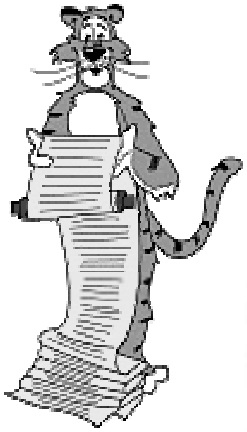
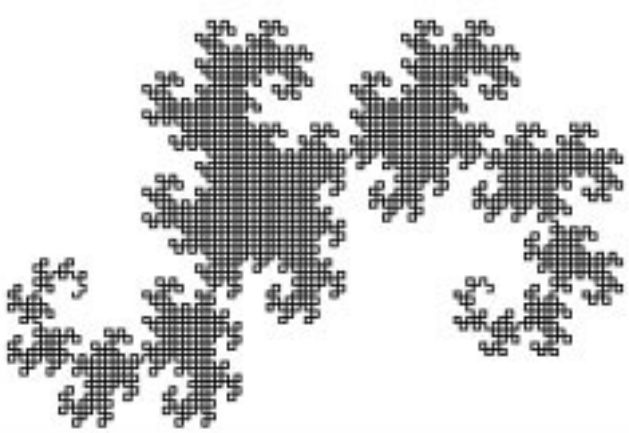
Lecture P7: Advanced Recursion





Overview

What is recursion?

- When one function calls ITSELF directly or indirectly.

Why learn recursion?

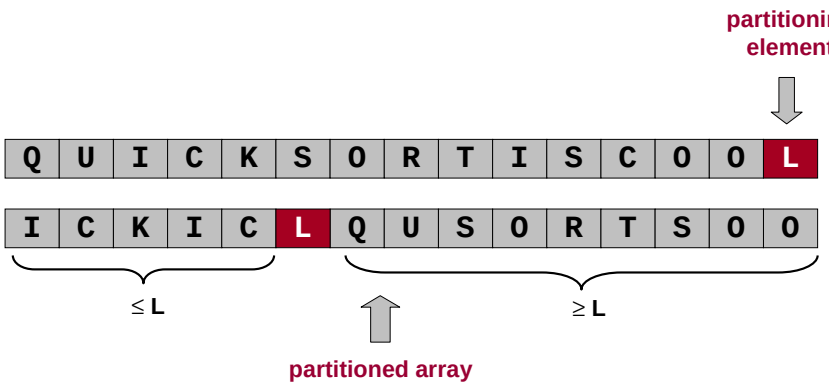
- New mode of thinking.
- Powerful programming tool to solve a problem by breaking it up into one (or more) smaller problems of similar structure.
  - *" Divide et impera "*
  - *" Veni, vidi, vici "*

Julius Caesar (100BC - 44BC)

Quicksort

Quicksort.

- Partition array so that:
  - some partitioning element `a[m]` is in its final position
  - no larger element to the left of `m`
  - no smaller element to the right of `m`



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- Partition array so that:
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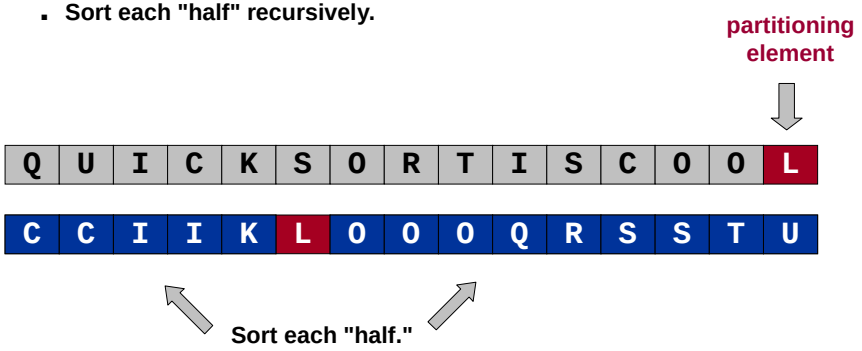
- Sort each "half" recursively.



Quicksort

Quicksort.

- Partition array so that:
  - some partitioning element `a[m]` is in its final position
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- Sort each "half" recursively.

quicksort.c (see Sedgewick Program 7.1)

```
void quicksort(char a[], int left, int right) {
  int m;
  if (right > left) {
     m = partition(a, left, right);
     quicksort(a, left, m - 1);
     quicksort(a, m + 1, right);
  }
}
```

base case???

