Lecture Notes #13 - Global Illumination

Reading: Angel: 16.10; Foley: p.792

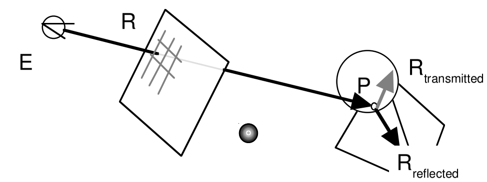
Topics:

- Rendering equation
- <u>Approximations</u>

  Ray tracing

  Radiosity

Ray Tracing

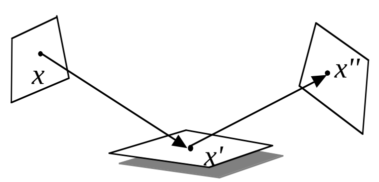


Let I(b,r) = intensity seen along direction r from point b.

$$I(E,R) = I_{direct} + I_{indirect} \approx I_{direct} + I_{reflected} + I_{transmitted}$$

$$I_{direct} = \text{computed from the Phong model}$$
$$I_{reflected} = Reflectance * I(P, R_{reflected})$$
$$I_{transmitted} = Transmittance * I(P, R_{transmitted})$$

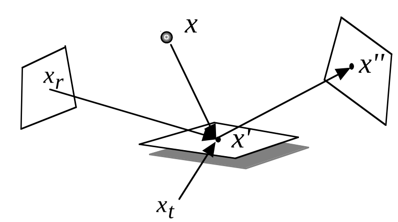
The Rendering Equation (Kajiya '86)



$$\boxed{L(x' \to x'') = E(x' \to x'') + \int_x f_r(x,x',x'')L(x \to x')V(x,x')G(x,x')dx}$$

- L(x'->x'') is the total *radiance* from x' to x''
- E(x'->x'') is the *emitted* radiance from x' to x''
  - non-zero for light sources
- f_r is the BRDF
- V(x,x') is a *visibility* term:
  - 1 if x is visible from x'; 0 otherwise
- G(x,x') is a *geometry* term

Ray Tracing as Approximation to Rendering Equation



$$L(x' \to x'') = E(x' \to x'') + \int_x f_r(x,x',x'')L(x \to x')V(x,x')G(x,x')dx$$

- Sample the integrand where it is likely to be large: at points x such that:
  - x is at a light source
  - x is in the reflected direction
  - x is in the refracted direction

Ray Tracing

Gross approximation to the rendering equation.

<u>accounts for</u>:

shadows

refraction

inter-object reflection

<u>doesn't account for</u>:

caustics (focusing)

finite light sources

diffuse inter-object reflection

Relatively easy to implement.

Produces pretty nice images, although they're a little "too perfect".

Radiosity

- Radiosity = energy/unit area leaving a surface:

$$B(x) = \int_{\omega} L(x \to x')\cos\theta \, d\omega$$

The Radiosity Equation

- *Radiosity assumption*: all surfaces totally diffuse
- Simplifications to rendering equation:
  - BRDF comes outside the integral
  - Radiance leaving each surface is equal in all directions
  - => Radiosity describes everything
- Rendering equation becomes the radiosity equation:

$$\boxed{B(x) = E(x) + \rho(x)\int_{x'} B(x')\frac{V(x,x')G(x,x')}{\pi}dx'}$$

$$\rho(x) = \text{"}\mathit{hemispherical\ \ reflectance}\text{"} = k_d\,\pi \qquad 0 \le \rho(x) \le 1$$

Solving the Radiosity Equation

$$B(x) = E(x) + \rho(x)\int_{x'} B(x')\frac{V(x,x')G(x,x')}{\pi}dx'$$

- Relatively hard:
  - Unknown B(x) appears on both sides, once inside the integral, once outside.
  - Analytic solutions generally don't exist.
  - Must approximate using numerical methods.
- The "finite element" approach
  - Divide scene up into small surface patches (discretize).
  - Assume on each patch i=1,...,n:
    - Radiosity B_i is constant.
    - Emission E_i is constant.
    - Reflectance ρ_i is constant.

