Solutions to Homework Set 9

Section 9.5, Problem 7

The total number of edges in a bipartite graph G equals the number of edges which meet vertices in X. So

$$|E| = \sum_{i \in X} deg(i) \geq |X| \cdot p$$

$|G|$ also equals the number of edges which meet vertices in Y. So

$$|E| = \sum_{i \in Y} deg(i) \leq |Y| \cdot p$$

Then $|X| \cdot p \leq |E| \leq |Y| \cdot p \Rightarrow |X| \leq |Y|$. It means Y has at least as many vertices as X. □

Section 9.5, Problem 10

(a) Left graph, we name all the vertices in left as $x_1, x_2, ..., x_7$, all the vertices in right as $y_1, y_2, ..., y_8$ from up to bottom.

$\rho(G) = 7$, S can be $x_1, x_2, x_3, x_4, x_5, x_6, x_7$.

(b) Right graph, we name all the vertices in left as $x_1, x_2, ..., x_7$, all the vertices in right as $y_1, y_2, ..., y_8$ from up to bottom.

$\rho(G) = 6$, S can be $x_1, x_2, x_5, x_6, x_7, y_3$.

Section 9.5, Problem 11

A bipartite graph G can be constructed by the given relations, and $\rho(G) = 7$ can be proven, so all 7 jobs can be filled.

Section 9.5, Problem 18

$$\begin{bmatrix} (1,2) & (2,1) \\ (2,1) & (1,2) \end{bmatrix}$$

For this preferential ranking matrix, both complete marriages are stable. Because either every man or every woman gets his/her first choice.

Section 9.5, Problem 20

Notice this matrix has the property that each entry (x, y) satisfies $x + y = n + 1$. If each woman gets her k_{th} choice, then every man is married to his $(n + 1 - k)_{th}$ choice. Suppose a woman A prefers another man b to her current husband, then b ranks higher on A's preference list, say $k - c$ where c is a positive integer. From the property of the matrix we'll know b ranks A $n + 1 - k + c$ on his list, that's lower than his current wife's rank $n + 1 - k$. Thus no "unstable" couple pairs exist, the marriage is stable.

Section 9.5, Problem 25

This is a completely mechanical process. We experience ten round when using the women-optimal algorithm and eventually reach the stable marriage combination: $A \to c, B \to d, C \to a, D \to b$. If you like to see men making proposal, you get the same result but it only takes four round.

Comments? Corrections? Suggestions? Mail to `qiexh@cs.princeton.edu`