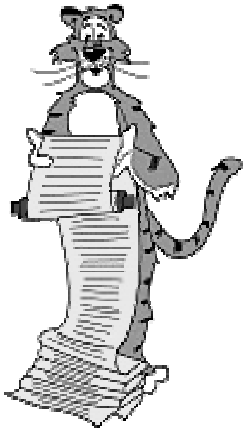
Lecture A1: The TOY Machine



What is TOY?

An imaginary machine similar to:

- Ancient computers.
- Today's microprocessors.

Why study?

- Simplified machine.
  - easier to digest
  - captures essence of modern microprocessors
- Machine language programming.
  - how do C programs relate to computer?
  - still (a few) situations today where it is really necessary

Inside the Box

Switches.

- Input data and programs.

Lights.

- View data.

Registers.

- Fastest form of storage.
- Use as scratch space during computation.
- 8 registers.
  - each stores 16 bits

Program counter (PC).

- An extra register.
- Keeps track of next instruction to be executed.

Memory.

- Store data and programs.
- 256 "words".
  - each word stores 16 bits

ALU (arithmetic-logic unit).

- Execute instructions and manipulate data.

Data and Programs Encoded in Binary

Each bit consists of two states:

- Switch is ON or OFF.
- High voltage or low voltage.
- 1 or 0.
- True or false.

How to represent integers?

- Use binary encoding.
- Ex: $6375_{10} = 0001100011100111_2$

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

$$6375 = +2^{12}+2^{11} \quad +2^{7}+2^{6}+2^{5} \quad +2^{2}+2^{1}+2^{0}$$
$$= 4096+2048 \quad +128+64+32 \quad +4+2+1$$

Shorthand Notation

Use hexadecimal (base 16) representation.

- Binary code, four bits at a time.
- Ex: $6375_{10} = 0001100011100111_2 = 18E7_{16}$

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1
1				8				E				7			

$$6375 = 1 * 16^3 + 8 * 16^2 + 11 * 16^1 + 7 * 16^0$$

$$= 4096 + 2048 + 224 + 7$$

Machine "Core" Dump

Everything is encoded in binary (hex).

- Integers, machine instructions, text, reals, etc.

Registers							
R7	R6	R5	R4	R3	R2	R1	R0
0000	0788	B700	0010	0401	0002	0003	00A0

Machine contents at a particular place and time.

- Record of what program has done.
- Determines (with PC) what program will do.

Main Memory								
00:	0000	0000	0000	0000	0000	0000	0000	0000
08:	0000	0000	0000	0000	0000	0000	0000	0000
10:	9222	9120	1121	A120	1121	A121	7211	0000
18:	0000	0001	0002	0003	0004	0005	0006	0007
20:	0008	0009	000A	000B	000C	000D	000E	000F
28:	0000	0000	0000	FE10	FACE	CAFE	ACED	CEDE
.								
.								
E8:	1234	5678	9ABC	DEF0	0000	0000	F00D	0000
F0:	0000	0000	EEEE	1111	EEEE	1111	0000	0000
F8:	B1B2	F1F5	0000	0000	0000	0000	0000	0000

Program and Data

Program:

- Sequence of instructions.

16 different instruction:

- 16-bit word (interpreted one way).
- Changes contents of registers, memory, and PC in specified, well-defined ways.

Data:

- 16-bit word (interpreted other ways).

Program counter (PC):

- Stores memory address of "next instruction."

Instructions	
0:	halt
1:	add
2:	subtract
3:	multiply
4:	system call
5:	jump
6:	jump if greater
7:	jump and count
8:	jump and link
9:	load
A:	store
B:	load address
C:	xor
D:	and
E:	shift right
F:	shift left

How to Use the TOY Machine

To run a program:

- Load the program and data.
  (set switches, press LOAD for each word)
- Set switches to address of first instruction.
- Press GO.

