COS 487: Theory of Computation **Fall 1998**

Second Exam

Due: January 11 *Sanjeev Arora*

Instructions: The test contains 4 questions (some with sub-parts) and a fifth optional question. Give yourself at most 24 hours to do the test.

Write answers briefly and legibly in the space provided. If your answer does not seem to fit in that space, check your answer for wordiness. If you need extra space, you may attach extra sheets, **which the instructor will read at his discretion**. You cannot consult another person in any way. You can however consult any notes/handouts from this class as well as the text. Feel free to quote, without proof, any results from class, homework, or the text.

DO NOT READ THE TEST BEFORE YOU ARE READY TO WORK ON IT.

Your Name:

Write and sign the honor code pledge[1] here:

[1]The pledge is "I pledge my honor that I have not violated the honor code during this exam."

Question 1 (48 points) Answer if the following statements are true, false or if their status is open. Give brief reasons for your answer.

(a) Every language in EXPTIME is polynomial-time reducible to TQBF (where EXPTIME $=\cup_{c>0} TIME(2^{O(n^c)})$).

(b) RP $\subseteq$ PSPACE. (The class RP was defined in class.)

(c) COMPOSITES $=\{n : n$ is a composite number$\}$ is NP-complete.

(d) If $A \leq_L B$ and $B \leq_L C$ then $A \leq_L C$.

(e) ISO $=\{< G_1, G_2 >:$ graphs G_1 and G_2 are isomorphic$\}$ is NP-complete.

(f) Let a 1-Graph be a directed graph in which the *out-degree* of each vertex (i.e., the number of edges directed out of the vertex) is at most 1. Then the language

$$\text{1-PATH} = \{< G, s, t >: G \text{ is a 1-graph in which there is a path from } s \text{ to } t\}$$

can be decided deterministically in logspace.

Question 2 (32 points) We say that language A is a *projection* of language B if

$$A = \{z : \exists\ y \text{ such that } z\#y \in B\}\,.$$

Characterize the set of projections of the following classes of languages. Give brief reasons.

(a) All regular languages.

(b) All context-free languages.

(c) All decidable languages.

(d) All languages in P.

Question 3 (30 points) A 2-way finite automaton differs from a DFA in that it is allowed to sweep back and forth on the input tape. It is said to *accept* if it ever goes into accept state after reading the last symbol of the input. Show that the set of languages accepted by 2-way finite automata is exactly the set of regular languages.

Question 4: (30 points) Show the NP-completeness of the DISJOINT PATHS problem, in which we are given an undirected graph G and k pairs of vertices $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$. We have to decide if there exists a set of k vertex-disjoint paths in the graph, where the ith path connects s_i and t_i. (Two paths are said to be *vertex disjoint* if they do not share any vertices.)

THAT'S ALL FOLKS. NEXT QUESTION IS OPTIONAL AND MUST BE COMPLETED WITHIN THE ALLOTED 24 HOURS

(5) (*optional*) (25 points) The complexity class DP contains every language L such that there are languages $L_1, L_2 \in$ NP such that $L = L_1 - L_2$.

(a) (4 points) Argue that CLIQUE and $\overline{\text{CLIQUE}}$ are in DP.

(b) (21 points) Show that the following language is complete for DP under polynomial-time reductions.

$$\text{EXACT-CLIQUE} = \{<G, k>: \text{ the size of the largest clique in } G \text{ is exactly } k\}.$$