COS 341, Fall 1998
October 26, 1998
Handout No. 4

Last Year's Midterm Exam

Problem 1[30 points] In the very first class of the course, we discussed the *covering* problem. We showed that an 8 x 8 chessboard (consisting of 64 squares with unit-length sides) can be covered perfectly with dominos; but if we cut off two opposite corners from the chessboard, then the resulting chessboard can no longer be covered perfectly with dominos.

For this exam, consider a 9 x 9 board (consisting of 81 squares with unit-length sides). Clearly, it can be covered perfectly with *triminos*, where a trimino is a piece of dimension 1 x 3 or 3 x 1.
Question: Prove that if we cut off any three (out of four) corners of the 9 x 9 board, the resulting board cannot be covered perfectly with triminos. Give a rigorous argument.

Problem 2 [30 points] Let n be any positive integer.
(a) Evaluate

$$\sum_{1\le k\le n} \binom{2n}{2k-1} 2^{2k-1}.$$

(b) Evaluate

$$\sum_{1\le k\le n} \binom{2n}{2k-1} k2^{k}.$$

Give your answers in closed form.

Problem 3 [30 points] Solve the following recurrence relation for a_n:
(a)

$$\begin{aligned} a_0 &= 3 \\ na_n &= (n-1)a_{n-1}+1 \end{aligned}$$

for $n \ge 1$.

(b)

$$\begin{aligned} a_0 &= 3 \\ na_n &= (n-2)a_{n-1}+1 \end{aligned}$$

for $n \ge 1$.
Your answers must be in closed form.