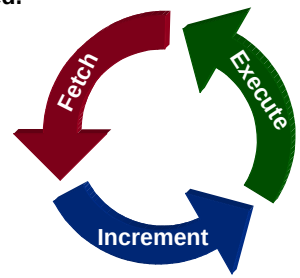
Basic Cycle

GO button:

- Loads PC from address switches.
- Initializes fetch-increment-execute cycle.
- Machine runs until halt instruction reached.

Basic cycle:

- FETCH (get instruction from memory into ALU).
- INCREMENT program counter.
- EXECUTE (may require data from or to memory).



Output:

- System call can write output to output device (tty).
- Read contents of memory word in lights.

TOY Instructions

Each instruction consists of 16 bits.

- Leftmost four bits (first hex digit) encode instruction type or opcode.
- Divide up remaining 12 bits to denote component of each instruction.
- Format 1 and Format 2 instructions.
  - different ways of dividing up the 12 bits

Instructions	
0:	halt
1:	add
2:	subtract
3:	multiply
4:	system call
5:	jump
6:	jump if greater
7:	jump and count
8:	jump and link
9:	load
A:	store
F:	shift left

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	1	1	0	0	1	0	0	0	1	1	0	1	0	0
opcode				dest				regA				regB			
opcode				dest				addr							

Format 1

Register-register.

- Bits 12-15 encode opcode.
- Bits 8-11 encode destination register.
- Bits 4-7 encode source register A.
- Bits 0-3 encode source register B.

Ex: 1234 means

- Add registers R3 and R4.
- Put result in register R2.
- R2 ← R3 + R4

Format 1 Instructions	
0:	halt
1:	add
2:	subtract
3:	multiply
4:	system call
C:	xor
D:	and

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0	0	0	1	0	0	1	0	0	0	1	1	0	1	0	0
1_{16}				2_{16}				3_{16}				4_{16}			
opcode				dest				regA				regB			

Format 2

Register-memory / register-immediate.

- Bits 12-15 encode opcode.
- Bits 8-11 encode destination register.
- Bits 0-7 encode memory address or arithmetic constant.

Ex: B234 means

- Load the value 34_{16} into register R2.
- R2 ← 0034

Format 2 Instructions	
5:	jump
6:	jump if greater
7:	jump and count
8:	jump and link
9:	load
A:	store
B:	load address
E:	shift left
F:	shift right

15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
1	0	1	1	0	0	1	0	0	0	1	1	0	1	0	0
B_{16}				2_{16}				34_{16}							
opcode				dest				addr							

Format 2

Ex: B234 means

- Load the value 34_{16} into register R2.
- `R2 ← 0034`

Ex: 9234 means

- Load contents of memory location 34_{16} into register R2.
- `R2 ← mem[34]`

Ex: A234 means

- Store the contents of register R2 into memory location 34_{16}.
- `mem[34] ← R2`

Format 2 Instructions	
5:	jump
6:	jump if greater
7:	jump and count
8:	jump and link
9:	load
A:	store
B:	load address
E:	shift left
F:	shift right

Don't confuse opcodes '9' and 'B'.

Sample TOY Program 0

TOY code for C expression: t = b*b - 4*a*c.

- Suppose memory locations D0-D3 used to store data:
  - D0 stores a
  - D1 stores b
  - D2 stores c
  - D3 stores t
- Suppose memory locations 10-19 used to store instructions:
  - `10: 91D1 3111 B204 93D0 94D2 3223 3224 2112`
  - `18: A1D3 0000`

Set PC to 10; Press GO.

- TOY computes the value.

Sample TOY Program 0

Step-by-step trace:

quadratic.toy

```
10: 91D1     R1 <- b
11: 3111     R1 <- b*b
12: B204     R2 <- 4
13: 93D0     R3 <- a
14: 94D2     R4 <- c
15: 3223     R2 <- 4*a
16: 3224     R2 <- (4*a)*c
17: 2112     R2 <- (b*b) - (4*a*c)
18: A1D3     t  <- (b*b - 4*a*c)
19: 0000     halt
```

C compiler produces code like this.