Quicksort

Quicksort.

- Partition array so that:
  - some partitioning element `a[m]` is in its final position
  - no larger element to the left of `m`
  - no smaller element to the right of `m`
- Sort each "half" recursively.
- How do we partition efficiently?
  - N - 1 comparisons
  - straightforward with auxiliary array
  - better solution: uses "no" extra space!

Quicksort : Implementing Partition

partition (see Sedgewick Program 7.2)

```
int partition(char a[], int left, int right) {
  int i = left-1;     /* left to right pointer */
  int j = right;      /* right to left pointer */

  while(1) {
     while (a[++i] < a[right])
        ;
     while (a[right] < a[--j])
       if (j == left)
          break;

     if (i >= j)
        break;
     swap(a, i, j);
  }

  swap(a, i, right);
  return i;
}
```

find element on left to swap

look for element on right to swap, but don't run off end

pointers cross

swap partition element

Quicksort : Implementing Partition

main( )

```
#include <stdio.h>
#define N 14

int main(void) {
   char a[] = "pseudomythical";
   printf("Before: %s\n", a);
   quicksort(a, 0, N-1);
   printf("After:  %s\n", a);
   return 0;
}
```

swap( )

```
void swap(char a[], int i, int j) {
   char t;
   t = a[i]; a[i] = a[j]; a[j] = t;
}
```

Quicksort : Performance

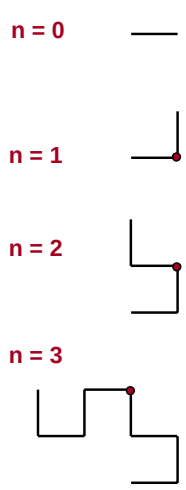
Quicksort vs. Insertion sort.

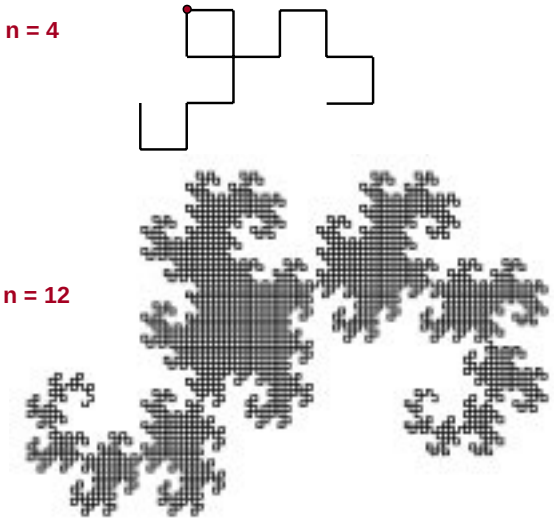
	Insertion Sort			*Quicksort*		
computer	thousand	million	billion	thousand	million	billion
home pc	instant	2 hour	310 years	instant	0.3 sec	6 min
super	instant	1 sec	1.6 weeks	instant	instant	instant

Stay tuned: Lecture T5.

Dragon (Jurassic Park) Curve

Fold a wire in half n times. Unfold to right angles.





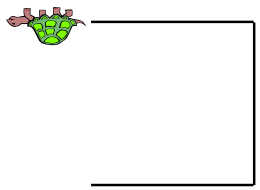
Drawing a Dragon Curve

Use simple "turtle graphics."

- F: move turtle forward one step (pen down).
- L: turn left 90°.
- R: turn right 90°.

Example.

- F L F L F



Drawing a Dragon Curve

Use simple "turtle graphics."

- F: move turtle forward one step (pen down).
- L: turn left 90°.
- R: turn right 90°.

Example.

- dragon(0): F
- dragon(1): F L F
- dragon(2): F L F L F R F
- dragon(3): F L F L F R F L F L F R F R F
- dragon(4): F L F L F R F L F L F R F R F L F L F L F R F R F L F R F R F

dragon(3) nogard(3)

"backwards" dragon(3):
reverse string, switch L and R

Recursive Dragon Curve Program

A dragon curve of order n is:

- Dragon curve of order n-1.
- Move left.
- Dragon curve of order n-1 backwards (switch L and R).

dragon( )

```
void dragon(int n) {
   if (n == 0)
      F();
   else {
      dragon(n-1);
      L();
      nogard(n-1);
   }
}
```

need implementation of `nogard( )`

drawing in PostScript

```
void F(void) {
  printf("10 0 rlineto\n");
}

void L(void) {
  printf("90 rotate\n");
}

void R(void) {
  printf("-90 rotate\n");
}
```