Finite Element Approach

$$\frac{1}{A_i}\int_{x\in i} B(x)\,dx = \frac{1}{A_i}\int E(x)\,dx + \frac{1}{A_i}\int_{x\in i} \rho \int_{x'\notin i} B(x)\frac{V(x,x')G(x,x')}{\pi}\,dx'\,dx$$

$$B_i = E_i + \rho_i \frac{1}{A_i}\sum_{j=1}^{n}\int_{x\in i}\int_{x'\in j} B_j \frac{V(x,x')G(x,x')}{\pi}\,dx'\,dx$$

$$= E_i + \rho_i \sum_{j=1}^{n} B_j \left[\int_{x\in i}\int_{x'\in j}\frac{V(x,x')G(x,x')}{\pi}\,dx'\,dx\right]$$

$$= E_i + \rho_i \sum_{j=1}^{n} B_j \qquad F_{ij}$$

"form factor"

Finite Element Approach

$$B_i = E_i + \rho_i \sum_j B_j F_{ij} \qquad F_{ij} = \frac{1}{A_i}\iint_{x\in i, x'\in j}\frac{V(x,x')G(x,x')}{\pi}\,dx\,dx'$$

- In matrix form:

$$\begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix} + \begin{bmatrix} \rho_1 F_{11} & \rho_1 F_{12} & \cdots & \rho_1 F_{1n} \\ \rho_2 F_{21} & \rho_2 F_{22} & \cdots & \vdots \\ \vdots & & \ddots & \\ \rho_n F_{n1} & & \cdots & \rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$$

$$\begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & \vdots \\ \vdots & & \ddots & \\ -\rho_n F_{n1} & & \cdots & 1-\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_n \end{bmatrix}$$

Issues

- How to compute form factors?
- How many patches to use?
- How best to solve the linear system?
    - It is large and dense

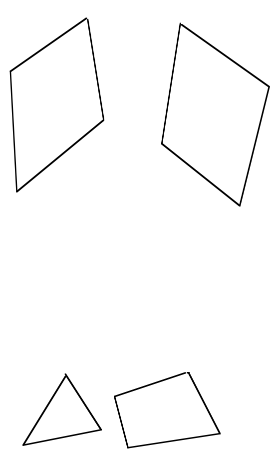
Computing Form Factors

$$F_{ij} = \frac{1}{A_i}\iint_{x\in i, x'\in j}\frac{V(x,x')G(x,x')}{\pi}\,dx\,dx'$$

- Quadrature using ray casting for visibility
- Use z-buffer hardware (hemi-cube [Cohen&Greenberg '85])

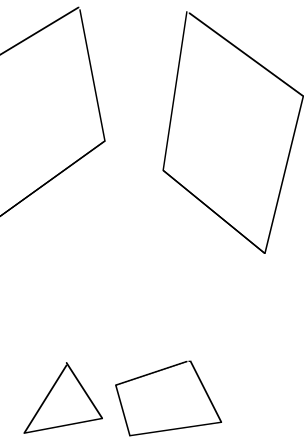
Standard Radiosity

- Each entry of form factor matrix represents an "interaction"
- n^2 interactions
- Small patches needed for accurate close interaction
- Small patches are overkill for distant interactions



Hierarchical Radiosity

- Hanrahan, Salzman, Aupperle, Siggraph '91
- Hierarchically decompose
  - patches (into a quadtree)
  - FF matrix (matrix never explicitly formed)
- Claim: O(f(e) n) interactions for accuracy tolerance e



Basic Algorithm

- Compute interactions:

```
Refine(Patch *r, *s, eps) {
  Frs = FormFactor( r, s)
  Fsr = FormFactor( s, r)
  if (accurate to within eps) {
    Link(r,s);
  } else {
    if (SplitAndRefine(s)) {
      Subdiv(s)
      Refine(r, s->ne, eps)
      Refine(r, s->nw, eps)
      Refine(r, s->se, eps)
      Refine(r, s->sw, eps)
    } else {...}
}
```

- Solve:

```
Solve() {
  repeat {
    foreach top level patch r {
      Gather(r)
    }
    foreach top level patch r {
      Push(r)
      Pull(r)
    }
  until converged;
}
```


Basic Algorithm

```
Gather(Patch *r) {
 /* Gather radiosity to r */
 if (r) {
  r->B = r->emission
  for each link L btw r & s {
   r->B += L->FF * r->rho * s->B
  }
  Gather(r->nw)
  Gather(r->ne);
  Gather(r->sw)
  Gather(r->se);
 }
}
```

```
Push(Patch *r) {
 /*
  Add radiosity at r to radiosities of
 children
 */
}

Pull(Patch *r) {
 /*
  Set radiosity of each node to
  area weighted average of
  children
 */
}
```