CS 341 — Discrete Math — Fall 1999

Problem Set 1 Answer Key

by Jace Kohlmeier

1 Section 1.8, Ex. 1

Please note: When proving and "if and only if" statement, it is necessary to show *both* the "if" direction *and* the "only if" direction. Therefore, you should generally organize your answer into two parts.

1.1 "If" direction

We show that if for some m-by-n chessboard at least one of m and n is even, then the chessbaord has a perfect cover. We may assume, without loss of generality, that m is even (since the case for an even n is exactly similar). Then $m = 2i$ for some integer i, and we may place i dominoes in each column, filling the squares in every column. And by alternating the colors in the highest squares, we will achieve a perfect cover for our chessboard.

1.2 "Only if" direction

Here, we must show that if a m-by-n chessboard has a perfect cover, then at least one of m and n is even. We prove this by contradiction. Assume that m and n are both odd, and that the chessboard has a perfect cover by d dominoes. Since m and n are odd, the total number of squares on the board (mn) is also odd. But we also have that the total number of squares is $2d$, which is even—a contradiction! This shows that our assumptions are false, which completes our proof.

2 Section 1.8, Ex. 16

Based on the equation given in your text, the sum of any row or column is 34. Therefore, the reader can check that the two additional numbers placed in the top row must be 14 and 15, and the two numbers in the left column must be 12 and 16. Furthermore, if we look at the diagonal from the lower left to the upper right, we note that we must place 12 (and not 16) in the lower left, because otherwise we would not have small enough numbers remaining to complete the diagonal with a sum of 34. Now with

12 in the lower left and 14 or 15 in the upper right, we note that we must use 1 and a 7 or 8 to complete the aforementioned diagonal, where the 7 or 8 depends on whether 14 or 15 is in the upper right. In either case, we must now have a 1 in a row or column containing 3 or 4 (see figure). And the reader can check that this row or column would be impossible to complete with the remaining number to obtain a large enough sum to equal 34. Thus the "magic square" can not be completed.

2	3	$14or15$	$14or15$
4		$1or7$	
16	$1or8$		
12			

3 Problem 3

The algorithm produces the following matrices, which do indeed have the magic sums of 34 and 260.

16	2	13	3
5	11	10	8
9	7	6	12
4	14	15	1

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	33
32	34	35	29	28	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

4 Section 1.8, Ex. 31

As per section 1.7, we construct a table showing the heap totals as their binary equivalents.

Heap of size 10	0	0	1	0	1	0
Heap of size 20	0	1	0	1	0	0
Heap of size 30	0	1	1	1	1	0
Heap of size 40	1	0	1	0	0	0
Heap of size 50	1	1	0	0	1	0
Totals	2	3	3	2	3	0

As you can see the game is unbalanced due to the odd totals in three columns. There are several possible moves to balance the game. One example would be to remove be to remove 26 from the pile of 30. This will balance the game by changing the game state as follows:

Heap of size 10	0	0	1	0	1	0
Heap of size 20	0	1	0	1	0	0
Heap of size 30	0	0	0	1	0	0
Heap of size 40	1	0	1	0	0	0
Heap of size 50	1	1	0	0	1	0
Totals	2	2	2	2	2	0

5 Section 1.8, Ex. 32

Note that the binary expansion of an odd number always has a 1 in the rightmost digit, and therefore there will be a 1 for every odd heap in that heap's rightmost column of our table to determine whether the game is balanced. Conversely, every even heap would have a 0 in the rightmost column. Therefore, if there is an odd number of odd heaps, then the total for the rightmost column will also be odd. By definition, then, the game is unbalanced. Then by the results of section 1.7, player I can always such games.

6 Four-door Monte Hall problem

Assume you are using the switch strategy. Then the winning event is exactly the event the the following three events occur in sequence: you pick a door that does not contain a donkey, Monte picks a door that has a donkey, and you switch to a door not picked by Monte that contains a good prize. The probabilities of these three events, respectively, are 3/4, 1, and 1/2. Therefore, multiplying we get the probability of winning is 3/8.