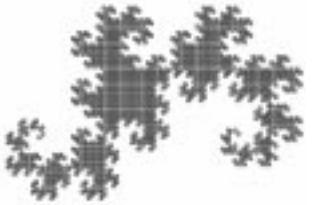

Drawing a Dragon Curve

To get nogard(n):

- dragon(2): F L F **L** F R F
- nogard(2): F L F **R** F R F
- dragon(3): F L F L F R F **L** F L F R F R F
  dragon(2) nogard(2)
- nogard(3): F L F L F R F **R** F L F R F R F
  dragon(2) nogard(2)

nogard( )

```
void nogard(int n) {
   if (n == 0)
      F();
   else {
      dragon(n-1);
      R();
      nogard(n-1);
   }
}
```



Unwinding Tail Recursion

Replace nogard() with its results.

nogard( )

```
void nogard(int n) {
   if (n == 0)
      F();
   else {
      dragon(n-1);
      R();
      nogard(n-1);
   }
}
```

nogard( )

```
void nogard(int n) {
   int k;
   for (k = n-1; k >= 0; k--) {
      R();
      dragon(k);
   }
   F();
}
```


Alternate Dragon

Replace call to nogard() by non-recursive version.

dragon( )

```
void dragon(int n) {
   int k;
   if (n == 0)
      F();
   else {
      dragon(n-1);
      L();
      for (k = n-2; k >=0; k--) {
         dragon(k);
         R();
      }
      F();
   }
}
```

nogard(n-1)

Enumerating All Permutations

Enumerate all permutations of a set of elements.

- N elements ⇒ N! possibilities
- If elements named a, b, c, then 6 possible permutations are: abc, acb, bac, bca, cab, cba.

Inelegant Solution (for N = 3)

```
#include <stdio.h>
#define N 3

int main(void) {
  char a[] = "abc";
  int i, j, k;
  for (i = 0; i < N; i++)
     for (j = 0; j < N; j++)
        for (k = 0; k < N; k++)
           if (i != j && i != k && j != k)
              printf("%c%c%c\n", a[i], a[j], a[k]);
  return 0;
}
```

3 implicitly hardwired everywhere

Enumerating All Permutations

Enumerate all permutations of a set of elements.

- N elements ⇒ N! possibilities
- If elements named a, b, c, then 6 possible permutations are: abc, acb, bac, bca, cab, cba.

Key idea: permutations of abcde are one of the followig:

- End with a preceded by one of 4! permutations of bcde.
- End with b preceded by one of 4! permutations of acde.
- End with c preceded by one of 4! permutations of abde.
- End with d preceded by one of 4! permutations of abce.
- End with e preceded by one of 4! permutations of abcd.

Reduces enumerating permutations of N elements to enumerating permutations of N-1 elements.

Enumerating All Permutations

Recursive solution for trying all permutations:

- Array `a[]` stores current permutation.
- Initially `a[] = "abcde"`

Enumerating all Permutations

```
void enumerate(char a[], int n) {
  int i;
  if (0 == n)
     printf("%s\n", a);
  else
     for (i = 0; i < n; i++) {
        swap(a, i, n-1);
        enumerate(a, n-1);
        swap(a, n-1, i);
     }
}
```

base case

swap elements i and n-1

Decide position of remaining n-1 cities.

restore order

Enumerating All Permutations

Recursive solution for trying all permutations:

Enumerating all Permutations

```
#include <stdio.h>

void swap(char a[], int i, int j) {
  char t;
  t = a[i]; a[i] = a[j]; a[j] = t;
}

void enumerate(...) { . . . }

int main(void) {
  char a[] = "abcde";
  enumerate(a, 5);
  return 0;
}
```

Unix

```
% a.out
bca
cba
cab
acb
bac
abc
```

Application: Traveling Salesperson Problem

Given N points, find shortest tour connecting them.

- **Brute force: try all N! possible permutations.**

Recursive solution for finding best TSP tour.

- **Store coordinates of points in `a[]`.**
- **Replace `printf()` with `checklength()`.**
- **Takes N! steps.**
- **No computer can run this for N ≥ 100.**
  - **100! > 10^{150}.**

Is there an efficient way to do this computation?