

COS 341, Oct 13, 1999

Due: October 20, 1999

Homework Set 4

Reading Assignments Start reading Chapter 7.

Written Assignments

Do exercises 18 and 20 in Section 5.7.

Do the following problems:

Special Problem 1 (counted as 1 exercise) Evaluate $\sum_{1 \leq k \leq n} k 5^k \binom{n}{k}$.

Special Problem 2 (counted as 1 exercise) Show that

$$\sum_{0 \leq k \leq n} \frac{(2n)!}{(k!)^2 ((n-k)!)^2} = \binom{2n}{n}^2.$$

Special Problem 3 (counted as 2 exercises) Consider a sequence of $2n$ distinct people in a line at a cashier. Suppose n of the people pay \$1 and n of the people get \$1. Show that the number of arrangements in which the cashier never goes in debt (ie, at every stage at least as many people have paid in \$1 as were paid out \$1) is equal to $\binom{2n}{n} - \binom{2n}{n+1}$. **Hint:** Show a one-to-one correspondence between sequences where at some stage the cashier goes at least \$1 in debt and all sequences of $2n$ people in which $n+1$ of the people pay \$1.