had 0001
- restart search on next text char

More complicated in general, but
- can always move to next text character
- next action uniquely determined

Nonobvious algorithm
- solved open theoretical and practical problems

KMP finite state automaton

``next action uniquely determined``
- Build FSA from pattern
- Run FSA on text (inner loop: S = FSA[S][a[i++]])

N+M running time
NO BACKUP in text

Ex: FSA for 000001

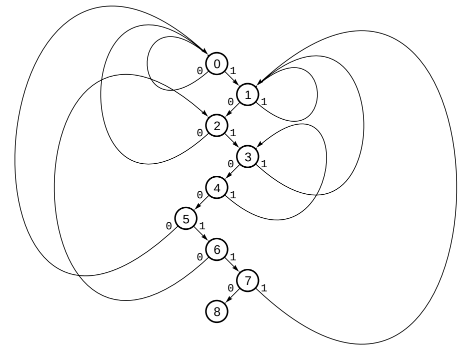
.		0	1	2	3	4	5
.	0	1	2	3	4	5	5
.	1	0	0	0	0	0	6

worst-case text
.   000000000000000000000000000001
.   012345555555555555555555555556

random text
.   100111010010100010100111000111
.   001200010120101230101200012300

KMP FSA typical example

Finite-state automaton for 10100110



.		0	1	2	3	4	5	6	7
.	0	0	2	0	4	5	0	2	8
.	1	1	1	3	1	3	6	7	1

Search example:
.   100111010010101010100110000111
.   01201112345623434343345678

KMP automaton construction

FSA builds itself (!)
- mismatch state is defined earlier in FSA

Ex: FSA for 10100110

State 6
- 1010011: go to state 7
- 1010010: go to state for 010010 (state 2)
  remember state for 010011 (X = state 1)

State 7
- 10100110: go to state 8
- 10100111: go to state for 0100111 (state 1)
  remember state for 0100110 (X = state 2)

Main point: states needed are successors of old X
- X: mismatch state
- match: FSA[match][i] = i+1
- mismatch: FSA[mismatch][i] = FSA[mismatch][X]
  X = FSA[match][X]

KMP automaton construction example

Pattern: 10100110

```
.        0   1
.    0   0   2      1
.    1   1   1      01

.        0   1   2
.    0   0   2   0      00
.    1   1   1   3      000

.        0   1   2   3
.    0   0   2   0   4      011
.    1   1   1   3   1      0011

.        0   1   2   3   4
.    0   0   2   0   4   5     0101
.    1   1   1   3   0   3     00123

.        0   1   2   3   4   5
.    0   0   2   0   4   5   0     01000
.    1   1   1   3   0   0   6     001200

.        0   1   2   3   4   5   6
.    0   0   2   0   4   5   0   2     010010
.    1   1   1   3   0   0   6   7     0012012

.        0   1   2   3   4   5   6   7
.    0   0   2   0   4   5   0   2   8     0100111
.    1   1   1   3   0   0   6   7   1     00120111
```


KMP implementation

Build FSA from pattern, run FSA on text string

Improvement:

- match state in FSA unnecessary (always i+1)
- use mismatch state only ("next" array)
- small hack: next[0] = -1

```
int kmpsearch(char *p, char *a)
{
   int i, j, M = strlen(p), N = strlen(a);
   initnext(p);
   for (i = 0, j = 0; j < M && i < N; i++, j++)
      while ((j>=0) && (a[i]!=p[j])) j = next[j];
   if (j == M) return i-M; else return i;
}
```

initnext function constructs automaton (see text)

KMP implementation (continued)

Easy to create specialized C program for pattern

```
int kmpsearch(char *a)
{ int i = 0;
s0: if (a[i] != '1') goto s0; i++;
s1: if (a[i] != '0') goto s1; i++;
s2: if (a[i] != '1') goto s0; i++;
s3: if (a[i] != '0') goto s1; i++;
s4: if (a[i] != '0') goto s3; i++;
s5: if (a[i] != '1') goto s0; i++;
s6: if (a[i] != '1') goto s2; i++;
s7: if (a[i] != '0') goto s1; i++;
     return i-8; }
```

Ultimate search program for pattern:
machine language version of FSA

Right-left pattern scan

SUBLINEAR ALGORITHMS

- move right to left in pattern
- move left to right in text

Ex: Find string of 9 consecutive 0s

```
.    100111010010100010100111000111
.            0
.           1
.                    1
.                             0
.                            0
.                           1
```

Same idea effective in large alphabet

Search time proportional to N/M for practical problems
Time decreases as pattern length increases (!)

Right-left pattern scan examples

Text character not in pattern: skip forward M chars

```
. now is the time for all good people to come
.       *      *      *      *      e
.                                  l
.                                 p
.                                o
.                               e
.                              p
```

Text character in pattern: skip forward to pattern end

```
. you can fool some of the people some of the time
.       *      *      *      o
.                               e
.                              l
.                             p
.                            o
.                           e
.                          p
```


Right-left pattern scan implementation

```
initskip(char *p)
  { int j, M = strlen(p);
    for (j = 0; j < 256; j++) skip[j] = M;
    for (j = 0; j < M; j++) skip[p[j]] = M-j-1;
  }
#define max(A, B) (A > B) ? A : B;
int mischarsearch(char *p, char *a)
  { int i, j; int M = strlen(p), N = strlen(a);
    initskip(p);
    for (i = M-1, j = M-1; j >= 0; i--, j--)
      while (a[i] != p[j])
        {
          i += max(M-j, skip[a[i]]);
          if (i >= N) return N;
          j = M-1;
        }
    return i+1;
  }
```

Boyer-Moore: KMP-like computation to improve skip, if possible

Multiple patterns

DICTIONARY (symbol table ADT)

- build trie from text (preprocess)
- pattern lookups in O(lgN) steps

EXCEPTION DICTIONARY

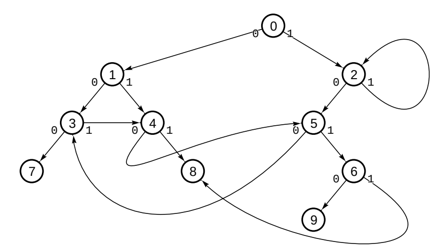
- build trie from set of patterns (preprocess)
- find patterns in given text in N lg N steps

generalization of KMP

- build trie from set of patterns
- convert to FSA with KMP-like computation
- find patterns in given text in N steps

Multiple patterns example

Ex: FSA for 000, 011, and 1010



	0	1	2	3	4	5	6
0	1	3	5	7	5	3	9
1	2	4	2	4	8	6	8

Simultaneous search for all patterns in text

```
.    111100100100101110100000
.    02222534534534568
```