Stay tuned for what happens if:

- Result of subtraction is negative.
- Numbers are too big.
- We need the square root (!)

Sample TOY Program 1: More Arithmetic

TOY code to compute $1 + 2 + 3 + 4 + 5 + 6 = 21_{10} = 15_{16}$.

- Suppose memory locations 10-1E used to store instructions:
  - `10: B001 B200 B101 1221 1110 1221 1110 1221`
  - `18: 1110 1221 1110 1221 1110 1221 0000 0000`

Set PC to 10; Press GO.

- TOY computes the value.

Sample TOY Program 1: More Arithmetic

Step-by-step trace of sum1.toy.

```
                                    R1      R2
10: B001      R0 <- 0001
11: B200      R2 <- 0000                    0000
12: B101      R1 <- 0001            0001
13: 1221      R2 <- R2 + R1                 0001
14: 1110      R1 <- R1 + R0         0002
15: 1221      R2 <- R2 + R1                 0003
16: 1110      R1 <- R1 + R0         0003
17: 1221      R2 <- R2 + R1                 0006
18: 1110      R1 <- R1 + R0         0004
19: 1221      R2 <- R2 + R1                 000A
1A: 1110      R1 <- R1 + R0         0005
1B: 1221      R2 <- R2 + R1                 000F
1C: 1110      R1 <- R1 + R0         0006
1D: 1221      R2 <- R2 + R1                 0015
1E: 0000      halt
```

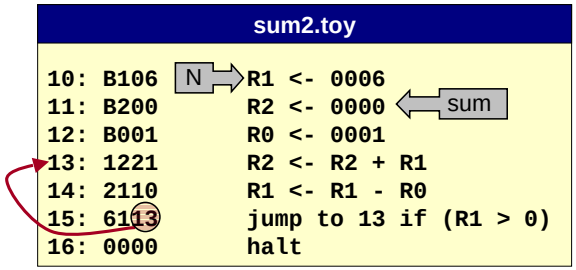

Sample TOY Program 1: Loop

TOY code to compute N + (N -1) + . . . + 2 + 1 for any value of N loaded into R1 initially.

- Suppose memory locations 10-17 used to store instructions:
  - 10: B106 B200 B001 1221 2110 6113 0000 0000

Set PC to 10; Press GO.

- TOY computes the value.



Sample TOY Program 1: Loop

Step-by-step trace of sum2.toy.

```
                                    R1      R2
10: B106    R1 <- 0006            0006
11: B200    R2 <- 0000                    0000
12: B001    R0 <- 0001
13: 1221    R2 <- R2 + R1                 0006
14: 2110    R1 <- R1 - R0         0005
15: 6113    jump if (R1 > 0)
13: 1221    R2 <- R2 + R1                 000B
14: 2110    R1 <- R1 - R0         0004
15: 6113    jump if (R1 > 0)
13: 1221    R2 <- R2 + R1                 000F
14: 2110    R1 <- R1 - R0         0003
15: 6113    jump if (R1 > 0)
13: 1221    R2 <- R2 + R1                 0012
14: 2110    R1 <- R1 - R0         0002
15: 6113    jump if (R1 > 0)
13: 1221    R2 <- R2 + R1                 0014
14: 2110    R1 <- R1 - R0         0001
15: 6113    jump if (R1 > 0)
13: 1221    R2 <- R2 + R1                 0015
14: 2110    R1 <- R1 - R0         0000
15: 6113    jump if (R1 > 0)
16: 0000    halt
```


Program 2: Horner's Method

Goal: evaluate $2x^3 + 3x^2 + 9x + 7$ at x = 10.

- Assume "data" stored in locations 30 - 34
  - x a b c d
  - 30: 000A 0002 0003 0009 0007 0000 0000 0000

First try:

- Compute x^3, multiply by a; compute x^2, multiply by b, … (cumbersome, inefficient)

Efficient algorithm (Horner's method):

- Rewrite $ax^3 + bx^2 + cx + d$ as $((a x + b) x + c) x + d$.
- Does polynomial evaluation for arbitrary x.
- Many applications (e.g., convert from decimal to hex).
- One raison d'être for early machines.

Program 2: Horner's Method

Step-by-step trace.

- Converts from decimal to hex.
- $2397_{10} = 95D_{16}$.

horner.toy

```
10: 9430     R4 <- mem[30]    000A x
11: 9531     R5 <- mem[31]    0002 a
12: 3554     R5 <- R5 * R4  0014 a*x
13: 9632     R6 <- mem[32]    0003 b
14: 1556     R5 <- R5 + R6  0017 a*x+b
15: 3554     R5 <- R5 * R4  00DC (a*x+b)*x
16: 9633     R6 <- mem[33]    0009 c
17: 1556     R5 <- R5 + R6  00E5 (a*x+b)*x + c
18: 3554     R5 <- R5 * R4  0956 ((a*x+b)*x+c)*x
19: 9634     R6 <- mem[34]    0007 d
1A: 1556     R5 <- R5 + R6  095D ((a*x+b)*x+c*x)+d
1B: 4502     write R5 to tty
1C: 0000    halt
```


A Little History

ENIAC. (Eckert and Mauchly, 1946)

- First general purpose electronic computer.
- 30 x 50 x 8.5 ft.
- 17,468 vacuum tubes.
- 300 multiplication per second.
- Conditional jumps, programmable.
  - code: set switches
  - data: punch cards
  - used to compute artillery firing tables

Basic Characteristics of TOY Machine

TOY is a general purpose computer.

- Sufficient power to perform and computation.
- Limited only by amount of memory (and time).

Stored-program computer. (von Neumann memo, 1944)

- Data and instructions encoded in binary.
- Data and instructions stored in SAME memory.
- Can change program (control) without rewiring.
  - immediate applications
  - profound implications
- EDSAC (Wilkes 1949).
  - first stored-program computer

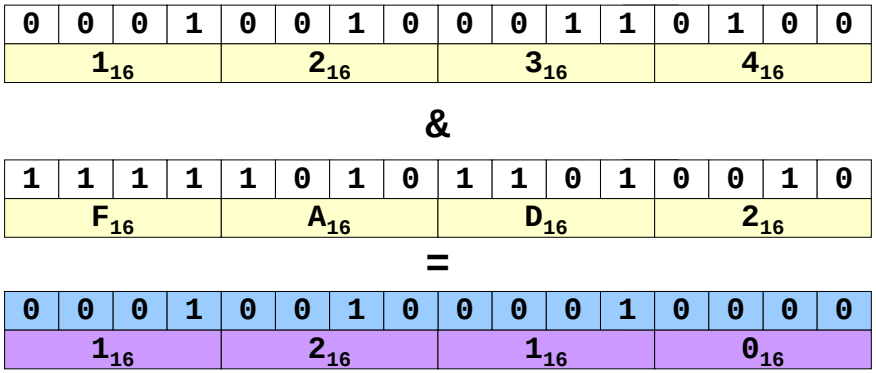
Lecture A1: Extra Slides

Format 1: AND

x	y	AND
0	0	0
0	1	0
1	0	0
1	1	1

Logic operations are BITWISE:

- 1234_{16} & $FAD2_{16}$ = 1210_{16}

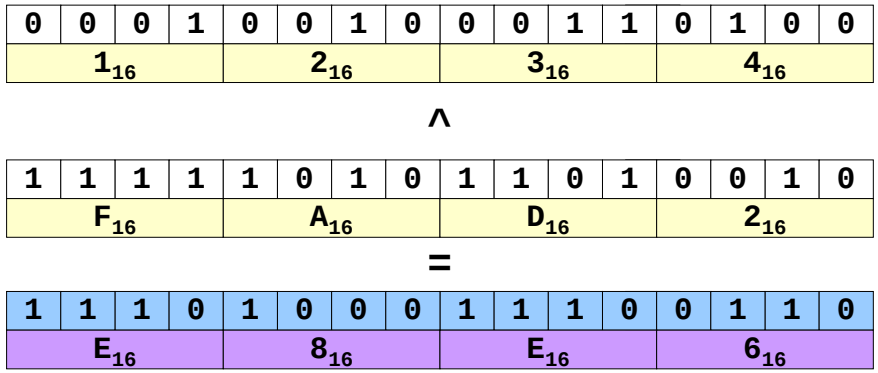


Format 1: XOR

x	y	XOR
0	0	0
0	1	1
1	0	1
1	1	0

Logic operations are BITWISE:

- 1234_{16} ^ $FAD2_{16}$ = $E8E6_{16}$



Other Logical Operations

Any logical operation can be implemented with AND and XOR.

- See Boolean circuit lecture.

Build OR from AND and XOR.

- (x & y) ^ (x ^ y)

x	y	OR
0	0	0
0	1	1
1	0	1
1	1	1

Build NOT from XOR.

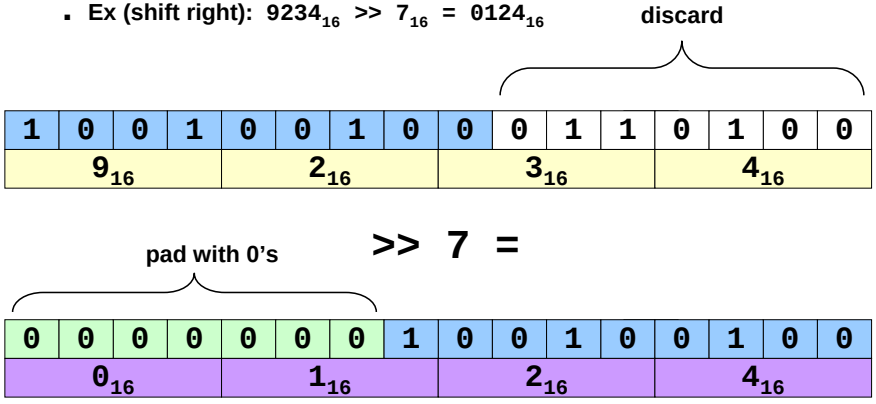
- 1 ^ x = x'
- FFFF ^ x = x' (bitwise NOT)

x	NOT
0	1
1	0

Format 2: Shift

Shift:

- Shift bits left or right.
- Pad with zeros.
- Ex (shift right): 9234_{16} >> 7_{16} = 0124_{16}



Program 3: Bit Manipulation

Example 3.

- Suppose memory location D0 is used to store LFBSR value:
  - `D0: 0684`
- Suppose memory locations 10-1B used to store instructions:
  - `10: 92D0 93D0 B001 E203 D220 E30A D330 C323`
  - `18: 92D0 F201 1223 A2D0 0000 0000 0000 0000`

Set PC to 10. Press GO. What happens?

- TOY simulates 1 step of LFBSR.

Program 3: Bit Manipulation

Step-by-step trace.

lfbsr.toy

```
10: 92D0    R2 <- mem[D0]    0000011010000100
11: 93D0    R3 <- mem[D0]    0000011010000100
12: B001    R0 <- 1
13: E203    R2 <- R2 >> 3       0000011010000
14: D220    R2 <- R2 & 1                    0
15: E30A    R3 <- R3 >> 10             000001
16: D330    R3 <- R3 & 1                    1
17: C323    R3 <- T2 ^ R3                   1
18: 92D0    R2 <- mem[D0]    0000011010000100
19: F201    R2 <- R2 << 1    0000110100001000
1A: 1223    R2 <- R2 + R3    0000110100001001
1B: A2D0    mem[D0] <- R2
1C: 0000    halt
```

load shift register fill

bit 3

bit 10

updated value